Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

CIRCUITS ET SIGNAUX QUANTIQUES

QUANTUM SIGNALS AND CIRCUITS

Sixième Leçon / *Sixth Lecture*

08-VI-1

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PDF FILES OF LECTURES AND SEMINARS ARE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

08-VI-2

CALENDAR OF SEMINARS

May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)
Continuous dispersive quantum measurement of an electrical circuit

May 20: Bertrand Reulet (LPS Orsay)
Current fluctuations : beyond noise

June 3: Gilles Montambaux (LPS Orsay)
Quantum interferences in disordered systems

June 10: Patrice Roche (SPEC-CEA Saclay)
Determination of the coherence length in the Integer Quantum Hall regime

June 17: Olivier Buisson, (CRTBT-Grenoble)
A quantum circuit with several energy levels

June 24: Jérôme Lesueur (ESPCI)
High Tc Josephson Nanojunctions: Physics and Applications

08-VI-3

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signal

Lecture IV: Quantum fluctuations in transmission lines

Lecture V: Introduction to non-linear active circuits

Lecture VI: Amplifying quantum signals with dispersive circuits

08-VI-4

LECTURE VI : AMPLIFYING QUANTUM SIGNALS WITH DISPERSIVE CIRCUITS

OUTLINE

1. Main ideas introduced in last lecture, purpose of this lecture
2. Minimal symplectic circuits are maximally efficient
3. Characteristics of amplifiers
4. Scattering matrix of minimal 1-port and 2-port active circuits
5. Ring modulator implementation of 2-port circuit

08-VI-5

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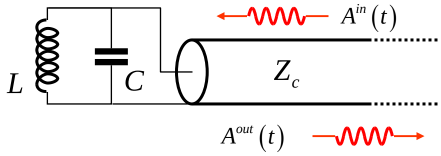
08-VI-5a

PASSIVE vs ACTIVE LINEAR, DISPERSIVE 1-PORT

Simplest example: LC. Both L and C are dispersive elements (no internal dissipation)

PASSIVE



$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \Gamma = \frac{1}{2Z_c C}$$

generic frequency

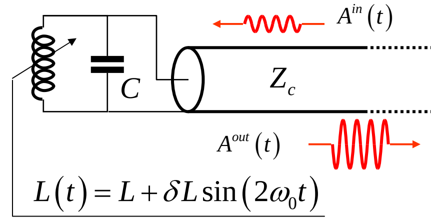
$$\begin{bmatrix} a^{out}[\omega] \\ a^{out}[-\omega] \end{bmatrix} = \begin{bmatrix} r[\omega] & 0 \\ 0 & r[-\omega] \end{bmatrix} \begin{bmatrix} a^{in}[\omega] \\ a^{in}[-\omega] \end{bmatrix}$$

$$S^{\dagger} I S = I$$

UNITARY=
ORTHOGONAL
+ SYMPLECTIC

$|r| = 1$

ACTIVE



$$L(t) = L + \delta L \sin(2\omega_0 t)$$

have treated only res. frequency case

$$\begin{bmatrix} a^{out}[\omega_0] \\ a^{out}[-\omega_0] \end{bmatrix} = \begin{bmatrix} r_0 & s_0 \\ s_0^* & r_0^* \end{bmatrix} \begin{bmatrix} a^{in}[\omega_0] \\ a^{in}[-\omega_0] \end{bmatrix}$$

$|r| > 1$

$${}^{t}SJS = J$$

GENERALIZED S MATRIX

ONLY SYMPLECTIC

ENERGY NOT CONSERVED: POWER GAIN

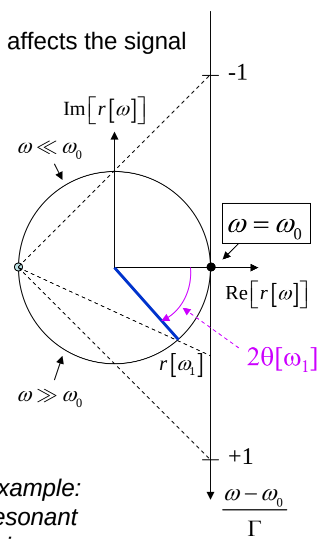
08-VI-6c

PASSIVE S IN ROTATING WAVE APPROXIMATION

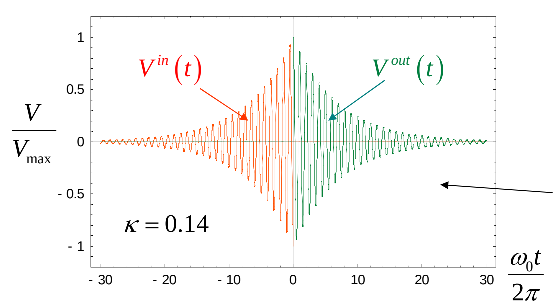
When $\frac{\Gamma}{\omega_0} = \kappa = \frac{1}{2Q} \ll 1$ we can consider only 1 pole affects the signal

$$r[\omega] = e^{i2\theta[\omega]} = -\frac{\omega_0^2 - \omega^2 - 2i\omega\Gamma}{\omega_0^2 - \omega^2 + 2i\omega\Gamma} \simeq \frac{1 - i\dfrac{\omega - sg(\omega)\omega_0}{\Gamma}}{1 + i\dfrac{\omega - sg(\omega)\omega_0}{\Gamma}}$$

$$V[\omega] = (1 + r[\omega])V^{in}[\omega] \simeq \frac{2}{1 + i\dfrac{\omega - sgn(\omega)\omega_0}{\Gamma}} V^{in}[\omega]$$



$$\tan(\theta[\omega]) = \frac{\omega - \omega_0}{\Gamma}$$

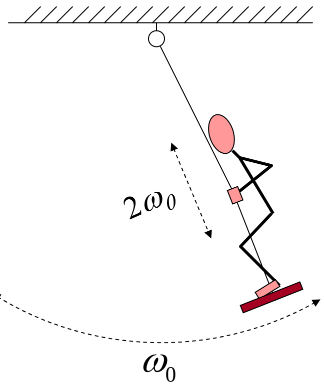


example: resonant drive

08-VI-7b

EXAMPLES OF PARAMETRICALLY PUMPED OSCILLATORS

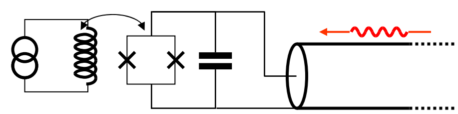
MECHANICAL SYSTEM



PUMP: WORK AGAINST CENTRIFUGAL FORCE

ELECTRICAL SYSTEM

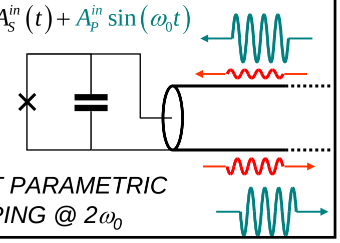
RF FLUX BIASED DC SQUID



$$I_B(t) = I_B + I_P \sin(2\omega_0 t)$$

EFFECTIVE PARAMETRIC OSCILLATOR

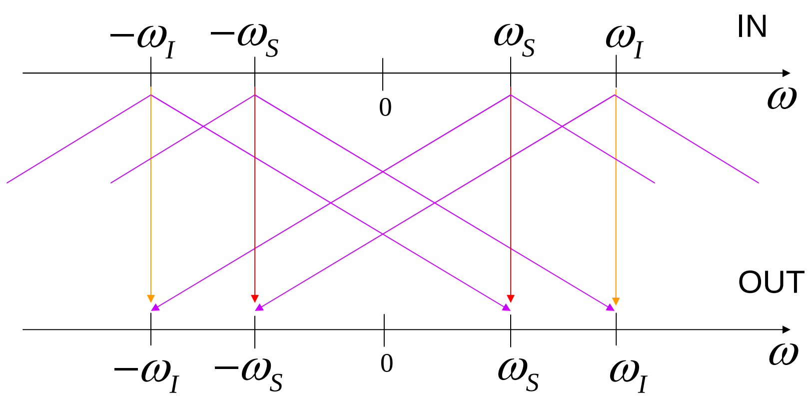
$$A^{in}(t) = A_S^{in}(t) + A_P^{in} \sin(\omega_0 t)$$



INDIRECT PARAMETRIC PUMPING @ $2\omega_0$

08-VI-8b

IN NON-LINEAR ACTIVE DEVICE, SIGNALS SCATTER BETWEEN DIFFERENT FREQUENCIES



Pump frequency Ω_p

$$\omega_S + \omega_I = \Omega_p$$

Signal Idler (Image)

IF $|\omega_S - \omega_I| = \Omega_p$ CONVERSION W/O AMPLIFICATION

08-VI-9a

PURPOSE OF THIS LECTURE: EXPLAIN HOW MINIMAL, ACTIVE DISPERSIVE CIRCUITS PERFORM AMPLIFICATION AT QUANTUM LIMIT

TWO MAIN ISSUES:

GAIN-BANDWIDTH TRADE-OFF

WHAT IS THE GENERALIZED SCATTERING MATRIX OF AN ACTIVE 1-PORT AND 2-PORT, BOTH AS A FUNCTION OF FREQUENCY OF SIGNALS AND AS A FUNCTION OF PUMP STRENGTH?

NOISE

HOW DO QUANTUM FLUCTUATIONS AFFECT THE NOISE OF THE AMPLIFIER?
WHAT IS THE MINIMAL AMOUNT OF NOISE ADDED BY THE AMPLIFIER?

IMPLEMENTATION ?

08-VI-10

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08-VI-5b

GENERAL SCATTERING MATRIX

$$\vec{a}^{in} = \begin{bmatrix} a_1^{in}[+\omega_1] \\ a_1^{in}[-\omega_1] \\ a_2^{in}[+\omega_2] \\ a_2^{in}[-\omega_2] \\ \cdots \\ \cdots \\ a_l^{in}[+\omega_l] \\ a_l^{in}[-\omega_l] \end{bmatrix} \qquad \vec{a}^{out} = \begin{bmatrix} a_1^{out}[+\omega_1] \\ a_1^{out}[-\omega_1] \\ a_2^{out}[+\omega_2] \\ a_2^{out}[-\omega_2] \\ \cdots \\ \cdots \\ a_l^{out}[+\omega_l] \\ a_l^{out}[-\omega_l] \end{bmatrix}$$

$$\vec{a}^{out} = S\,\vec{a}^{in}$$

$$\sqrt{\frac{\hbar|\omega_l|}{2}}\, a_l[\omega_l] = A_l[\omega_l]$$

HERE,
WAVE AMPLITUDES
HAVE (PHOTON NB.)$^{1/2}$
DIMENSION
BUT ARE TREATED
CLASSICALLY

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & .. & .. & 0 & 0 \\ -1 & 0 & 0 & 0 & .. & .. & 0 & 0 \\ 0 & 0 & 0 & 1 & .. & .. & 0 & 0 \\ 0 & 0 & -1 & 0 & .. & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & .. & .. & 0 & 1 \\ 0 & 0 & 0 & 0 & .. & .. & -1 & 0 \end{bmatrix}$$

POISSON BRACKET:

$$\{u,v\}_{P.B.} = {}^{t}\left[\partial_{\vec{a}} u\right] J \left[\partial_{\vec{a}} v\right]$$

see Goldstein, "Classical Mechanics" (Addison-Wesley1980)

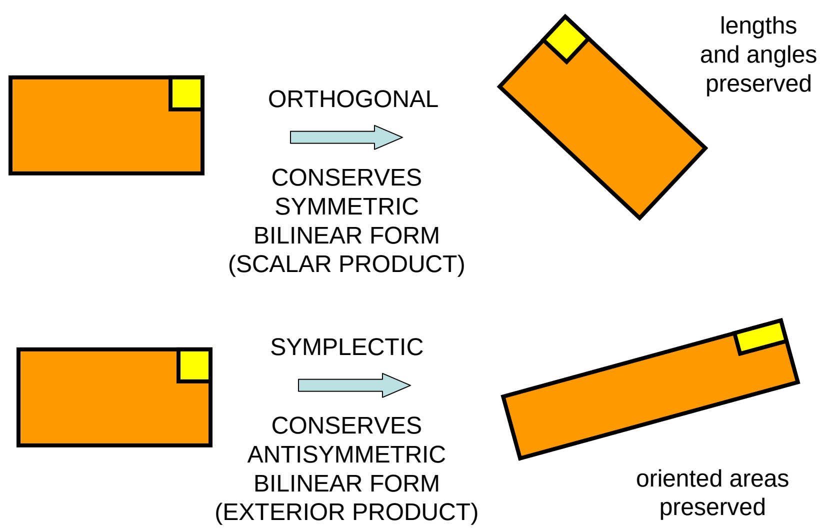
S MATRIX MUST CONSERVE ALL
POISSON BRACKETS: **S** IS SYMPLECTIC

$${}^{t}SJS = J$$

SYMPLECTICITY CAN BE UNDERSTOOD AS INFORMATION CONSERVATION

08-VI-11b

TWO TYPES OF MAPS



08-VI-12

SYMPLECTICITY IS A 1ST QUANTIZATION PROPERTY

HOWEVER

IT IS OFTEN EXPRESSED AS THE PROPERTY OF
CONSERVATION OF COMMUTATORS ...

$$\hat{X} = \overleftarrow{x} \cdot \vec{\hat{a}} \qquad \hat{Y} = \overleftarrow{y} \cdot \vec{\hat{a}} \quad \Longrightarrow \quad \left[\hat{X}, \hat{Y} \right] = \overleftarrow{x} \cdot \mathrm{J} \cdot \vec{y} \left[\vec{\hat{a}}, \overleftarrow{\hat{a}} \right]$$

$$\left[\hat{X}^{out}, \hat{Y}^{out} \right] = \left[\hat{X}^{in}, \hat{Y}^{in} \right] \quad \Longrightarrow \quad {}^{t}\mathrm{SJS} = \mathrm{J}$$

... WHICH CAN BE MISLEADING!

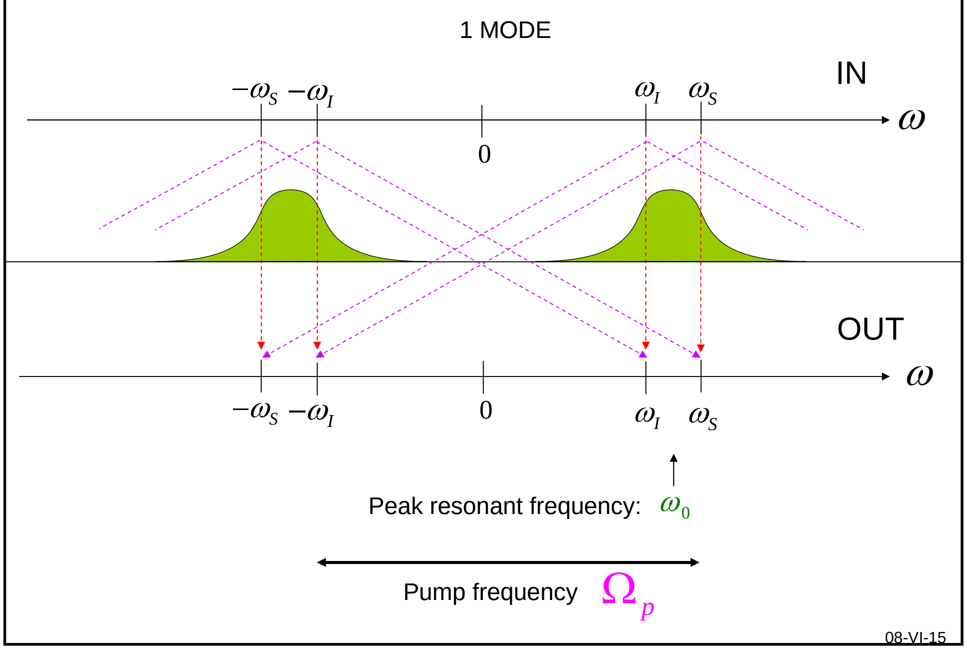
08-VI-13

A MAXIMALLY EFFICIENT
SIGNAL PROCESSING FUNCTION
MIX THE MINIMAL NUMBER OF MODES
COMPATIBLE WITH SYMPLECTICITY

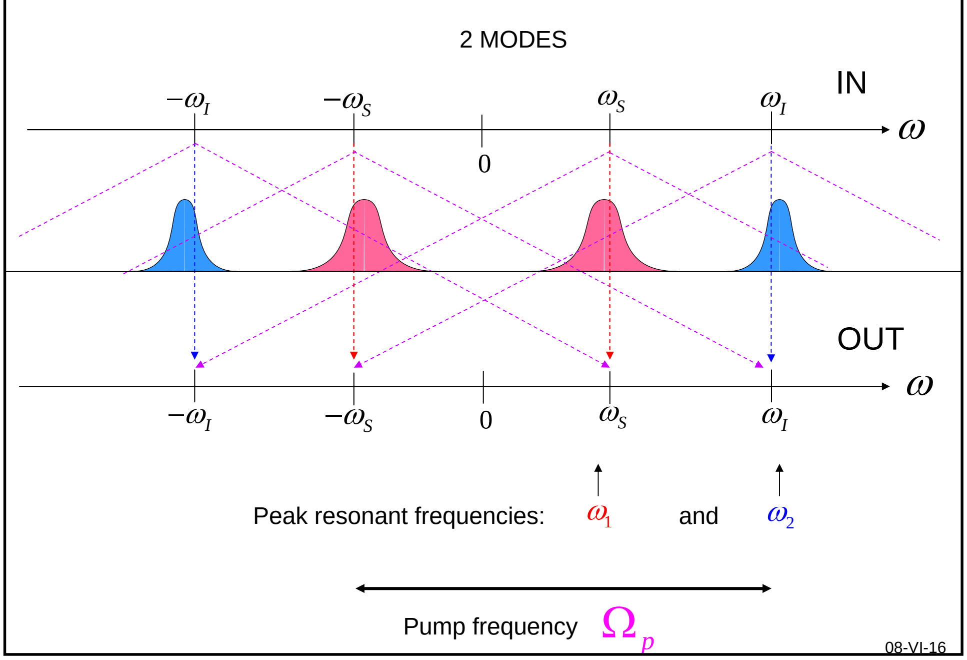
AMPLIFICATION: 2 CASES

08-VI-14a

DEGENERATE PARAMETRIC PUMPING



NON-DEGENERATE PARAMETRIC PUMPING



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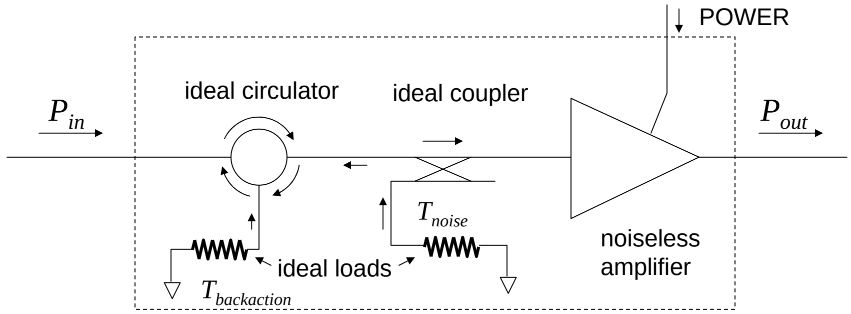
08-VI-5c

LINEAR AMPLIFIER CHARACTERISTICS

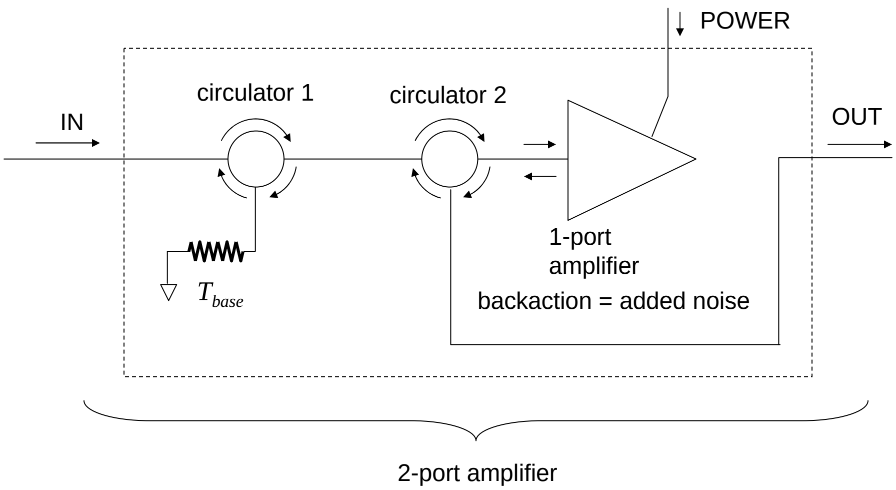


REPRESENTATION OF MATCHED 2-PORT

- Power gain (P_{out}/P_{in})
- Signal bandwidth
- Noise temperature T_{noise}
- Backaction $T_{backaction}$
- Dynamic range, intermodulation
- Tuning bandwidth

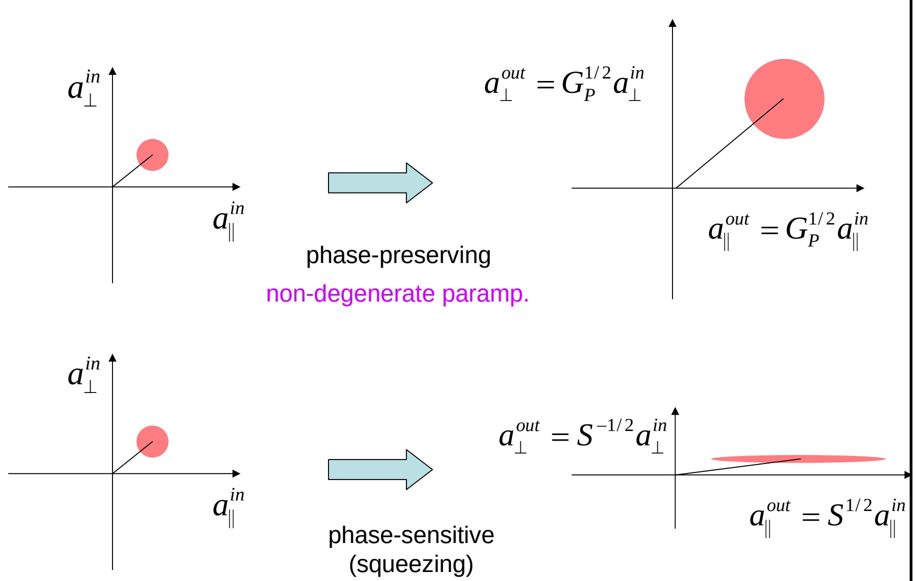
08-VI-17

2-PORT vs 1-PORT AMPLIFIER



08-VI-18

TWO TYPES OF LINEAR AMPLIFIERS



08-VI-19a

CRYOELECTRONIC AMPLIFIERS APPROACHING THE QUANTUM LIMIT

type	$kT_N/(\hbar\omega/2)$	power gain	out-of-band back-action noise	ease of use
HEMT	40-80	25-35dB	small	easy
SQUID	1-2	20-30dB	concern	OK
RF-SET	1-2	15-20 dB	concern	OK
QPC	1	~0dB	very small	difficult

HEMT: High Electron Mobility Transistor, SET: Single Electron Transistor, QPC: Quantum Point Contact

08-VI-20

LARGE GAIN LIMITS

PHASE-PRESERVING: $$k_B T_N \geq \frac{\hbar\omega_s}{2}$$ ← STANDARD QUANTUM LIMIT

(Caves, 1982)

PHASE-SENSITIVE: $$k_B T_N \geq 0$$ NOISELESS

08-VI-21

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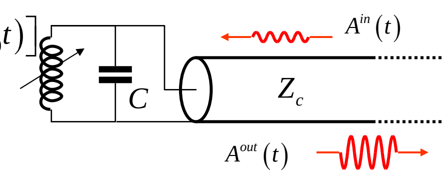
08-VI-5d

SCATTERING MATRIX OF DPA FOR ARBITRARY ω

$L^{-1}(t) = L_0^{-1}\left[1 + 2\varepsilon \sin\left(2\omega_0 t\right)\right]$



$\omega_0 = \frac{1}{\sqrt{LC}}$

$\Gamma = \frac{1}{2Z_c C}$

Stiff pump condition

$$\ddot{\Phi} + 2\Gamma\dot{\Phi} + \omega_0^2\Phi - i\omega_0^2\varepsilon\Phi\left(e^{2i\omega_0 t} - e^{-2i\omega_0 t}\right) = 4\Gamma V^{in}(t) \qquad V^{out}(t) = \dot{\Phi} - V^{in}(t)$$

Harmonic balance + RWA ⇨ $i\omega_0\left[\frac{(\omega_0 - \omega)}{i\Gamma} - 1\right]\Phi[\omega] - i\frac{\omega_0^2\varepsilon}{2\Gamma}\Phi[\omega - 2\omega_0] = 2V^{in}[\omega]$

After a few steps:

$$\begin{bmatrix} A^{out}[+\omega_S] \\ A^{out}[-\omega_S] \\ A^{out}[+\omega_I] \\ A^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & s \\ 0 & r^* & s^* & 0 \\ 0 & s^* & r^* & 0 \\ s & 0 & 0 & r \end{bmatrix} \begin{bmatrix} A^{in}[+\omega_S] \\ A^{in}[-\omega_S] \\ A^{in}[+\omega_I] \\ A^{in}[-\omega_I] \end{bmatrix}$$

$$r = \frac{1 + \vartheta^2 + \zeta^2}{(1 - i\vartheta)^2 - \zeta^2}$$

$$s = \frac{2\zeta}{(1 - i\vartheta)^2 - \zeta^2}$$

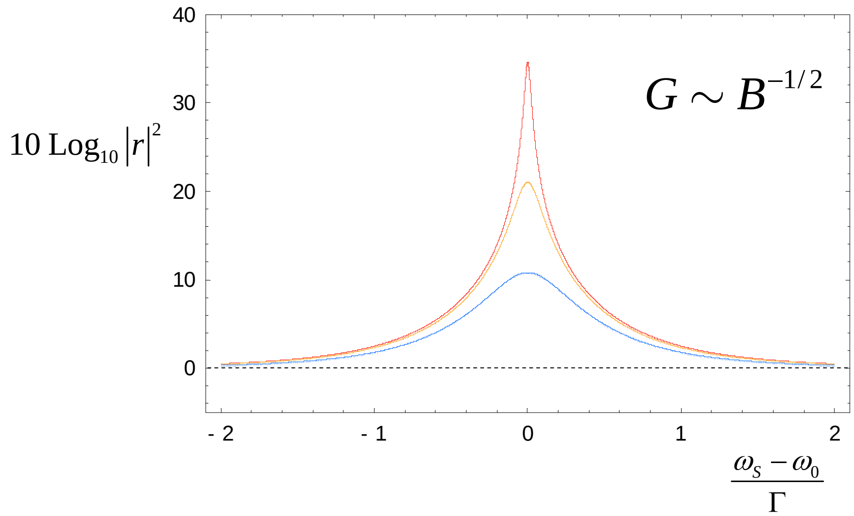
$$\zeta = \varepsilon\frac{\omega_0}{2\Gamma} < 1$$

SYMPLECTICITY: $\begin{cases} |r|^2 - |s|^2 = 1 \\ rs^* - r^*s = 0 \end{cases}$

$$\vartheta = \tan\theta = \frac{\omega_S - \omega_0}{\Gamma}$$

08-VI-21c

GAIN-BANDWIDTH COMPROMISE



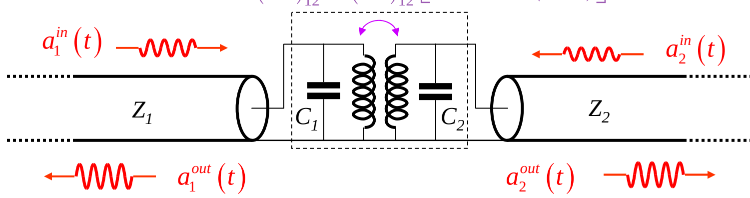
See experiment by MA Castellanos-Beltran & K. Lehnert, arXiv:0706.2373v

08-VI-22

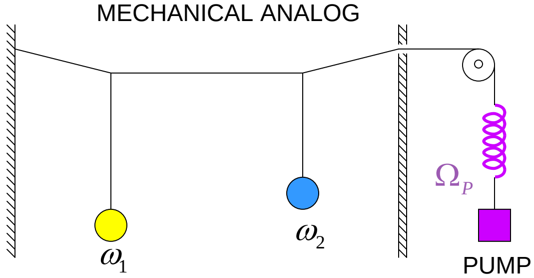
MINIMAL IMPLEMENTATION OF NON-DEGENERATE PARAMETRIC AMPLIFIER

TIME-DEPENDENT
COUPLING BETWEEN
2 OSCILLATORS

$$\left(\mathrm{L}^{-1}\right)_{12} = \left(\mathrm{L}_0^{-1}\right)_{12}\left[1+2\varepsilon\sin\left(\Omega_P t\right)\right]$$



MECHANICAL ANALOG



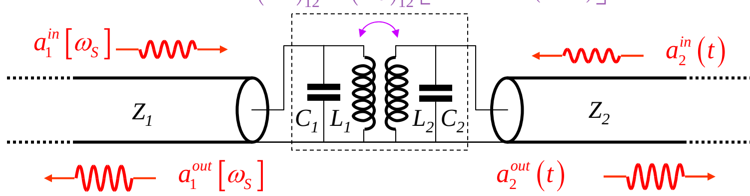
HERE,
WAVE AMPLITUDES
HAVE (PHOTON NB.)$^{1/2}$
DIMENSION
BUT ARE TREATED
CLASSICALLY

$$\sqrt{\frac{\hbar|\omega_l|}{2}}\, a_l[\omega_l] = A_l[\omega_l]$$

08-VI-23

SCATTERING MATRIX OF NDPA FOR ARBITRARY ω

$$\left(\mathrm{L}^{-1}\right)_{12} = \left(\mathrm{L}_0^{-1}\right)_{12}\left[1 + 2\varepsilon \sin\left(\Omega_p t\right)\right]$$



$$\omega_{1,2} = \frac{1}{\sqrt{L_{1,2}C_{1,2}}}$$

$$\Gamma_{1,2} = \frac{1}{2Z_{1,2}C_{1,2}}$$

phase-preservation

$$\begin{bmatrix} a^{out}[+\omega_S] \\ a^{out}[-\omega_S] \\ a^{out}[+\omega_I] \\ a^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} r_S & 0 & 0 & s \\ 0 & r_S^* & s^* & 0 \\ 0 & s^* & r_I^* & 0 \\ s & 0 & 0 & r_I \end{bmatrix} \begin{bmatrix} a^{in}[+\omega_S] \\ a^{in}[-\omega_S] \\ a^{in}[+\omega_I] \\ a^{in}[-\omega_I] \end{bmatrix}$$

$$r_{S,I} = \frac{(1 + i\vartheta_{S,I})(1 - i\vartheta_{I,S}) + \zeta^2}{(1 - i\vartheta_S)(1 - i\vartheta_I) - \zeta^2}$$

$$s = \frac{2\zeta}{(1 - i\vartheta_S)(1 - i\vartheta_I) - \zeta^2}$$

$$\zeta = \frac{\varepsilon}{2}\left(\frac{\omega_1\omega_2}{\Gamma_1\Gamma_2}\right)^{1/2} < 1$$

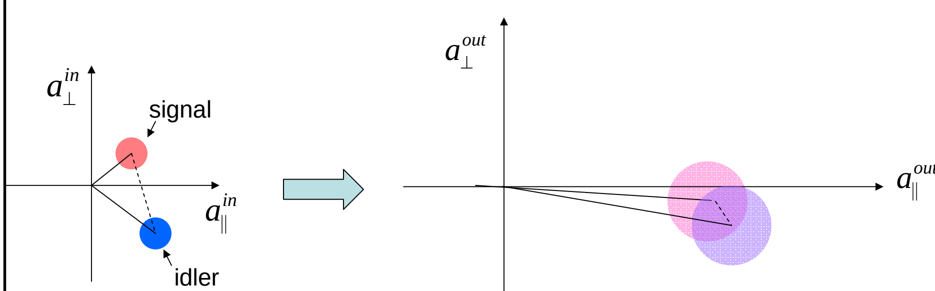
$$\vartheta_{S,I} = \tan\theta_{S,I} = \frac{\omega_{S,I} - \omega_{1,2}}{\Gamma_{1,2}}$$

AMPLIFICATION: $|r_S|^2 = |r_I|^2 \geq 1$

SYMPLECTICITY: $|r_S|^2 - |s|^2 = |r_I|^2 - |s|^2 = 1$

08-VI-24c

SIGNAL AND IDLER CO-AMPLIFICATION FOR NON-DEGENERATE PARAMETRIC AMPLIFIER

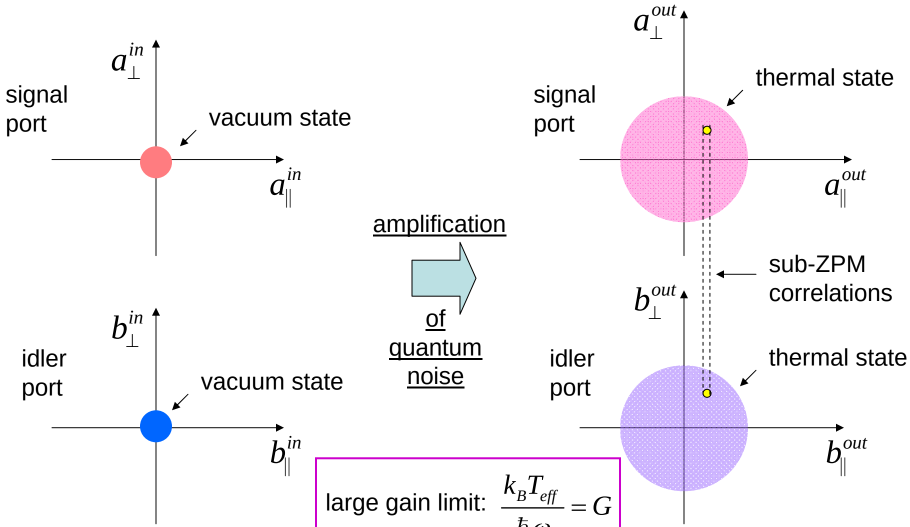


sum of signal and idler is amplified
difference of signal and idler is de-amplified

total volume in phase space is conserved

08-VI-25

SQUEEZING OF QUANTUM FLUCTUATIONS FOR FOR NON-DEGENERATE PARAMETRIC AMPLIFIER



large gain limit: $\frac{k_B T_{eff}}{\hbar\omega} = G$

see "Quantum Squeezing", Drummond and Ficek eds (Springer 2004)

08-VI-26d

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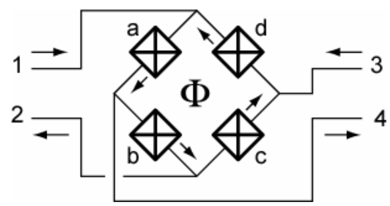
08-VI-5e

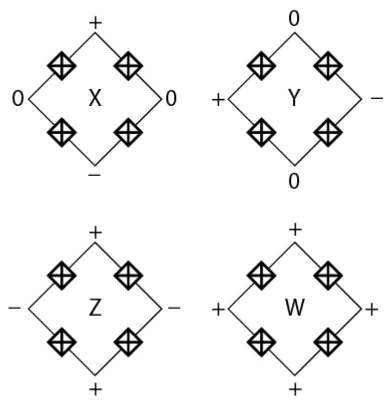
TOWARDS THE PUREST NON-LINEARITY: THE JOSEPHSON RING MODULATOR

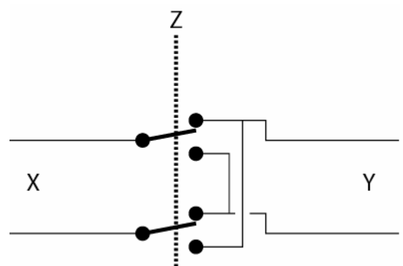
(Bergeal *et al.*, 2008, arXiv:0805.3452)



4 junctions in a ring threaded by flux

4 modes:





The Z mode can be understood as providing an invertible coupling between the X and Y mode

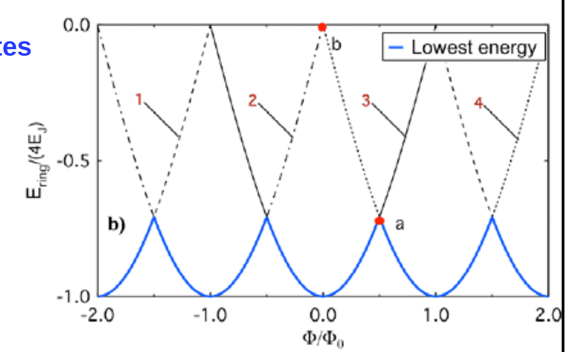
08-VI-27

CONSEQUENCES OF RING SYMMETRY

■ **Coexistence of 2 stable states with $4\Phi_0$-periodicity**

junction phases satisfy

$$\begin{cases} \phi_a + \phi_b + \phi_c + \phi_d = \dfrac{2\pi\Phi}{\Phi_0} \\ \sin\phi_a = \sin\phi_b = \sin\phi_c = \sin\phi_d \end{cases}$$



■ **Useful non-linearity with minimal number of spurious terms**

At $\Phi = \Phi_0/2$ and for small X, Y, Z:

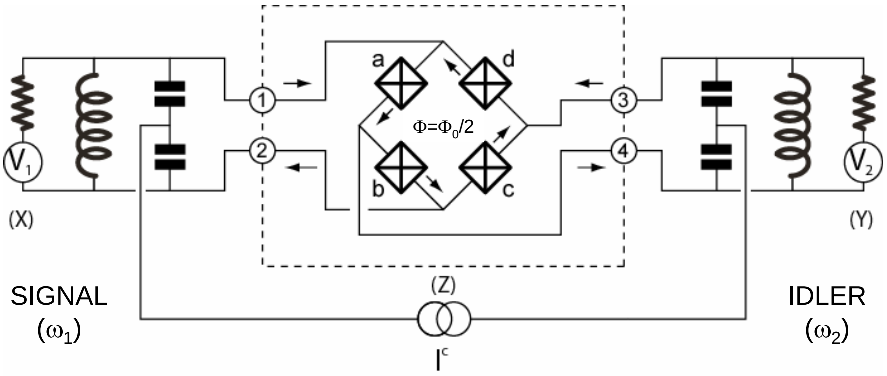
$$E_{ring} = \Xi XYZ + B\left(\frac{X^2}{4} + \frac{Y^2}{4} + \frac{Z^2}{2}\right) + \text{higher order terms}$$

Mix 3 orthogonal modes X, Y, Z (S. Girvin)

Spurious terms only renormalize mode frequencies

08-VI-28

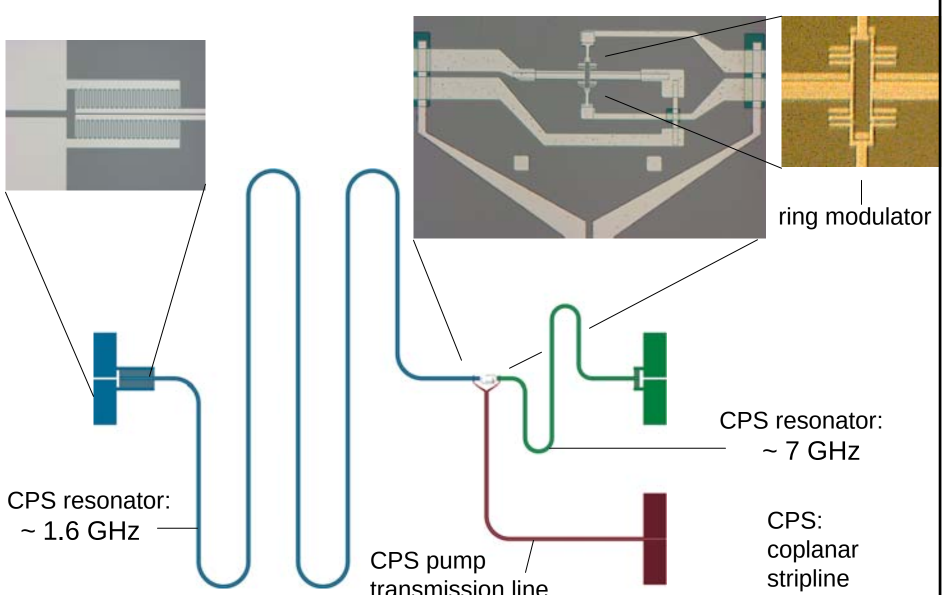
SCHEMATICS OF JOSEPHSON AMPLIFIER BASED ON RING MODULATOR



PUMP
(Ω_p)

08-VI-29

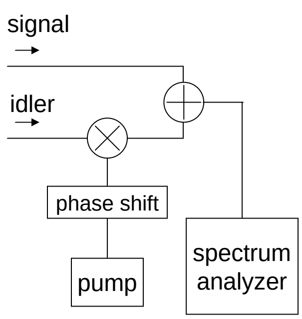
JOSEPHSON PARAMETRIC AMPLIFIER CHIP

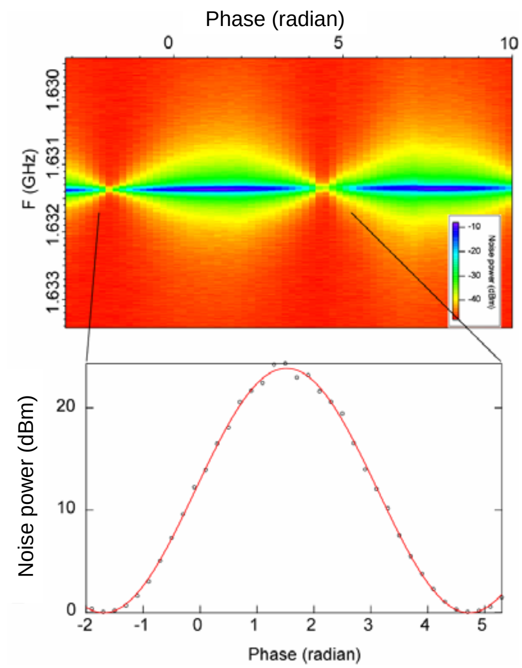


08-VI-30

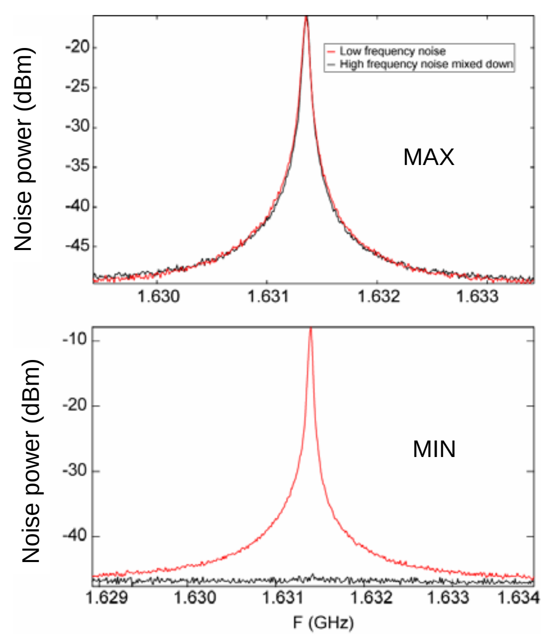
2-MODE NOISE SQUEEZING DATA

(Bergeal *et al.*, 2008)





2-MODE NOISE SQUEEZING DATA (CTND)



CONCLUSIONS

A PHASE PRESERVING AMPLIFIER ADDS AT LEAST HALF A PHOTON OF NOISE AT THE SIGNAL FREQUENCY

A PHASE SENSITIVE AMPLIFIER CAN BE NOISELESS

MINIMAL NUMBER OF MODES IN A DISPERSIVE ACTIVE CIRCUIT IS A NECESSARY CONDITION FOR AMPLIFICATION AT THE QUANTUM LIMIT

PARAMETRIC AMPLIFICATION USING 3-WAVE OR 4-WAVE MIXING OF JOSEPHSON JUNCTION IS MINIMAL

JOSEPHSON RING MODULATOR IS A CONVENIENT PURE 3-WAVE MIXING DEVICE FOR NON-DEGENERATE PARAMETRIC AMPLIFICATION

08-VI-30

NEXT YEAR :

1) NON-PERTURBATIVE ASPECTS OF QUANTUM CIRCUITS

2) PERIODICITY OF JOSEPHSON COSINE POTENTIAL AND ITS CONSEQUENCES

3) STRONG COUPLING BETWEEN JUNCTION AND REST OF CIRCUIT

4) STRONG DAMPING WITH NORMAL TUNNEL JUNCTIONS

08-VI-31

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END OF 2008 LECTURES