Chaire de Physique Mésoscopique
Michel Devoret
Année 2011, 10 mai - 21 juin

AMPLIFICATION ET RETROACTION QUANTIQUES

QUANTUM AMPLIFICATION AND FEEDBACK

Sixième Leçon / *Sixth Lecture*

LU:110624

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model open, out-of-equilibrium, non-linear quantum systems?

Lecture III: Can we maintain the noise at the quantum limit while increasing gain, bandwidth and dynamic range?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can a continuous quantum measurement be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?

CALENDAR OF SEMINARS

May 10: Fabien Portier, SPEC-CEA Saclay
The Bright Side of Coulomb Blockade

May 17, 2011: Jan van Ruitenbeek (Leiden University, The Netherlands)
Quantum Transport in Single-molecule Systems

May 31, 2011: Irfan Siddiqi (UC Berkeley, USA)
Quantum Jumps of a Superconducting Artificial Atom

June 7, 2011: David DiVicenzo (IQI Aachen, Germany)
Quantum Error Correction and the Future of Solid State Qubits

June 14, 2011: Andrew Cleland (UC Santa Barbara, USA)
Images of Quantum Light

June 21, 2011: Benjamin Huard (LPA - ENS Paris)
Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

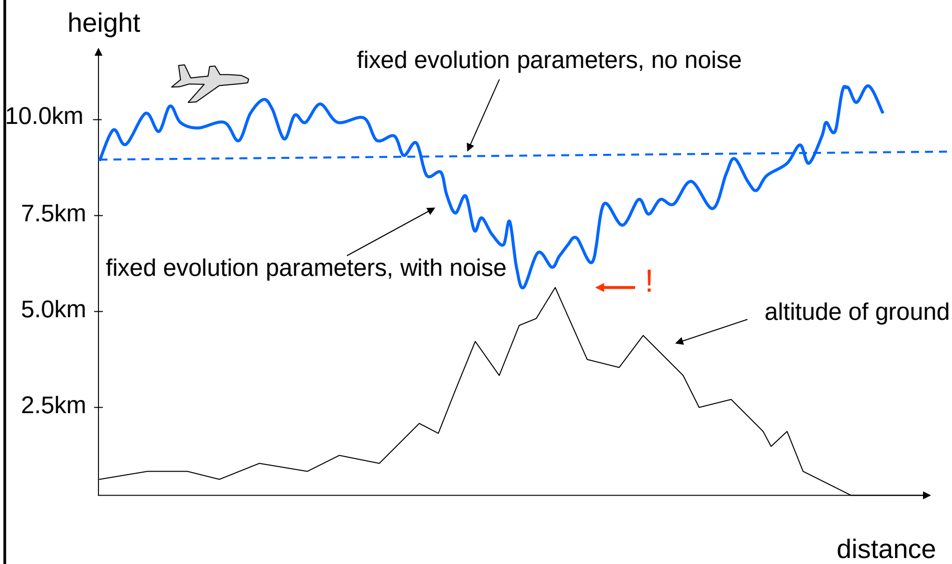
June 21, 2011 (3pm): Andrew Cleland (UC Santa Barbara, USA)
How to Be in Two Places at the Same Time ?

LECTURE VI : QUANTUM FEEDBACK CONTROL AND PERSISTENT RABI OSCILLATIONS

OUTLINE

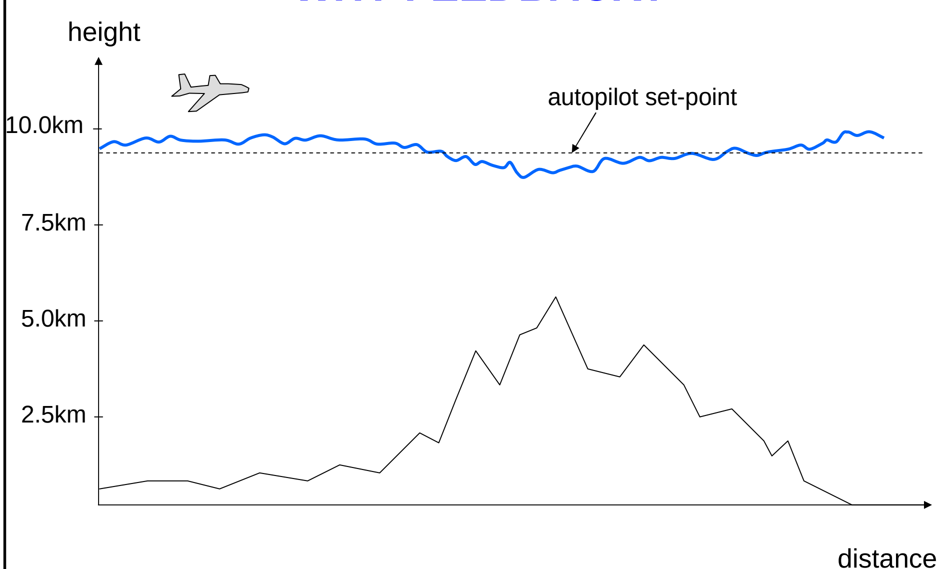
1. Classical and quantum feedback; persistent Rabi oscillations
2. Stochastic differential equations for quantum trajectories
3. Fidelity of quantum feedback control

WHY FEEDBACK?



Excursions away from perturbation-less trajectory must be bounded!

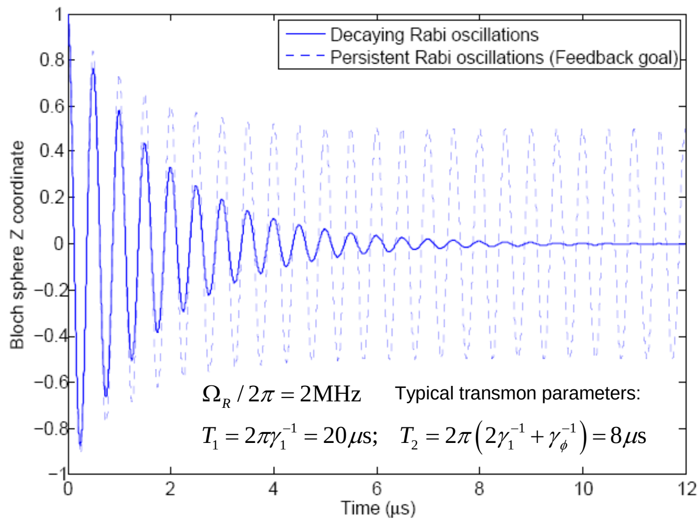
WHY FEEDBACK?



Feedback establishes effective potential and damping, limiting excursions

PERSISTENT RABI OSCILLATIONS: MOTIVATIONS

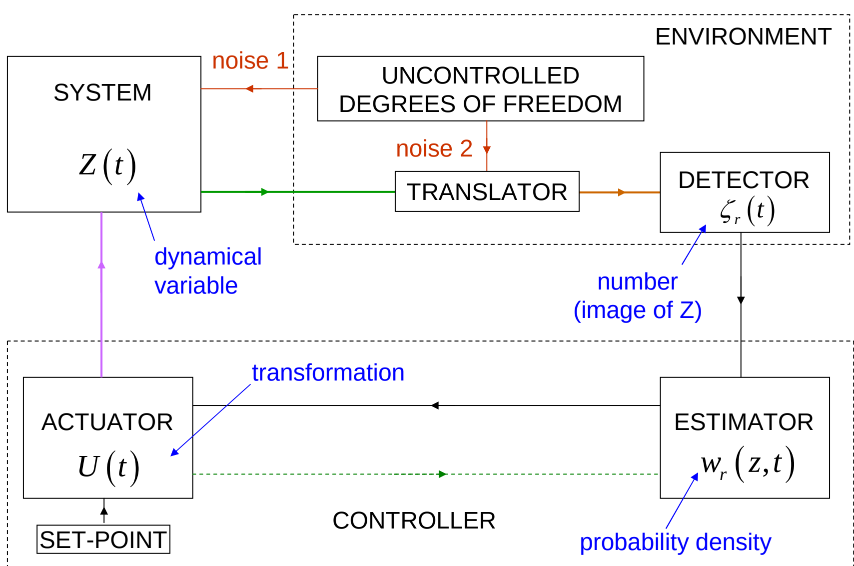
THY:
Ruskov &
Korotkov
(2002)



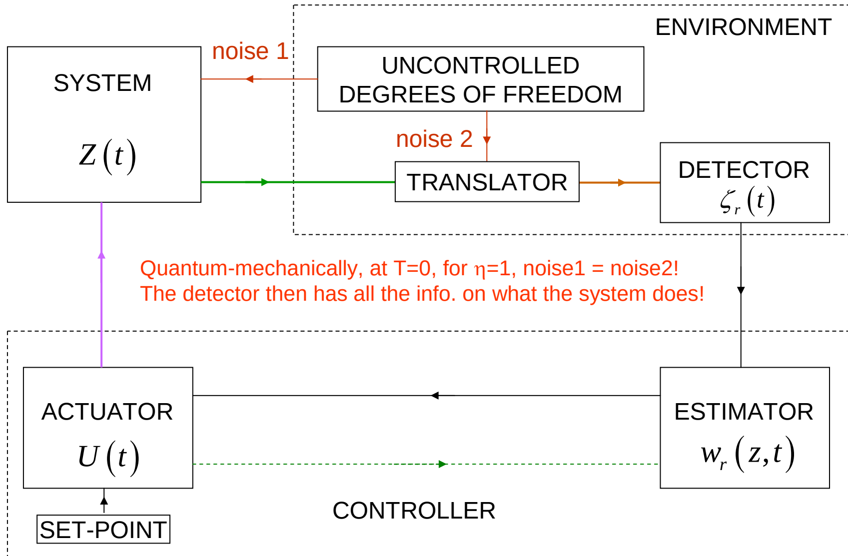
- Simplest and non-trivial quantum feedback demonstration for qubits
- Metrological application: RF amplitude-to-frequency converter

Courtesy of M. Mirrahimi

OPTIMAL FEEDBACK CONTROL



OPTIMAL FEEDBACK CONTROL



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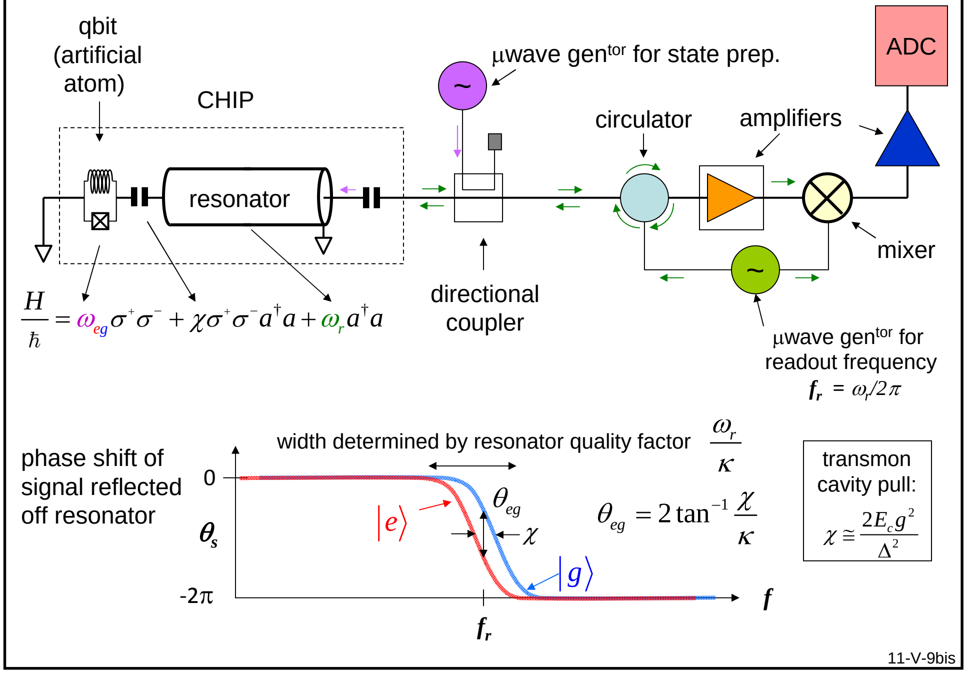
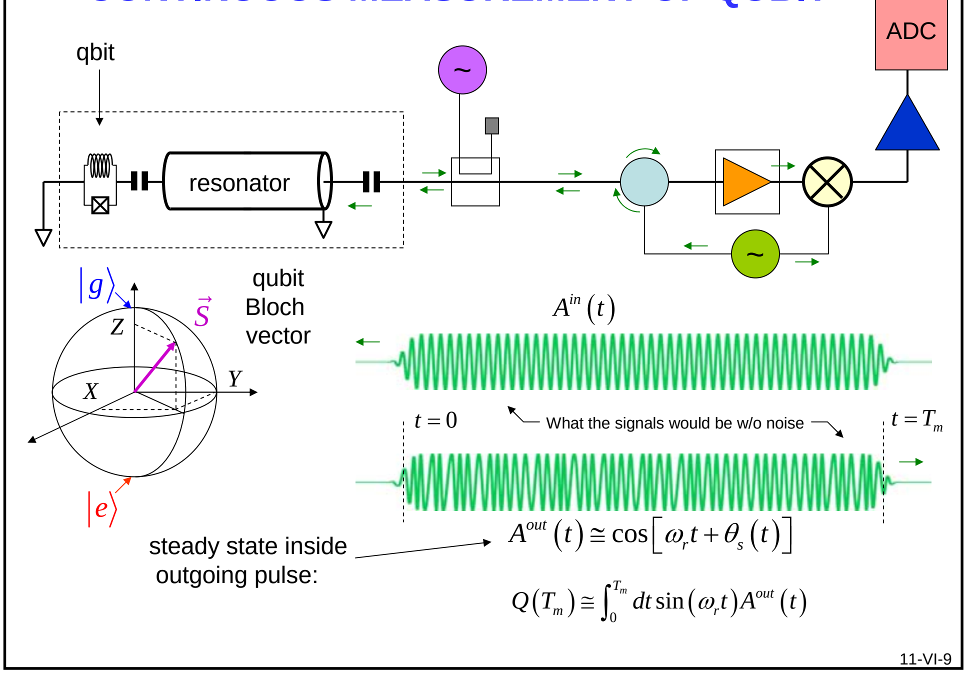
QUANTUM FEEDBACK IS CURRENTLY USED TO MAINTAIN A FOCK STATE IN A RESONATOR [see S. HAROCHE's 2010-2011 lectures and Dotsenko et al. Phys. Rev. A80, 013805 (2009)]

HERE WE ARE DISCUSSING QUANTUM FEEDBACK APPLIED TO THE PRESERVATION OF A QUBIT DYNAMICAL STATE

DISPERSIVE CQED QUBIT MEASUREMENT



CONTINUOUS MEASUREMENT OF QUBIT

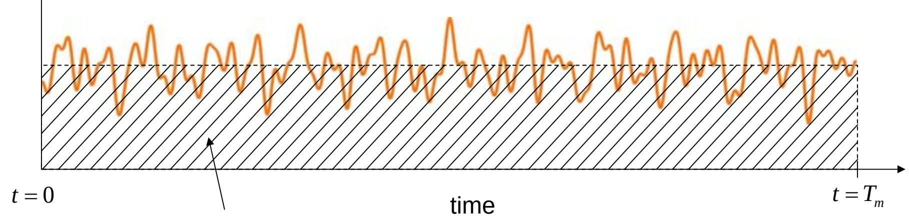


MEASUREMENT RECORD

Instantaneous growth rate of quadrature signal:

$$\zeta_r(t) = \int d\tau \left.\frac{dQ}{dt}\right|_\tau f(t-\tau)$$

filter function



$$Q_r(T_m) \cong \int_0^{T_m} dt\, \zeta_r(t)$$

Steady-state relation

$$dQ_r(t) = Z_r(t)\,dt + \frac{1}{\sqrt{\eta\gamma_m}}\,dW(t)$$

$$E\left[Z_r(t)\right] = \left\langle Z(t)\right\rangle = E\left[\frac{dQ_r}{dt}\right]$$

Wiener increment:

$$E\left[dW(t)\right] = 0$$

$$\left[dW(t)\right]^2 = dt$$

Idealized white noise

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OUTLINE

1. Classical and quantum feedback; persistent Rabi oscillations
2. Stochastic differential equations for quantum trajectories
3. Fidelity of quantum feedback control

11-VI-4

MASTER EQUATION OF THE QUBIT UNDER MEASUREMENT

$$\dot{\rho} = \mathcal{L}\rho = -\frac{i}{\hbar}\left[H_{qbit}, \rho\right] + \gamma_1 \mathcal{D}\left(\sigma^-\right)\rho + \frac{1}{2}\left[\gamma_\phi + \frac{\gamma_m}{2}\right]\mathcal{D}\left(\sigma_z\right)\rho$$

- Markov
- RWA
- T=0

where $\mathcal{D}(A)\rho = A\rho A^\dagger - A^\dagger A\rho/2 - \rho A^\dagger A/2$

relaxation rate: γ_ϕ

dephasing rate: γ_1

measurement rate: $\gamma_m = \bar{n}_r \frac{4\chi^2}{\chi^2+\kappa^2} = \frac{d(SNR)}{dt}$

Identical to the Bloch equations, but with one additional term:

Rotating frame at ω_{eg}

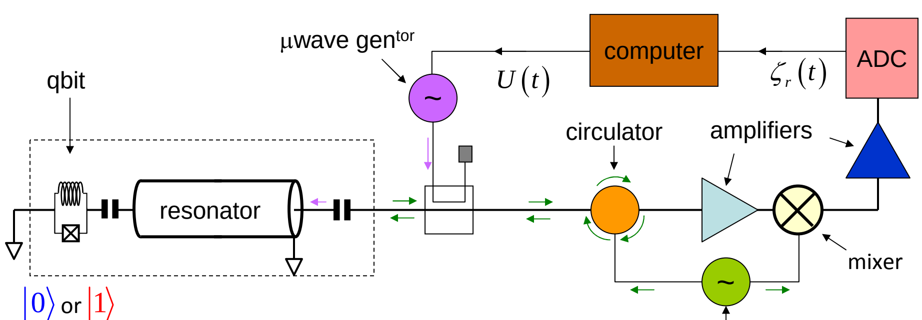
$$\begin{cases} \dot{X} = \Omega_R(t)Z(t) - \left(\frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2}\right)X \\ \dot{Y} = -\left(\frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2}\right)Y \\ \dot{Z} = -\Omega_R(t)X(t) - \gamma_1(Z-1) \end{cases}$$

measurement-induced dephasing

$\Omega_R(t)$: Rabi drive along Y, in freq. units

11-VI-11

DISPERSIVE QUBIT READOUT WITH FEEDBACK



µwave gen^{tor} for readout

Zhang, Ruskov and Korotkov (2005)
Jordan and Büttiker (2005)
Mirrahimi, Huard and Devoret, in preparation

Filter

$$U(t) = \int_{-\infty}^{t} d\tau\, \zeta_r(\tau) F(t-\tau)$$

Filter has delay

What should be the feedback law? How much delay is tolerable?
How pure is the state of the qubit with feedback?

11-VI-12

EQUATIONS OF QUANTUM TRAJECTORIES

Introduce ρ_r , the qubit density matrix conditioned by the string of measurement results and represents all the information the observer has accumulated on the qubit.

$\rho = E[\rho_r]$ is the usual qubit density matrix

It obeys:

$$d\rho_r = \mathcal{L}\rho_r dt + \eta\gamma_m \mathcal{M}(\sigma_z)\rho_r (dQ_r - Z_r dt)$$

an update equation, where

$$\mathcal{M}(A)\rho = \frac{1}{2}\left[\left(A - \langle A\rangle\right)\rho + \rho\left(A^\dagger - \langle A^\dagger\rangle\right)\right]$$

} Ito

This leads to the stochastic Bloch equations:

$$\begin{cases} dX_r = \Omega_R(t) Z_r - \left(\frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2}\right) X_r dt - \sqrt{\eta\gamma_m} X_r Z_r dW_r \\ dY_r = -\left(\frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2}\right) Y_r dt - \sqrt{\eta\gamma_m} Y_r Z_r dW_r \\ dZ_r = -\Omega_R(t) X_r - \gamma_1(Z_r - 1) + \sqrt{\eta\gamma_m}\left(1 - |Z_r|^2\right) dW_r \end{cases}$$

For simplicity we take here $\kappa = \chi$
No AC stark shift

The point $[X_r(t), Y_r(t), Z_r(t)]$ is in general inside the Bloch sphere. Its time evolution is the quantum trajectory of the system.

11-VI-13

THE QUANTUM TRAJECTORIES EQUATIONS ANSWER THE FOLLOWING QUESTIONS QUANTITATIVELY:

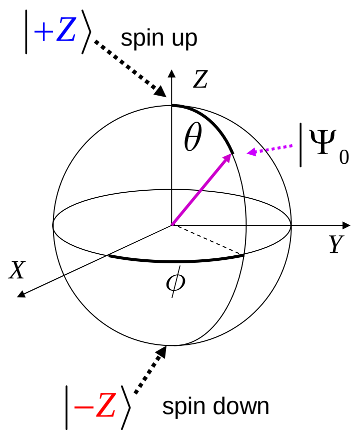
IN QUANTUM REGIME,
HOW MANY BITS OF INFORMATION DOES THE MEASUREMENT ACQUIRE PER UNIT OF TIME?
WHAT IS THE CORRESPONDING BACK-ACTION ON THE QUBIT?

A SIMPLER MODEL CAN ANSWER THESE QUESTIONS SEMI-QUANTITATIVELY.

11-VI-14

SPIN MODEL OF CONTINUOUS MEASUR^MENT

Bloch sphere
representation
of qubit
under measurement



$$|\Psi_0\rangle = \alpha|+Z\rangle + \beta|-Z\rangle$$

$$\alpha = \cos\frac{\theta}{2} e^{\frac{-i\phi}{2}}$$

$$\beta = \sin\frac{\theta}{2} e^{\frac{+i\phi}{2}}$$

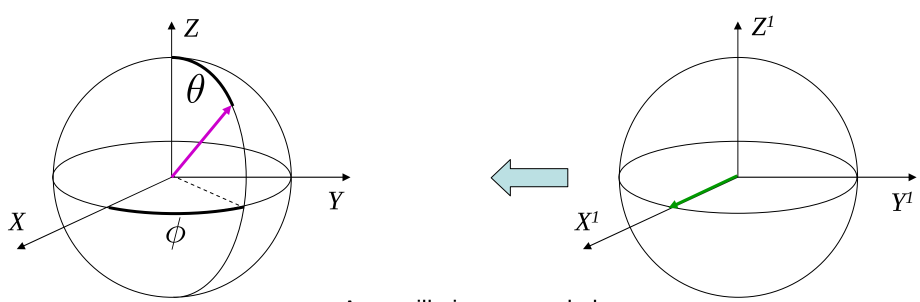
$$\langle\Psi_0|Z|\Psi_0\rangle = |\alpha|^2 - |\beta|^2 = Z_0$$

11-VI-15

SPIN MODEL OF CONTINUOUS MEASUR^MENT

Bloch sphere
representation
of qubit
under measurement

Bloch sphere
representation
of ancilla qubit
for **partial** measurement



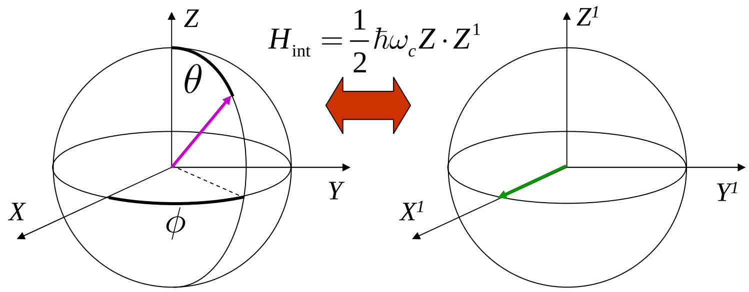
An ancilla is prepared along
X and moved towards qubit

11-VI-15a

SPIN MODEL OF CONTINUOUS MEASURMENT

Bloch sphere
representation
of qubit
under measurement

Bloch sphere
representation
of ancilla qubit
for partial measurement



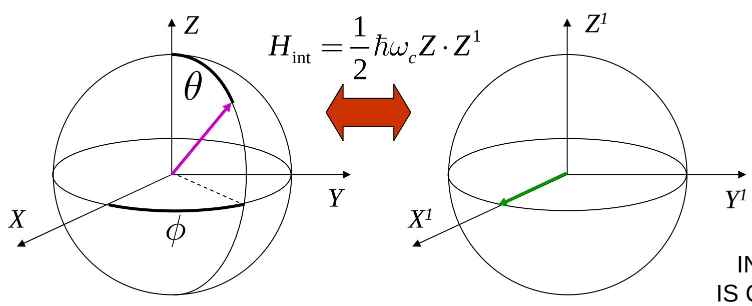
ancilla and qubit
interact during τ

11-VI-15b

SPIN MODEL OF CONTINUOUS MEASURMENT

Bloch sphere
representation
of qubit
under measurement

Bloch sphere
representation
of ancilla qubit
for partial measurement



INTERACTION
IS QND IF QUBIT1
ONLY SEES A FIELD
ALONG Z

ancilla and qubit
interact during τ

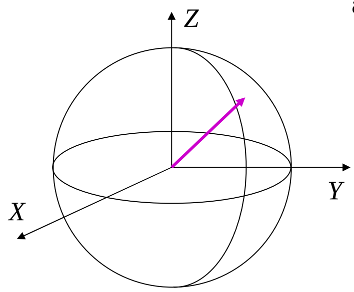
11-VI-15c

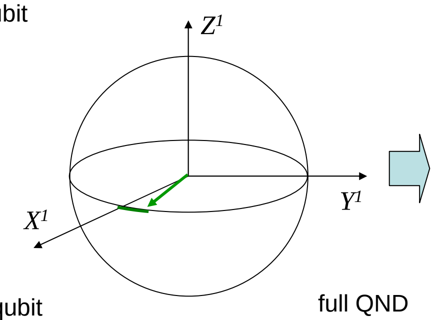
SPIN MODEL OF CONTINUOUS MEASUR$^{\text{MENT}}$

Interaction has produced: $U = \mathrm{e}^{-\frac{i\varepsilon}{2} Z \cdot Z^1}$

interaction strength: $\varepsilon = \omega_c \tau$

ancilla and qubit separate.





ancilla and qubit have become partially entangled (partial measurement). Bloch vectors are insufficient!

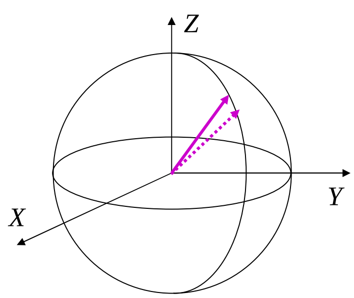
full QND projective measurement if $\varepsilon = \pi/2$

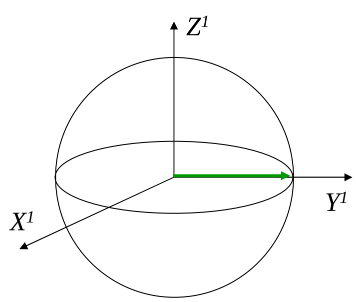
11-VI-15d

SPIN MODEL OF CONTINUOUS MEASUR$^{\text{MENT}}$

Next, ancilla is measured projectively along Y.

ONE GETS FOR INSTANCE, AFTER MEASUREMENT OF ANCILLA:



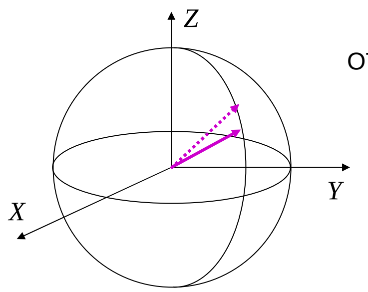


measurement record: $\zeta_1 = +1$

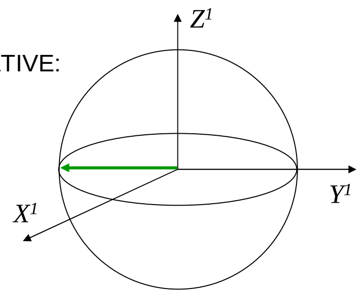
11-VI-15e

SPIN MODEL OF CONTINUOUS MEASUR$^{\text{MENT}}$

ancilla is measured projectively along Y



OTHER ALTERNATIVE:



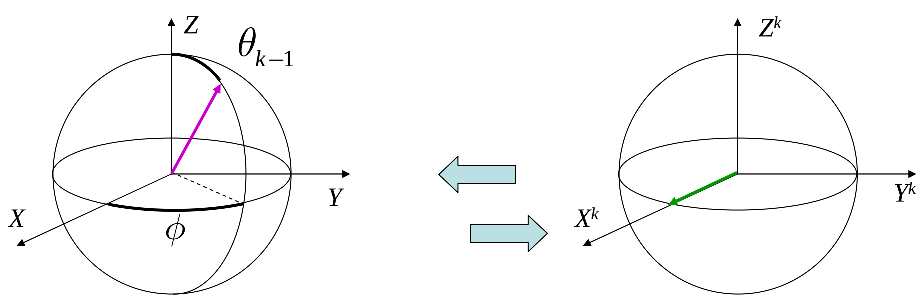
measurement record: $\zeta_1 = -1$

11-VI-15f

SPIN MODEL OF CONTINUOUS MEASUR$^{\text{MENT}}$

REPEAT N TIMES, TOTAL TIME = T_M

This generates measurement record $\left(\zeta_1, \zeta_2, .., \zeta_N\right)$



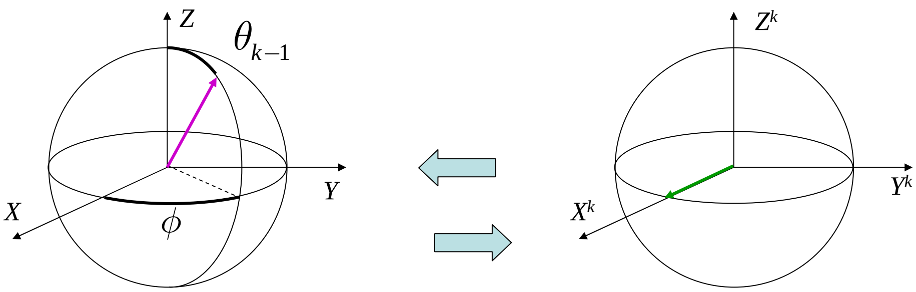
k-th ancilla is prepared along X and moved towards qubit etc,etc,...

11-VI-15g

SPIN MODEL OF CONTINUOUS MEASUR^MENT^

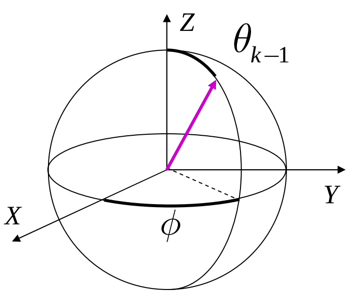
MAKE N TEND TOWARD INFINITY WHILE ε TENDS TOWARD ZERO

$$\left(\zeta_1, \zeta_2, .., \zeta_N\right) \rightarrow \zeta\left(t\right)$$



11-VI-15h

UPDATE FORMULA



After k-1 partial measurement, qubit <u>wavefunction</u> is:

$$\left|\Psi_{k-1}\right\rangle = \alpha_{k-1}\left|+Z\right\rangle + \beta_{k-1}\left|-Z\right\rangle$$

$$\begin{cases} \alpha_{k-1} = \cos\frac{\theta_{k-1}}{2} e^{\frac{-i\phi}{2}} \\ \beta_{k-1} = \sin\frac{\theta_{k-1}}{2} e^{\frac{+i\phi}{2}} \end{cases}$$

ϕ is unchanged, qubit state is pure!

state description: $\left\langle Z\right\rangle_{k-1} = \cos\theta_{k-1} = Z_{k-1}$

interaction strength $\sin\varepsilon = q \qquad 0 < q \le 1$

$$\gamma_m = \frac{Nq^2}{T_m}$$

append measurement record: $\zeta_k = \pm 1 \qquad p_k^{\pm} = \frac{1 \pm q \cdot Z_{k-1}}{2}$

$$\left(\zeta_1, \zeta_2, .., \zeta_{k-1}\right) \rightarrow \left(\zeta_1, \zeta_2, .., \zeta_{k-1}, \zeta_k\right) \qquad \left\langle \zeta_k \right\rangle_r = q \cdot Z_{k-1}$$

update qubit state conditioned by measurement result:

$$Z_k = \frac{q\zeta_k + Z_{k-1}}{1 + q\zeta_k Z_{k-1}}$$

stationary if $Z_{k-1} = \pm 1$

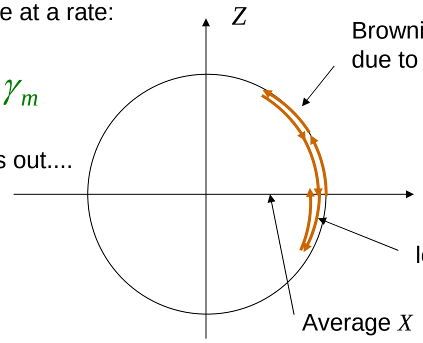
11-VI-16

HEISENBERG-LIKE RELATION

Auxiliary spins measure at a rate:

$$\frac{d(SNR)}{dt} = q^2 \frac{N}{T_m} = \gamma_m$$

Information on qubit goes out....



Measurement back-action dephasing rate: $\gamma_{\phi,ba} = \frac{q^2 N}{2T_m} = \frac{\gamma_m}{2}$

Measurement time

$$\gamma_m^{-1}\gamma_{\phi,ba} = \frac{1}{2}$$

similar to

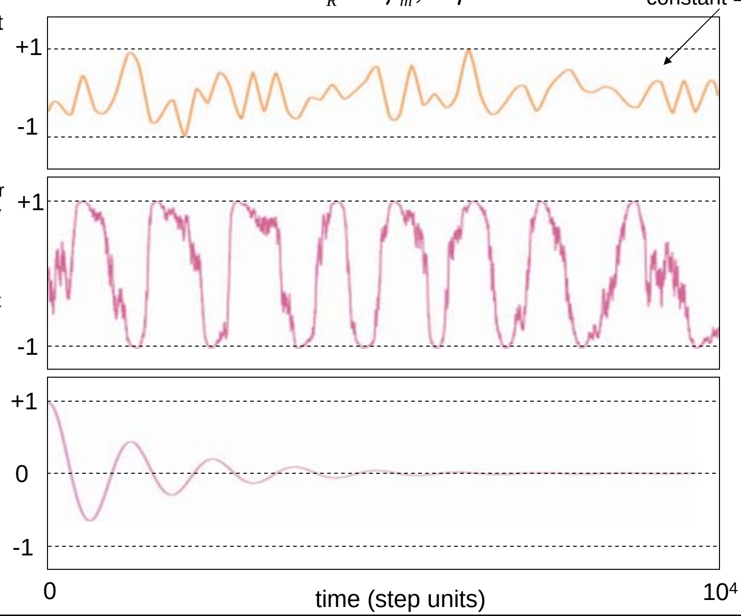
$$\Delta X_m \cdot \Delta P_{ba} = \frac{\hbar}{2}$$

11-VI-17

QUANTUM TRAJECTORY OF RABI OSCILLATIONS

$\Omega_R = 2\gamma_m; \quad \eta = 1$

Filter time constant = 10^2



Measurement record: $\zeta_r(t)$

Z component of Bloch vector conditioned by measurement: $Z_r(t)$ (What a perfect observer would reconstruct)

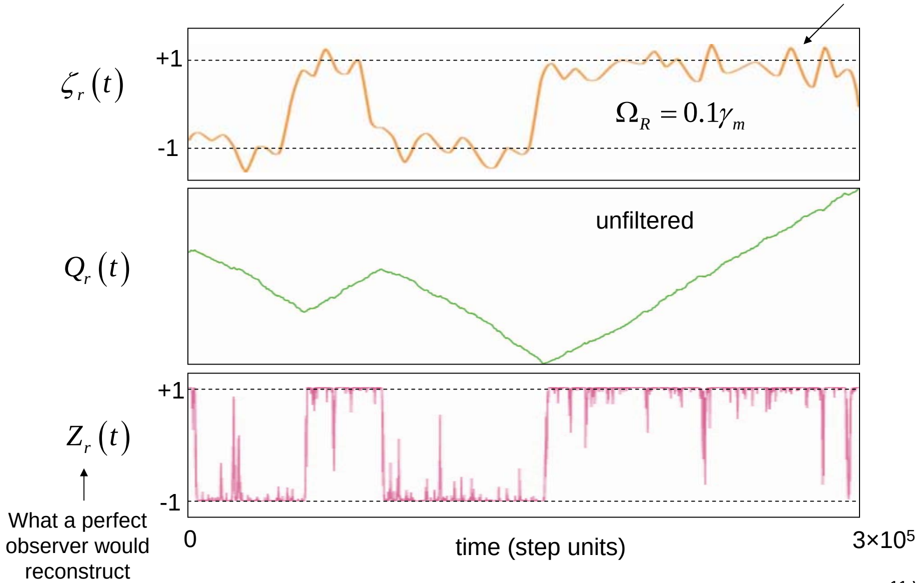
$Z(t)$ Z component of Bloch vector averaged over all realizations of trajectories

11-VI-18

QUANTUM JUMPS

For $\gamma_m \gg \Omega_R$ trajectories turn into "telegraphic" noise

Filter time constant = 5×10³



11-VI-19

OUTLINE

1. Classical and quantum feedback; persistent Rabi oscillations
2. Stochastic differential equations for quantum trajectories
3. Fidelity of quantum feedback control

11-VI-4

KOROTKOV'S PROPORTIONAL CONTROLLER

ESTIMATED DEPHASING OF RABI OSCILLATIONS:

$$\delta\theta(t) = \tan^{-1}\left(\frac{X_r(t)}{Z_r(t)}\right) + \frac{\pi}{2}\left[1 - \mathrm{sgn}\left(Z_r(t)\right)\right] - \bar{\Omega}_R t$$

$$-\pi < \delta\theta(t) < +\pi$$

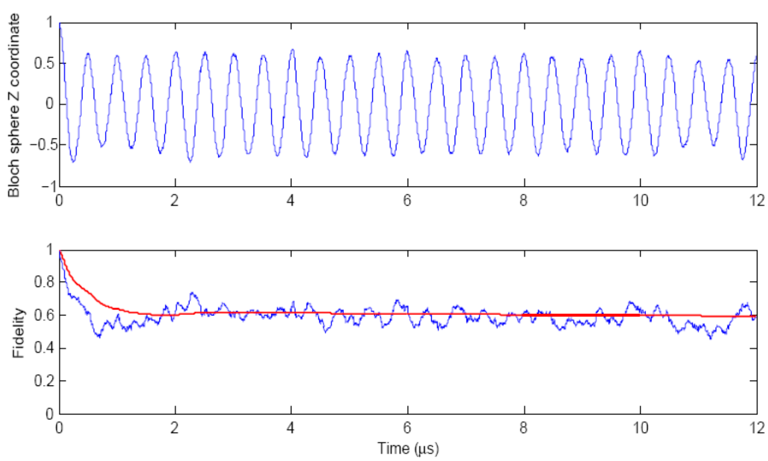
FEEDBACK LAW:

$$\Omega_R(t) = \bar{\Omega}_R t - G_{FB}\bar{\Omega}_R \delta\theta(t)$$

11-VI-20

FIDELITY OF PROPORTIONAL CONTROL

Simulations for transmon qubit

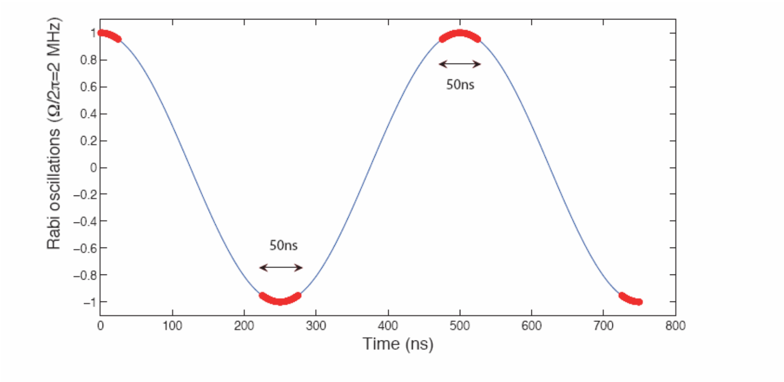


$$\eta = \frac{1}{2} \quad G_{FB} = \frac{1}{2} \quad \bar{n}_r = 0.13 \quad \kappa/2\pi = 8\mathrm{MHz} \quad \chi/2\pi = 1.2\mathrm{MHz}$$

Courtesy of M. Mirrahimi

11-VI-21

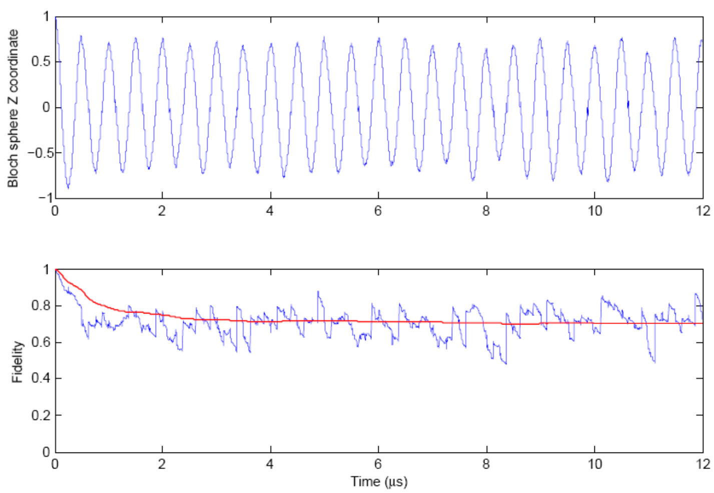
IMPROVEMENT OF FEEDBACK SCHEME USING STRONGER, PULSED MEASUREMENT



- Partial projection on the poles Z = ±1 (Quantum Zeno effect)
- Correction by a π-pulse around Y-axis if Z=-1 is detected (similar to adding a day on a leap year).

Courtesy of M. Mirrahimi

FIDELITY OF PULSED CONTROL



$\bar{n}_r = 2$ $\kappa/2\pi = 20\text{MHz}$ $\chi/2\pi = 4\text{MHz}$ delay of 100ns !

Courtesy of M. Mirrahimi

CONCLUSIONS

A quantum system driven out-of-equilibrium and in contact with several reservoirs can be seen from a scattering point of view emphasizing the notion of information channels.

Increasing the bit rate of monitored information channels over that of un-monitored ones is a necessary condition for increasing fidelity of quantum feedback control.

The key bi-directionality property of information channels manifests itself in powerful relations linking dissipation/amplification and noise, on one hand, as well as measurement precision/speed and the corresponding inevitable back-action, on the other hand.

11-VI-24

END OF 2011 COURSE ON QUANTUM AMPLIFICATION AND FEEDBACK

NEXT YEAR: NANOMECHANICAL RESONATORS IN QUANTUM REGIME

ACKNOWLEDGEMENTS

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COLLÈGE DE FRANCE 1530

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