Quantum dynamics in nano Josephson junctions

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Rutgers (USA)

Projects: ACI, EuroSQIP.

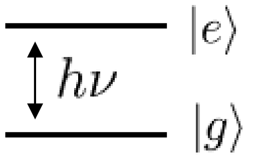
Introduction

In the last decade:

new experiences in quantum mechanics using

superconducting quantum circuits

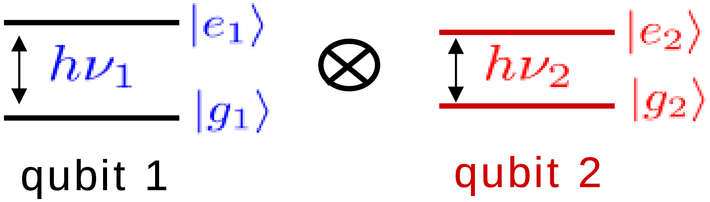
- realisation of a two level system



$$|\Psi\rangle = \alpha|g\rangle + \beta|e\rangle$$

- anharmonic quantum oscillator (multi-level system)

- two coupled qubits



$$|\Psi\rangle = (|g_1 e_2\rangle + |e_1 g_2\rangle)/\sqrt{2}$$

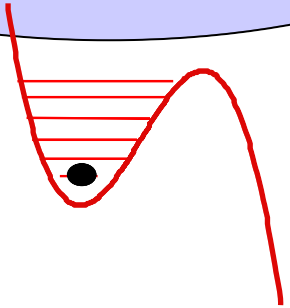
- two level system coupled to high Q cavity

Motivations:
- quantum dynamics in macroscopic system
- new quantum phenomena
  * very strong coupling with external field
  * strong coupling with environment
- quantum information
- model system for the quantum nano-electronics

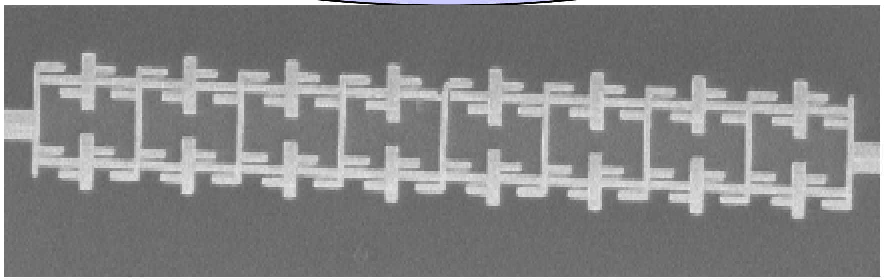
Research Topics in Grenoble

First part

Quantum dynamics of an anharmonic oscillator (DC SQUID)
Phase qubit

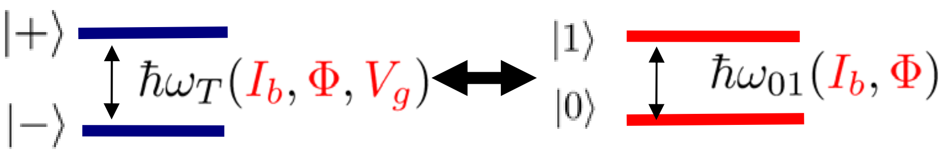


Rhombi chain:
A novel topologically protected qubit

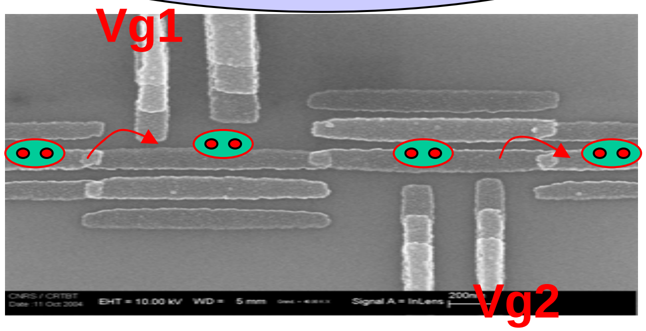


Second part

Phase qubit coupled to a charge qubit:
Tunable coupling



Cooper pair pumping through a double Island



Outline

Driven anharmonic oscillator

- Introduction on Josephson junction
- quantum dynamics in a dc SQUID
- multilevel quantum system
- quantum or classical dynamics

Coupled circuit between a charge and a phase qubit

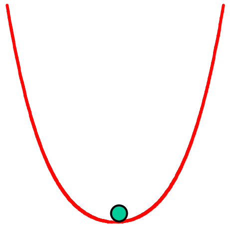
- asymmetry Cooper pair transistor
- entangled states
- tunable coupling
- resonant read-out

- Conclusion

Driven anharmonic oscillator

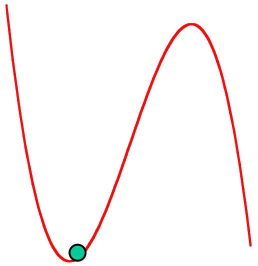
Harmonic oscillator:

$$H(t) = \frac{\hat{P}^2}{2m} + \frac{1}{2} m\omega_p^2 \hat{X}^2 + f_{ext}\cos(2\pi\nu t)\hat{X}$$



The quantum particle follows a motion very close to the classical one

By adding anharmonic terms → $-a\hat{X}^3 - b\hat{X}^4$



New physics appear which were extensively studied

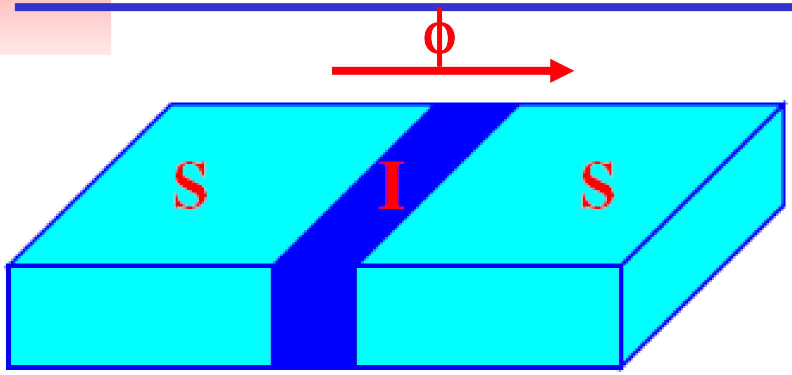
Classical mechanics: - Landau&Lifchitz
- modification of the resonance peak
- bi-stability (used as amplifier Siddiqqi 04, Ithier 05)

Quantum mechanics: many theoretical studies (Dykman88, Milburn86, Enzer97, Katz07, etc..)

Can we see quantum signature and cross-over between classical and quantum?

Non linear dynamics in superconducting quantum circuits

Basic building blocks: Josephson junction



Josephson relations:

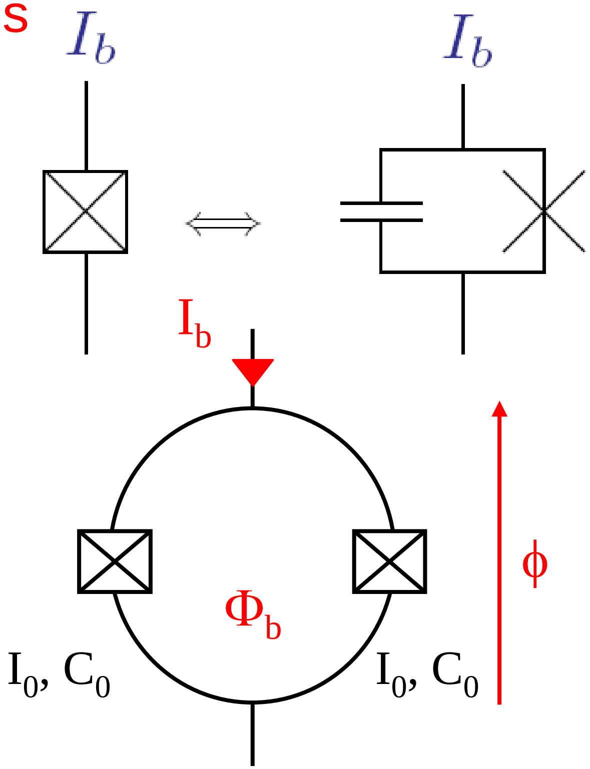
$$I = I_c \sin\phi$$

$$\dot{\phi} = 2eV/\hbar$$

Small Josephson junction: two energy scales

$$E_C = (2e)^2/2C$$

$$E_J = \hbar I_c/(2e)$$



Two junctions in parallel: SQUID

$$E_J = E_J(\Phi_b)$$

Equations of motion: current conservation $I_b = I_c \sin\phi + C d^2\phi/dt^2$

Current biased dc SQUID in the quantum limit: anharmonic oscillator

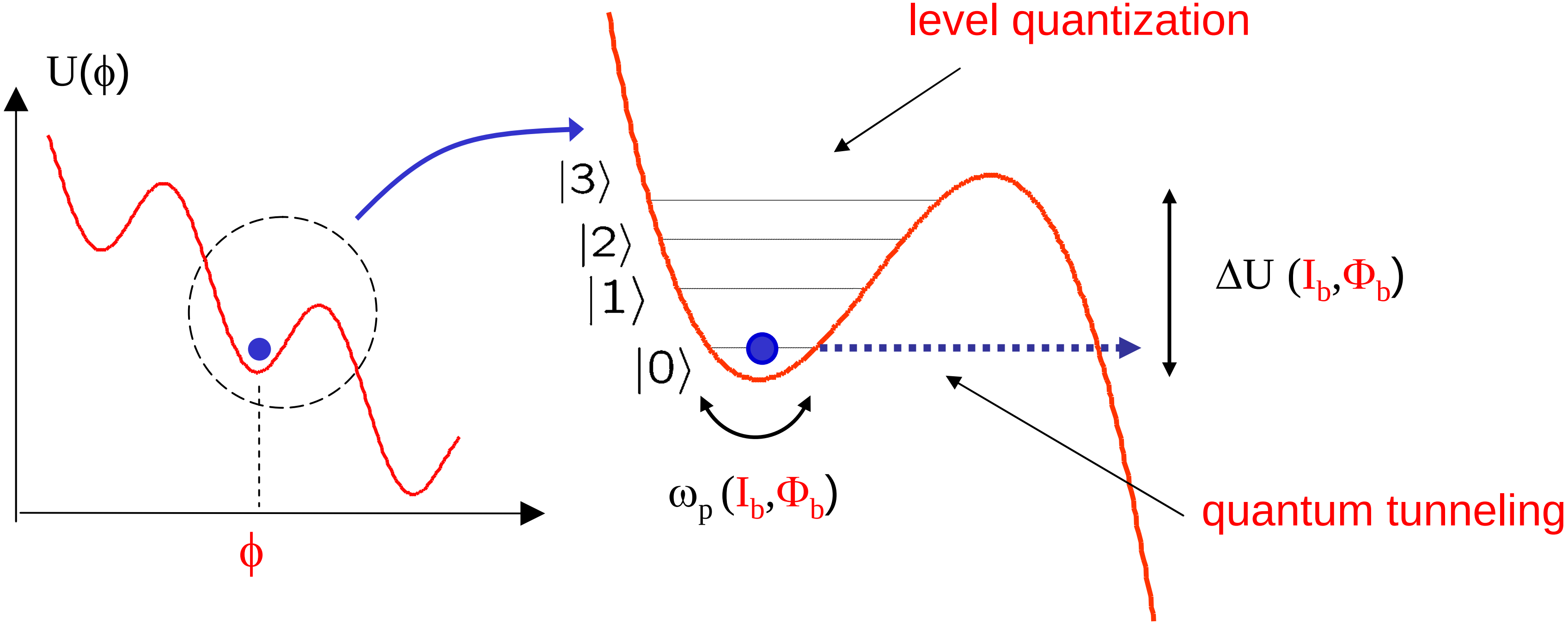
$$H = E_c(Q/2e)^2 - E_J \cos\phi - I_b\phi$$

$$[Q,\phi] = -2ie$$

Charging energy

Josephson potential

$E_J >> E_C$

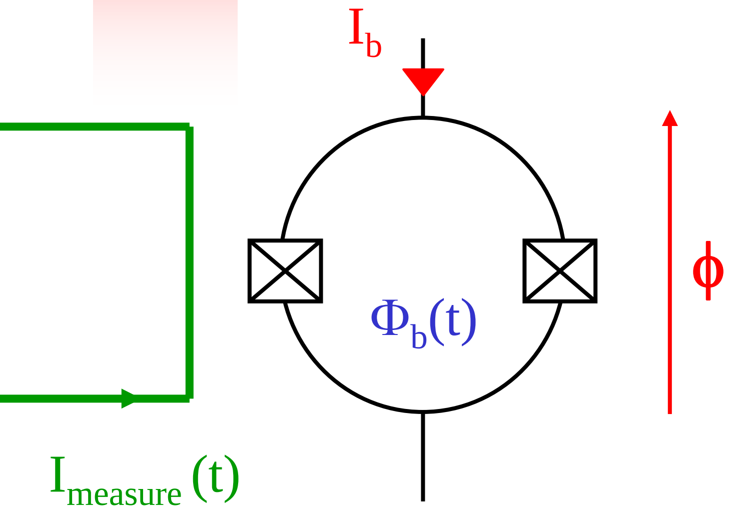


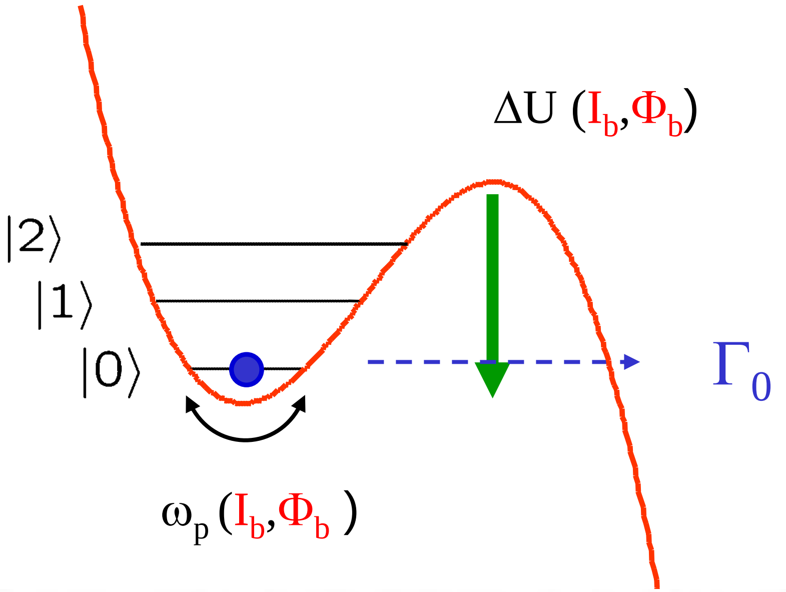
$$\frac{1}{2}\hbar\omega_p\left[\widetilde{P}^2 + \widetilde{X}^2\right] - \hbar\sigma\omega_p\widetilde{X}^3$$

Quantum anharmonic oscillator!

Quantum measurements

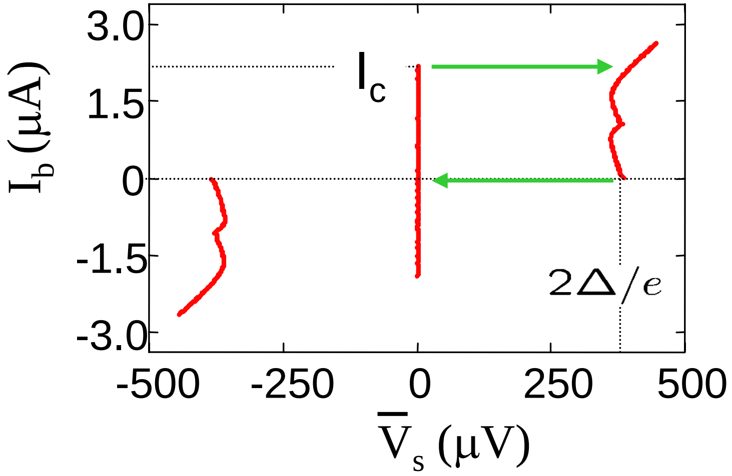
J. Claudon, A. Fay, E. Hoskinson, and O. Buisson, PRB2007

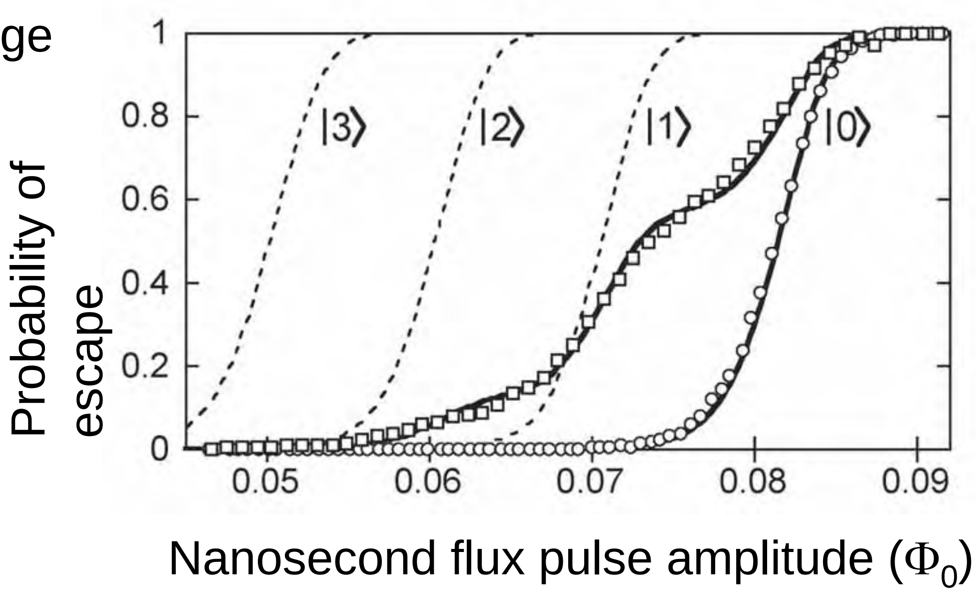




A nano-second flux pulse reduces the barrier

Hysteretic junction: escape leads to voltage





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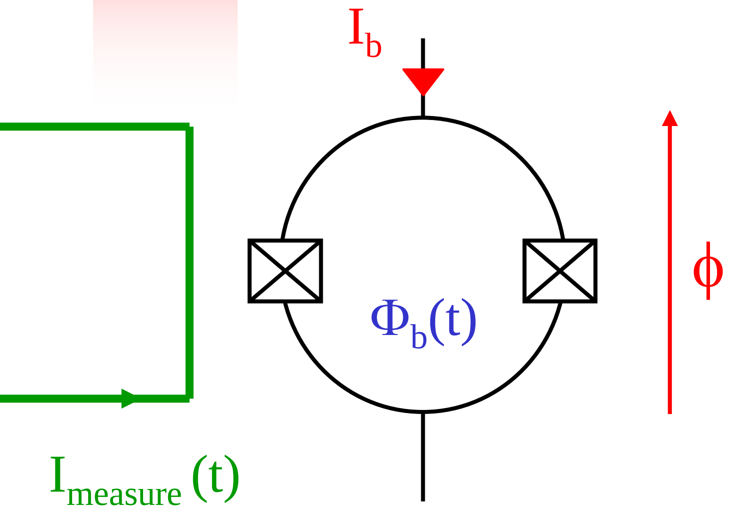

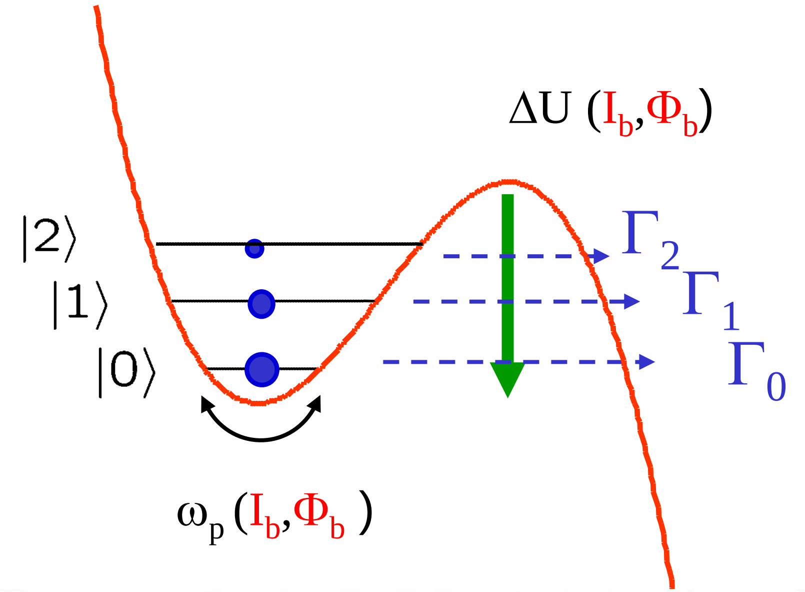

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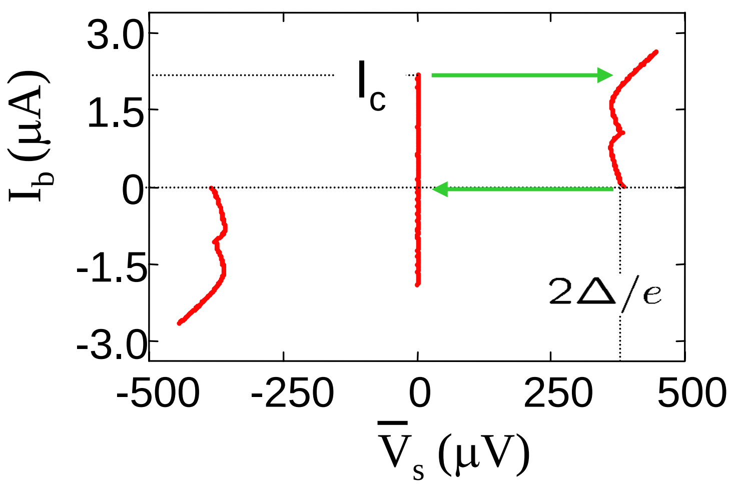
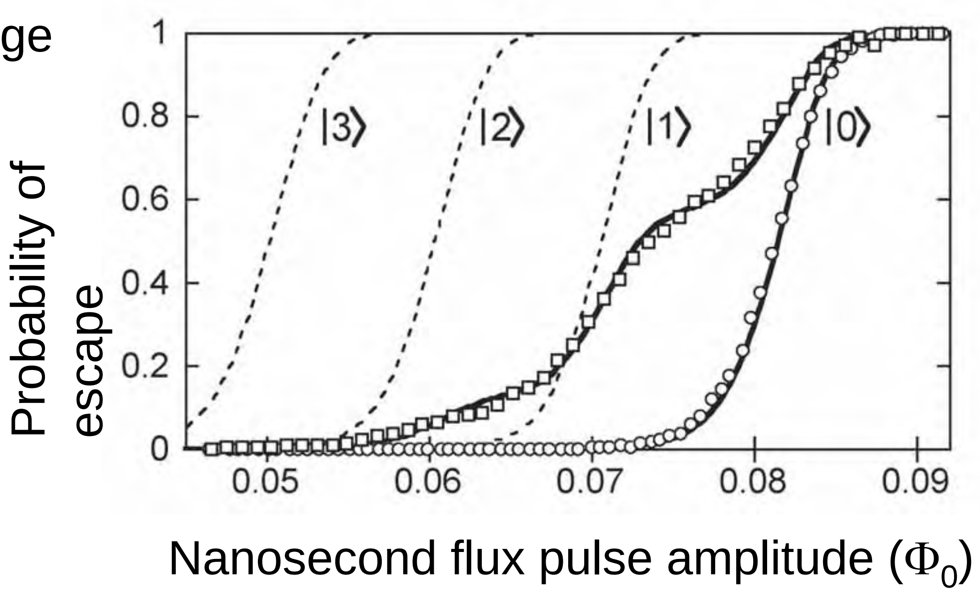
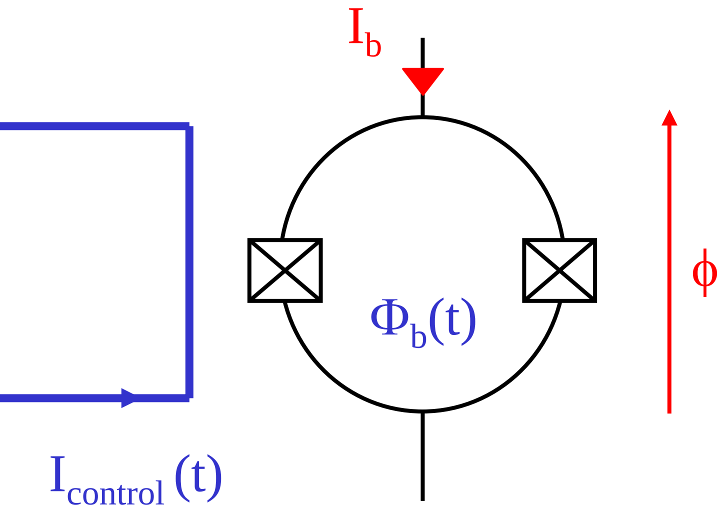
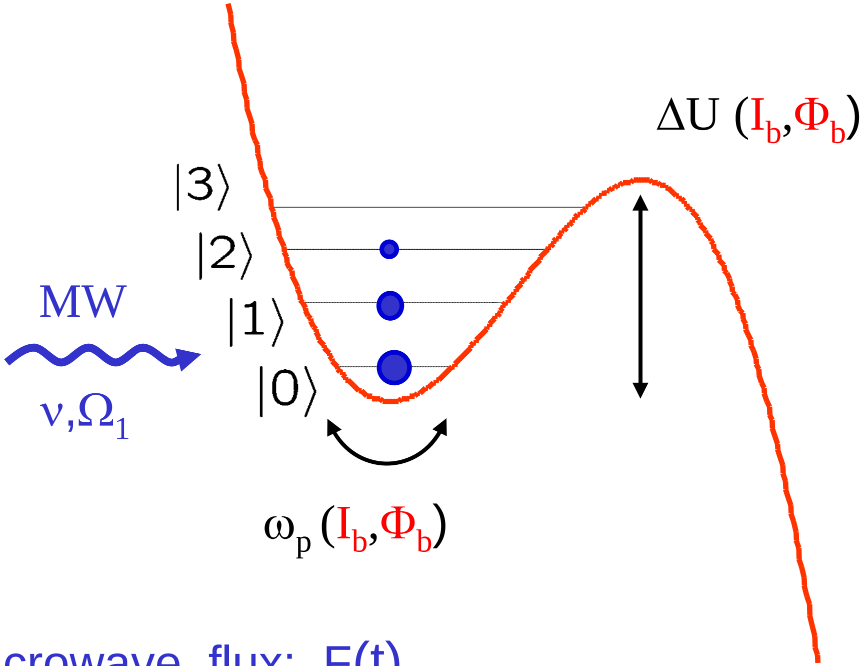




Quantum state manipulation

Deep well with quantized states





excitation ⟷ Microwave flux: F(t)

$$-\hbar\Omega_1 \cos(2\pi\nu t)\sqrt{2}\widetilde{X}$$

An external driving force!

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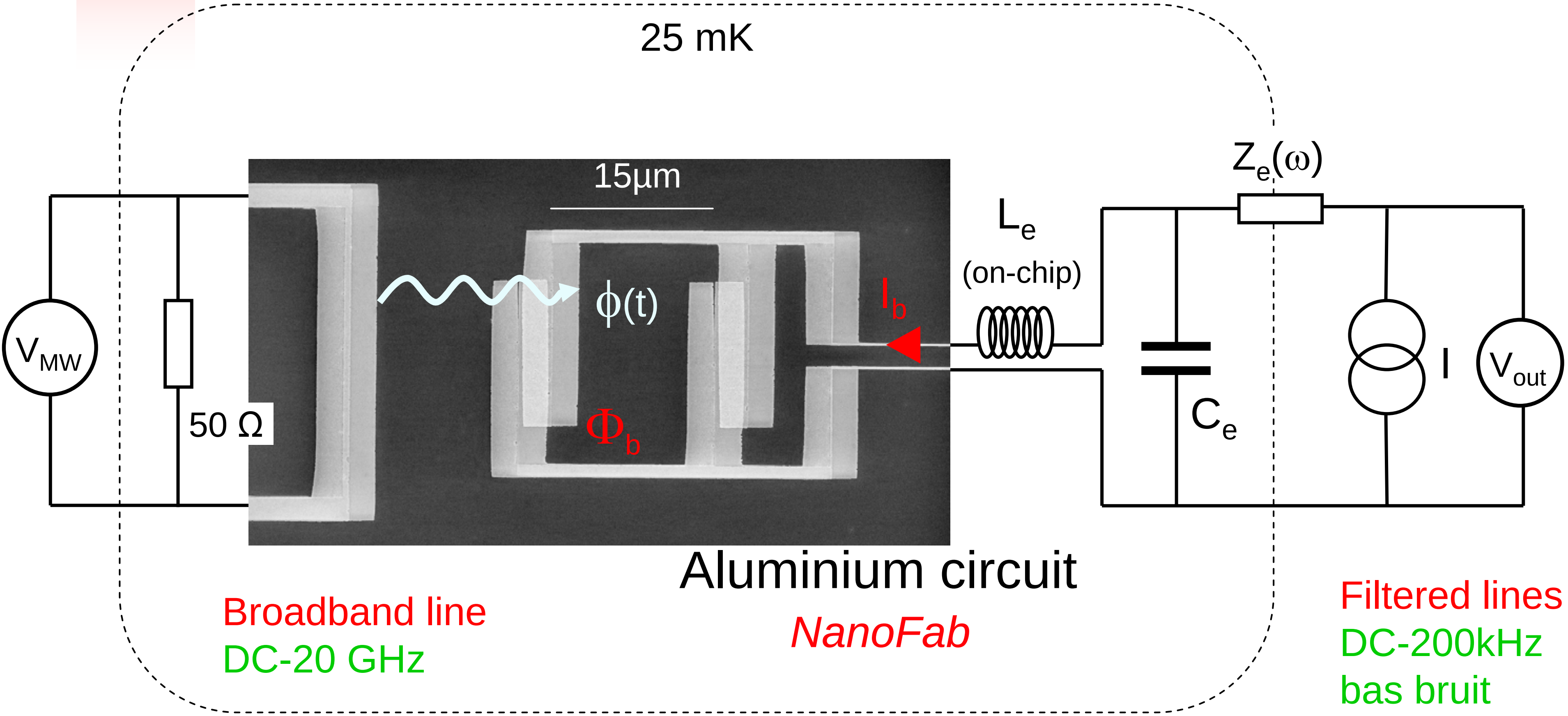
Experimental set-up



Broadband line
DC-20 GHz

Aluminium circuit
NanoFab

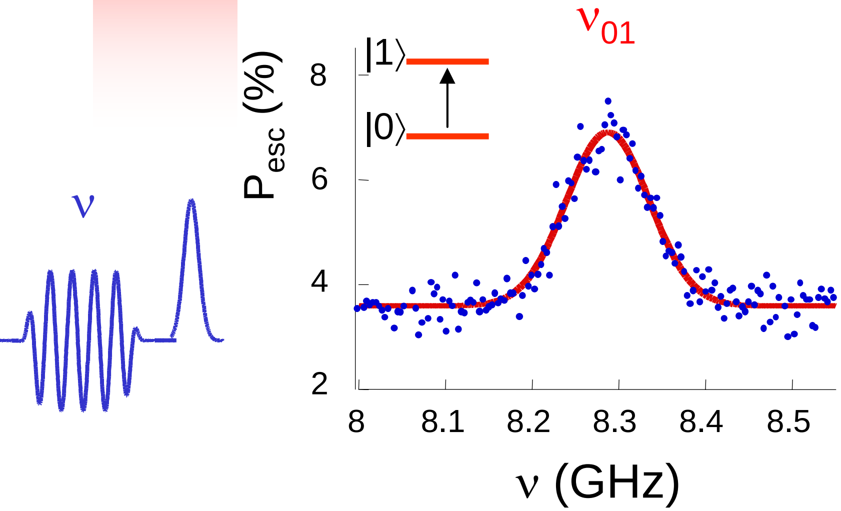
Filtered lines
DC-200kHz
bas bruit

- MW manipulation
- fast measurements

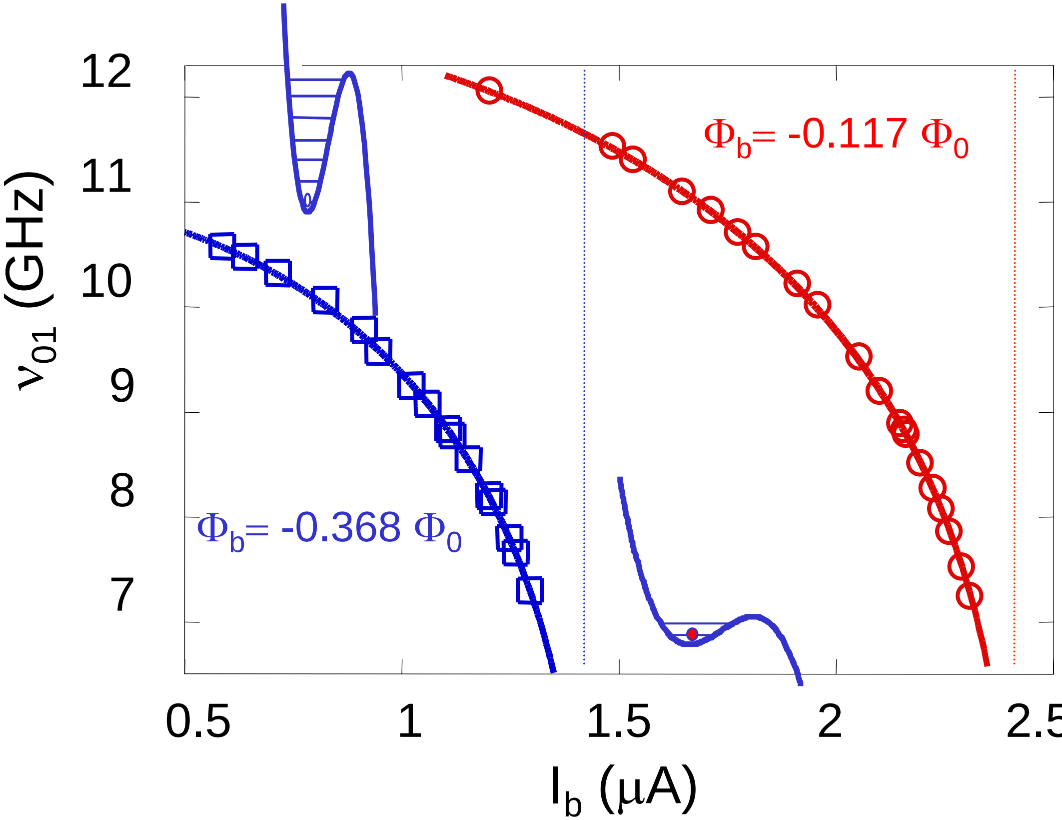
- courant bias
- voltage state of SQUID

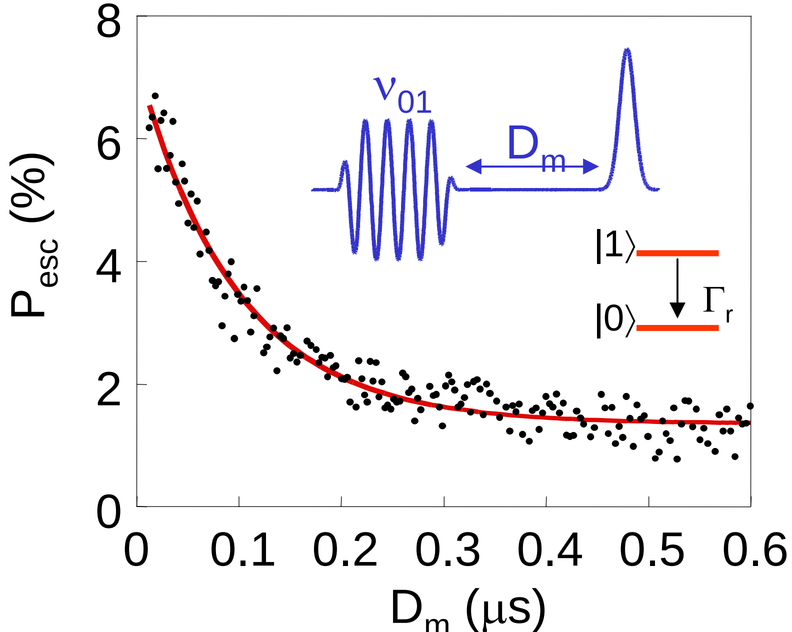
Spectroscopy and relaxation measurements

J. Claudon, A. Fay, L.P. Lévy, and O. Buisson (PRB2006)



I_b = 2.222 μA, Φ_b = -0.117 Φ_0





solid line $\propto \exp(-D_m / T_1)$

T_1 = 90 ns

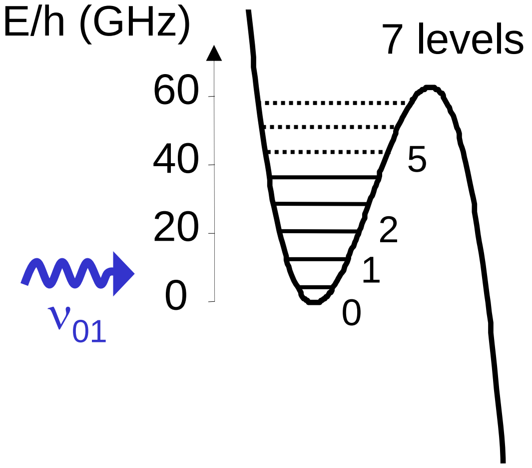
SQUID parameters

I_0 =1.242 μA
C_0= 560 fF
L_S= 280 pH

Coherent oscillations in a dc SQUID

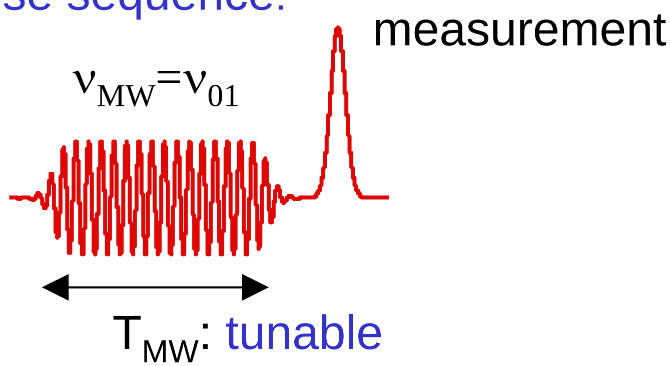
(J. Claudon F. Balestro, F. Hekking, and O. Buisson, PRL 2004)

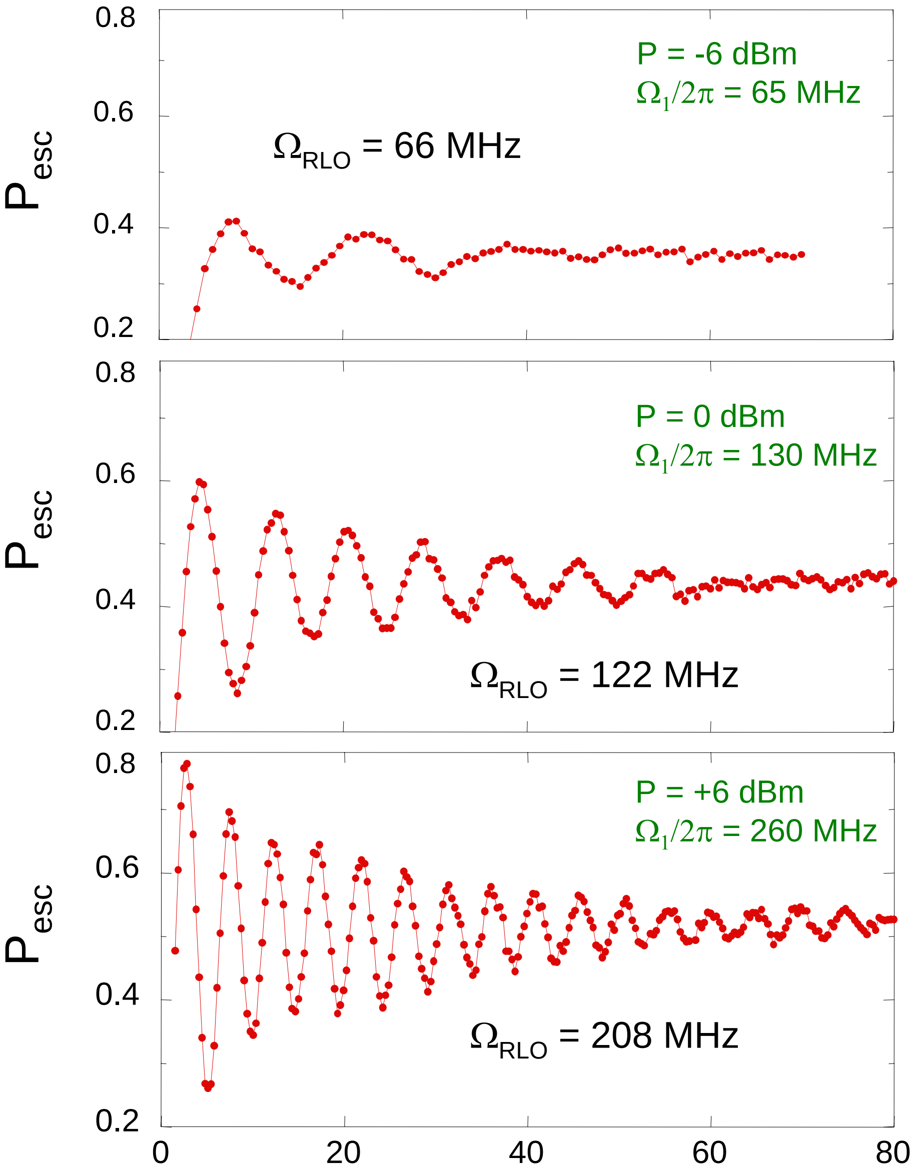
- Anharmonic oscillator:



Anharmonicity: $\nu_{01}-\nu_{12}$ = 160 MHz

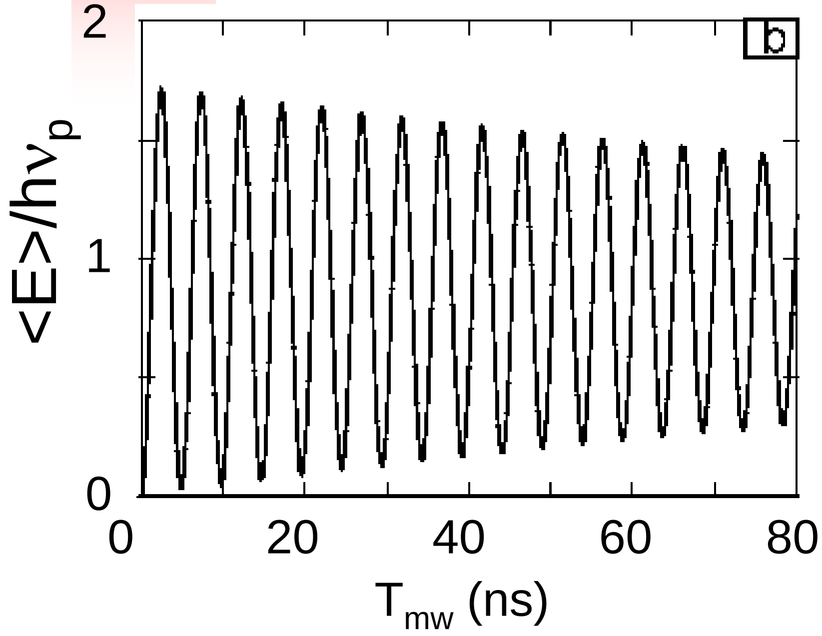
- Flux-pulse sequence:





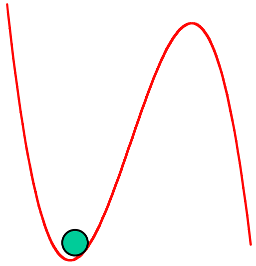
Are Rabi like oscillations quantum signature?

Classical dynamics



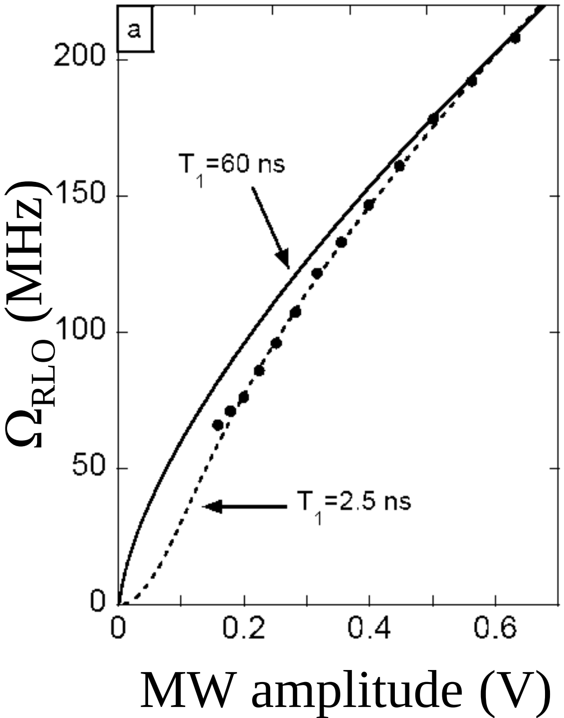
$$\ddot{\phi} + \alpha\dot{\phi} + \sin(\phi) = \eta + \varepsilon\sin(\omega\tau)$$

with $\dot{\phi} = \partial\phi / \partial\tau$



Beating phenomena exist
in the classical model!

Gronbech&Cirillo PRL2005
J. Marchese et al cond-mat0509729

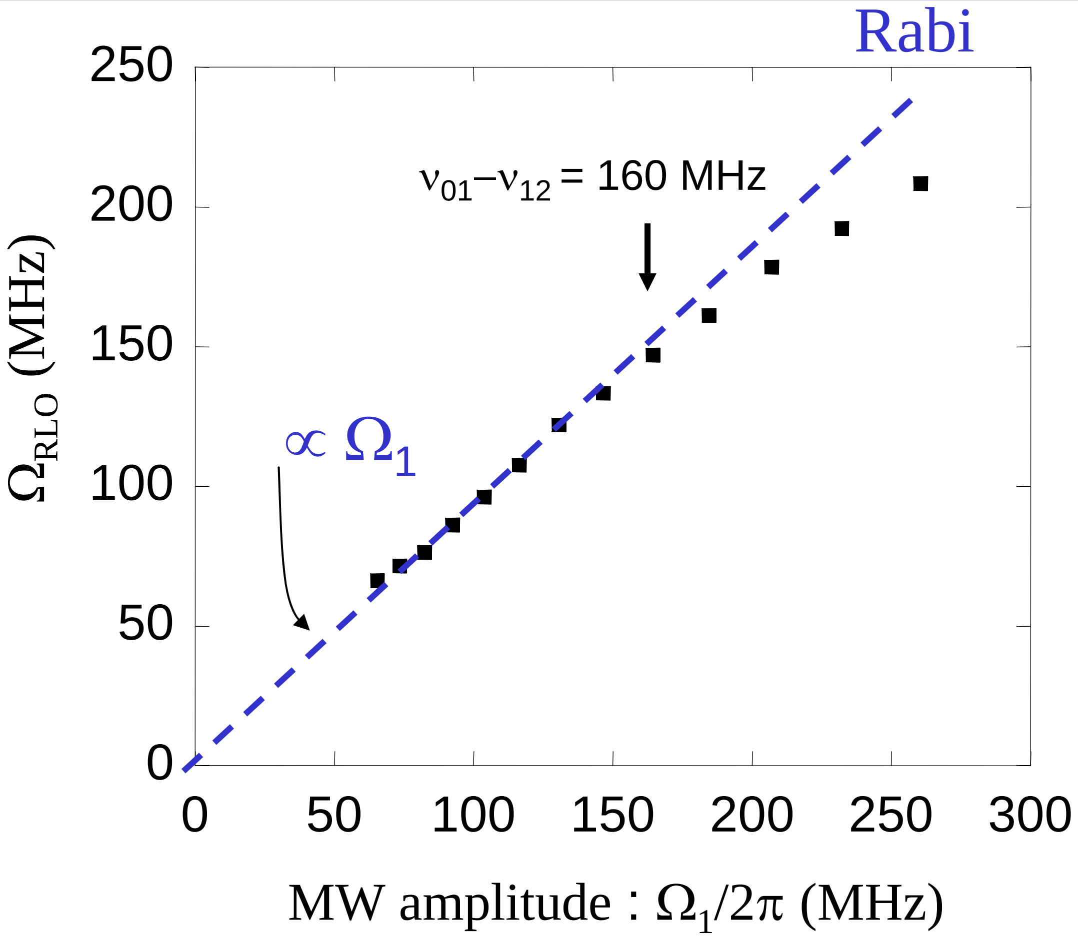


Classical model fails to describe the Ω_{RLO} versus MW amplitude in our device

A. Ratchov, PhD-thesis 2005

J. Claudon,A. Zazunov, F. Hekking, and O. B, arXiv:0709.3787

Rabi oscillations of a two level system



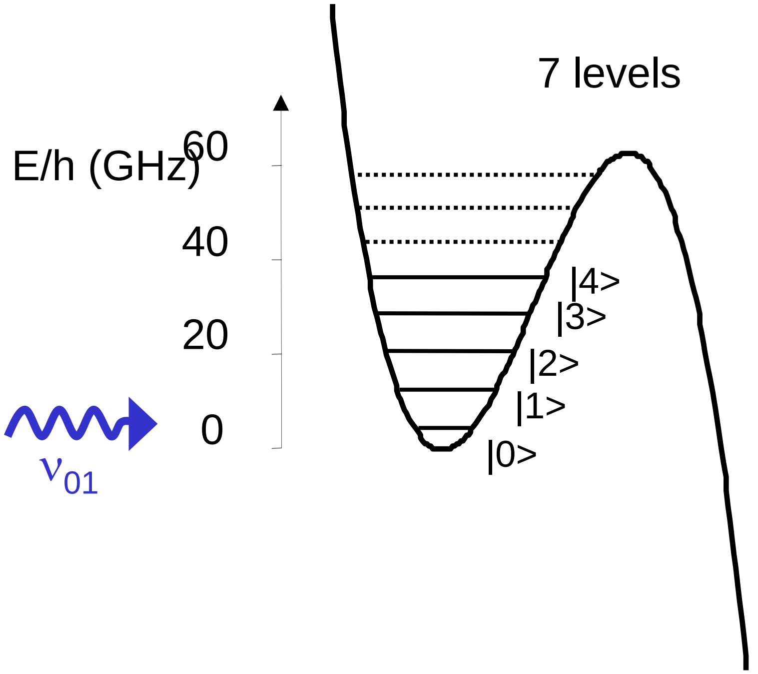
Strong deviation compare to Rabi prediction!

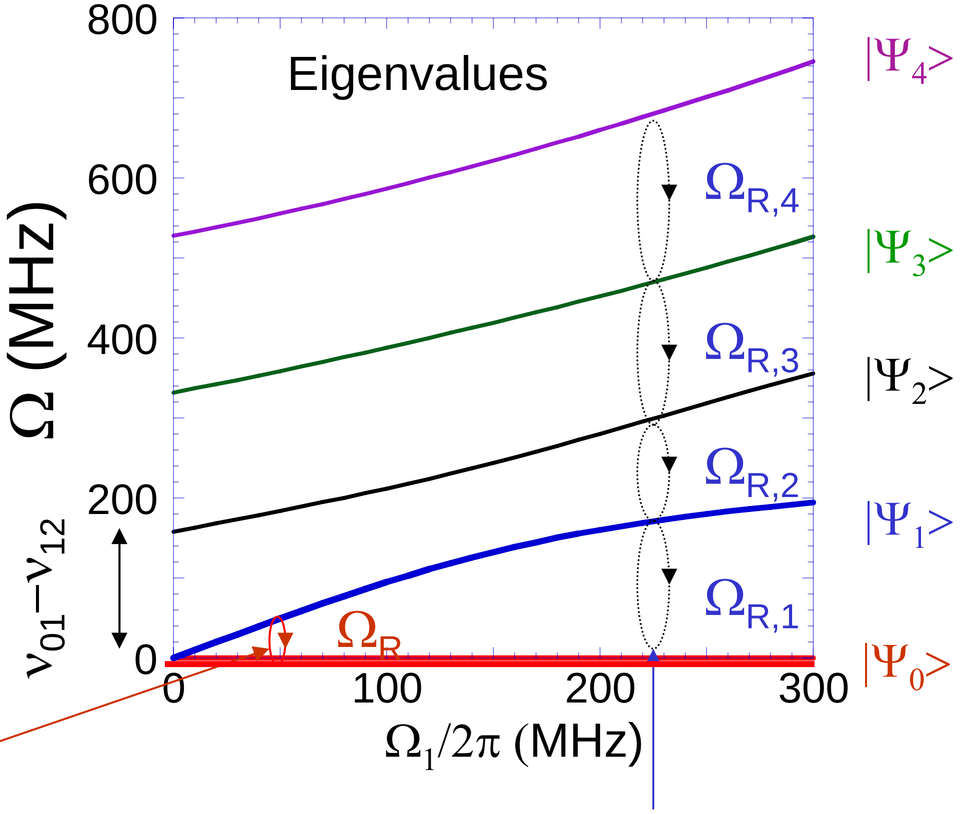
We must take into account the multi-level dynamics

Multilevel dynamics

$$\hat{H}(t)=\frac{1}{2}\hbar\omega_p(\hat{P}^2+\hat{X}^2)-\sigma\hbar\omega_p\hat{X}^3-\sqrt{2}\hbar\Omega_1\cos(2\pi\nu t)\hat{X} \longrightarrow$$ Time independent Hamiltonian

« Rotating wave » approximation





low amplitude:

Initial state: $\left|0(t=0^+)\right\rangle=(\left|\Psi_0\right\rangle+\left|\Psi_1\right\rangle)/\sqrt{2}$

Rabi oscillations

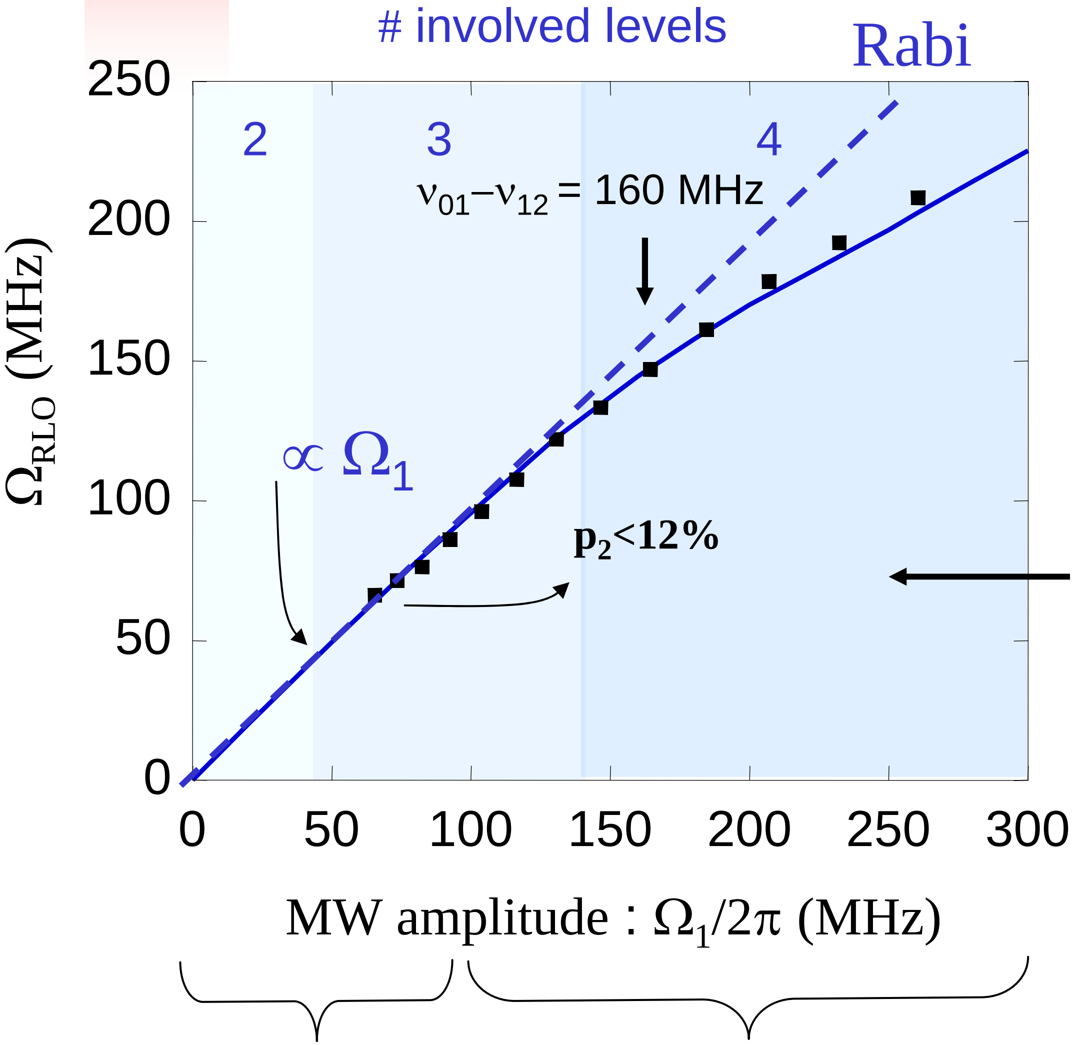
large amplitude:

$|0(t=0^+)>=a_0|\Psi_0>+a_1|\Psi_1>+a_2|\Psi_2>+a_3|\Psi_3>$

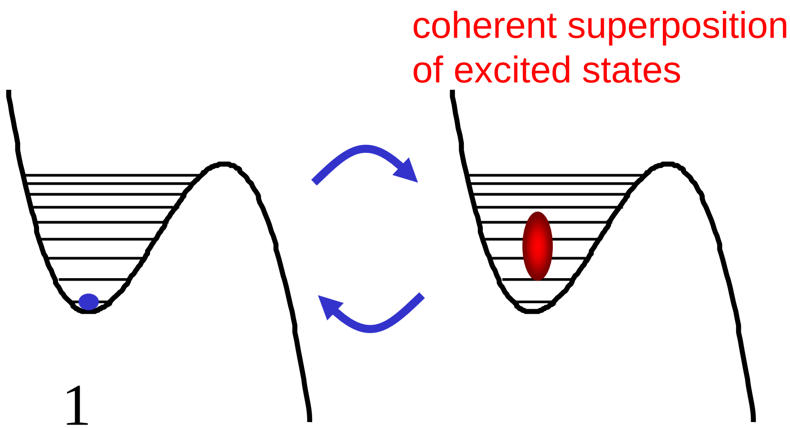
Multi-level dynamics

Cross-over between two and multi-level

(J. Claudon, A. Zazunov, F. Hekking, and O. Buisson, arXiv:0709.3787)



quantum theory with $\nu=\nu_{01}$



$$\frac{1}{T_{2,Rabi}}$$

Cross- over condition:

Anharmonicity ⟷ MW amplitude

$\nu_{01}-\nu_{12}$ Ω_1

- Low excitation power :
  two level description
- Intermediate power :
  multi- level description

Conclusion

Rabi like oscillations are not a quantum signature

Ω_{RLO} versus MW amplitude Ω_1 contains quantum signature

Classical theory does not explain the low amplitude dynamics when less than 4 levels are involved

Cross-over between two level and multi level dynamics

$$\nu_{01}-\nu_{12} = \Omega_1$$

Conditions to observe quantum effect in the Rabi oscillations:

$$\nu_{01}-\nu_{12} < 1/T_{2,Rabi}$$

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Introduction of the two coupled qubits circuit

Interaction between two quantum systems : very rich physics

Coupled qubits circuit to realize two qubits gate operations
(Control-NOT, i-SWAP,…)

Different circuits were considered recently:
Two charge qubits(NEC), flux qubits(Delft), phase qubits (Santa Barbara)
quantronium (Saclay), phase qubits coupled by cavity bus (Yale,Boulder)

Fixed coupling by capacitance, inductance, cavity

Ideal procedure:
- qubits stay at the optimal points
- single qubit operation with coupling off
- two qubits operation with coupling on

Tunable coupling at the optimal points

Tunable inductive coupling between two flux qubits (NEC, Berkeley)

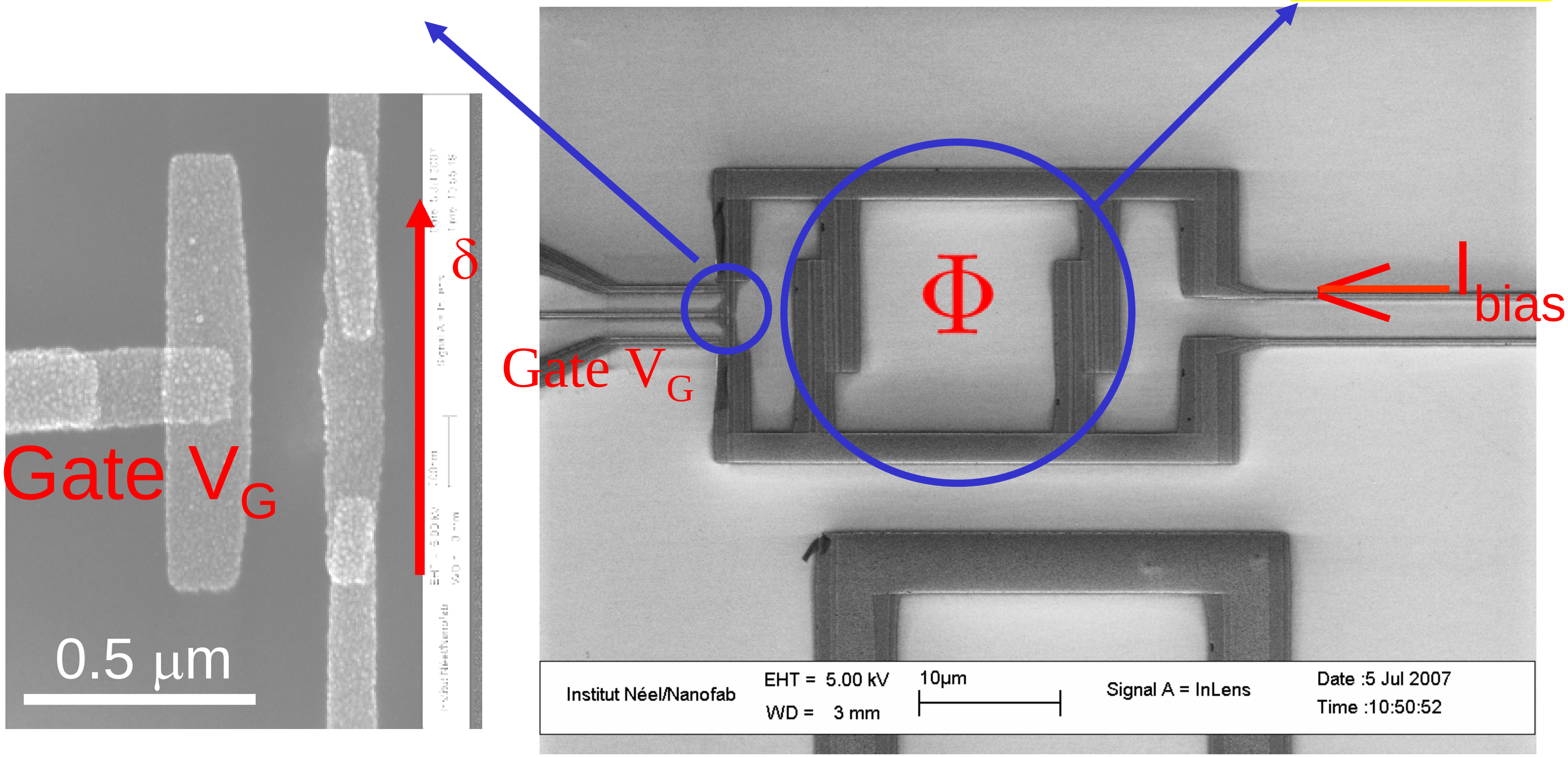
Tunable coupling between a phase qubit and a charge qubit

The coupled circuit

(A. Fay, W. Guichard, E. Hoskinson, F. Hekking, L. Lévy, and OB, PRL07)

Asymmetric Cooper pair transistor (Charge qubit)

dc SQUID
(Phase qubit)



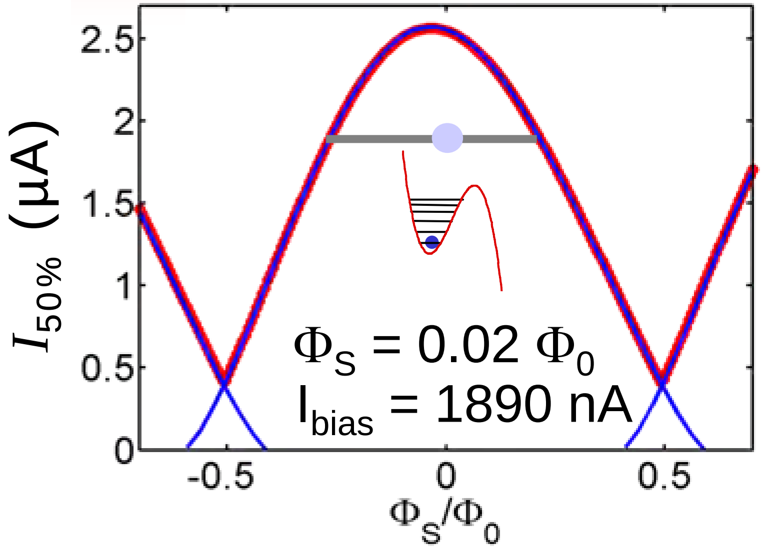
NanoFab

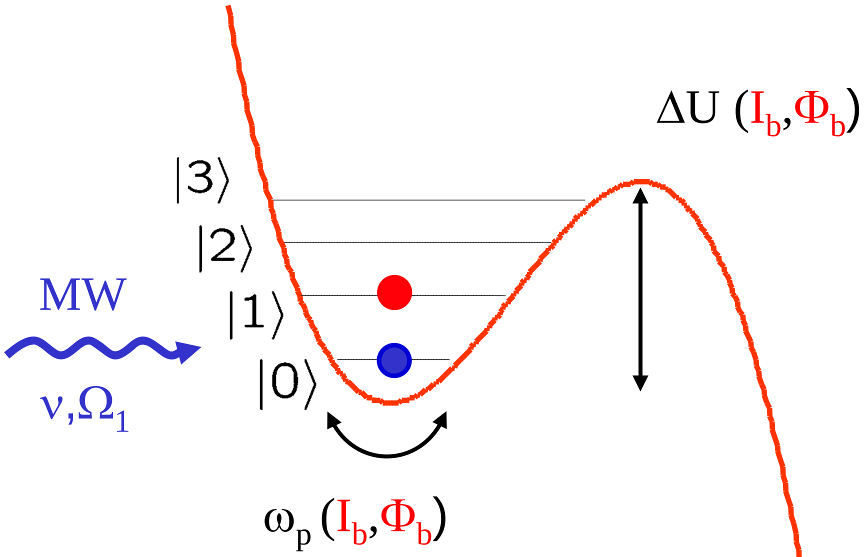
- SQUID Josephson junction size~ 10 μm²
- Transistor Josephson junction size~ 0.02 μm²

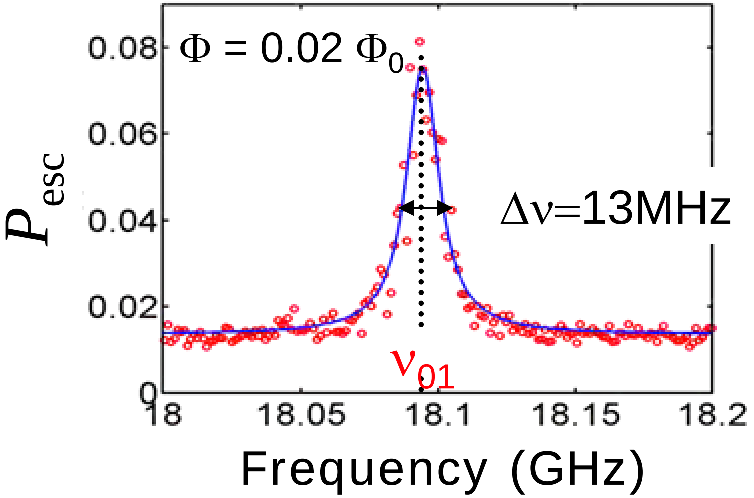
Asymmetry ⟶ coupling between the two qubits

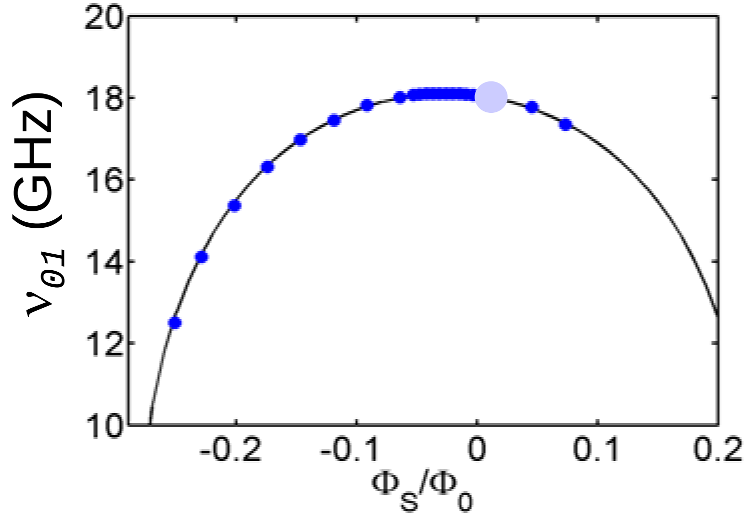
Current biased dc SQUID: a phase qubit

Low MW amplitude ➡ two level system









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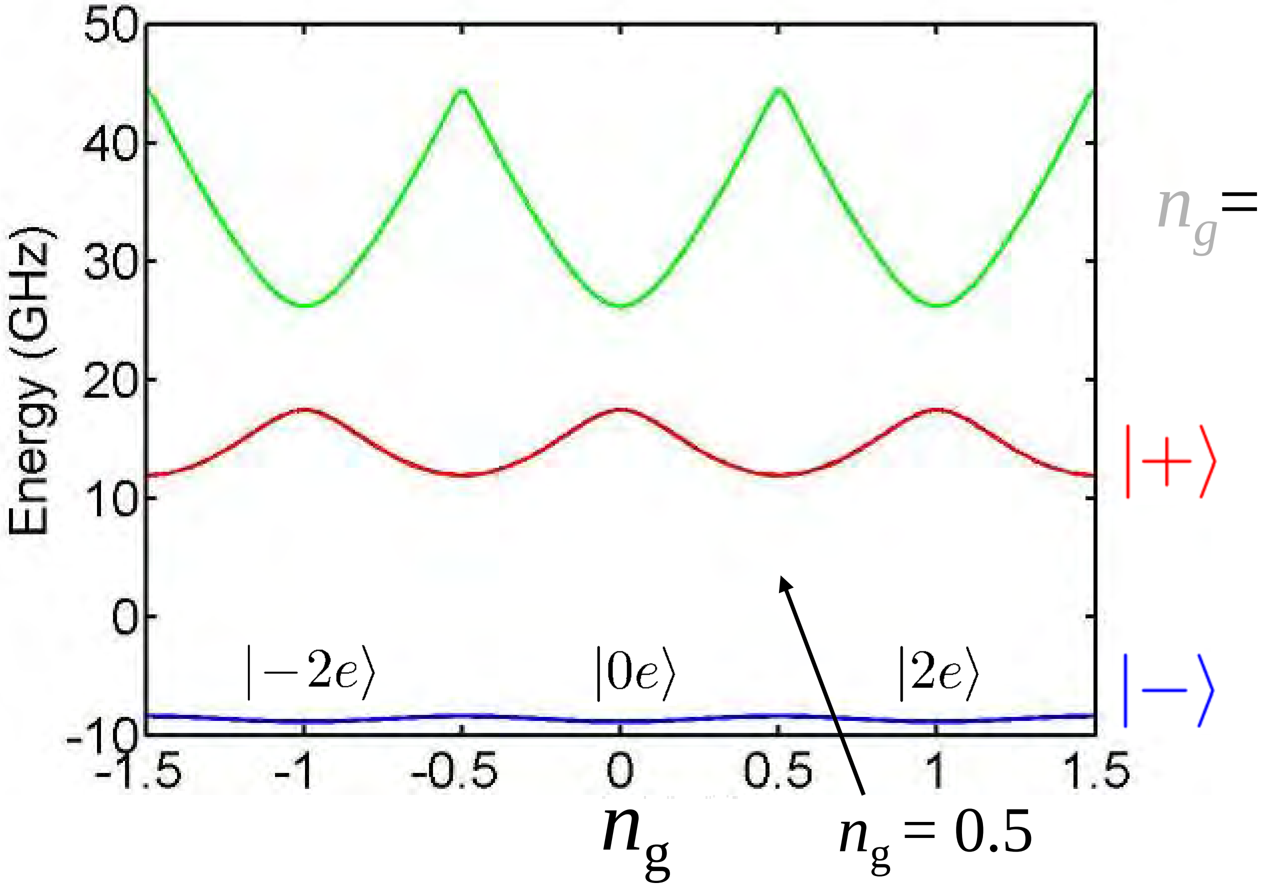
Asymmetric Cooper pair transistor: charge qubit

$$\hat{H}_T = \frac{(2e)^2(\hat{\mathbf{n}}-n_g)^2}{} - (E_J(\delta)|0e\rangle\langle 2e| + E_J^*(\delta)|2e\rangle\langle 0e|)$$

$$\boxed{\hat{H}_T = h\nu_T\hat{\sigma}_z^T}$$

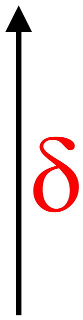
Charge energy

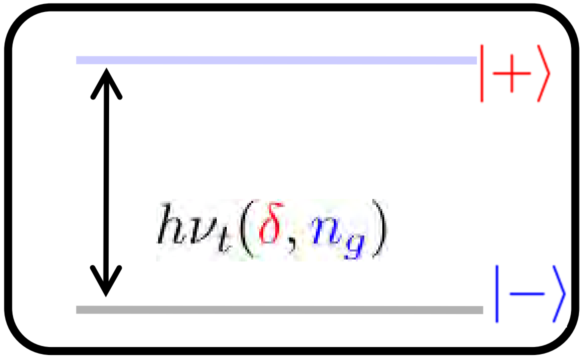
Josephson energy



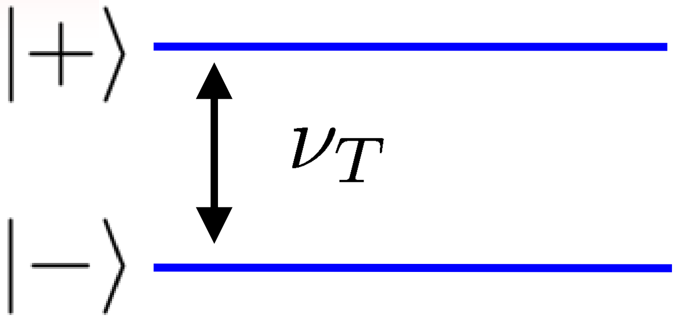
$n_g = 0.5$

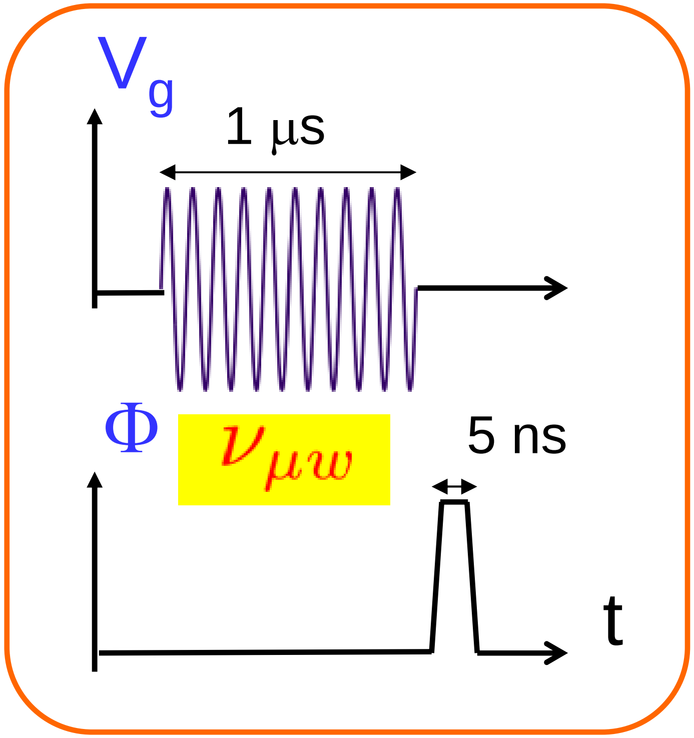
$n_g = C_gV_g/2e$

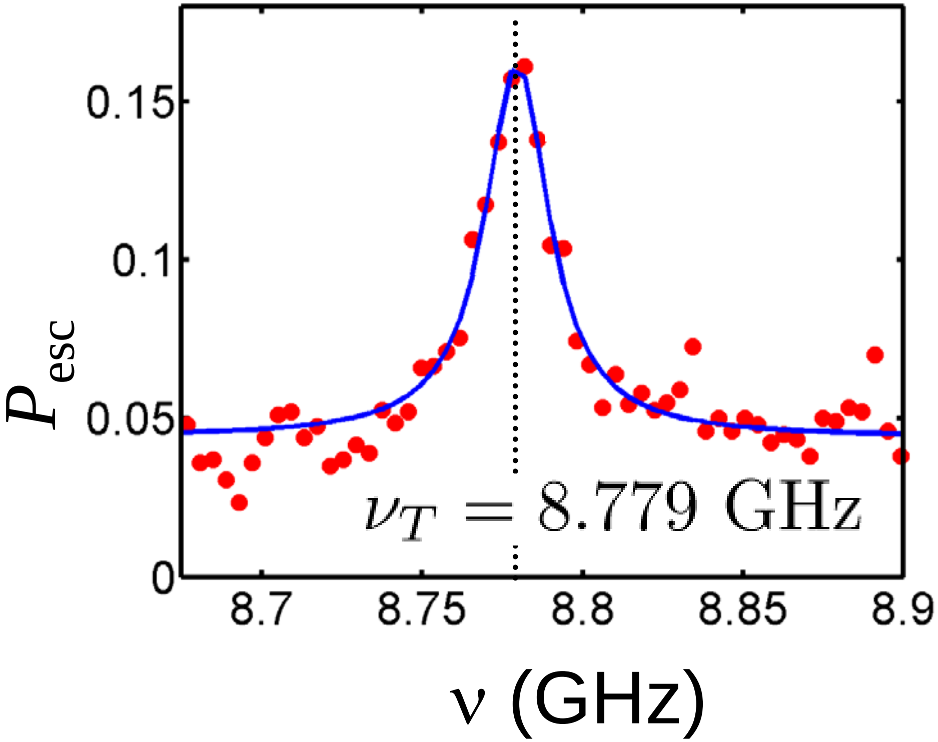




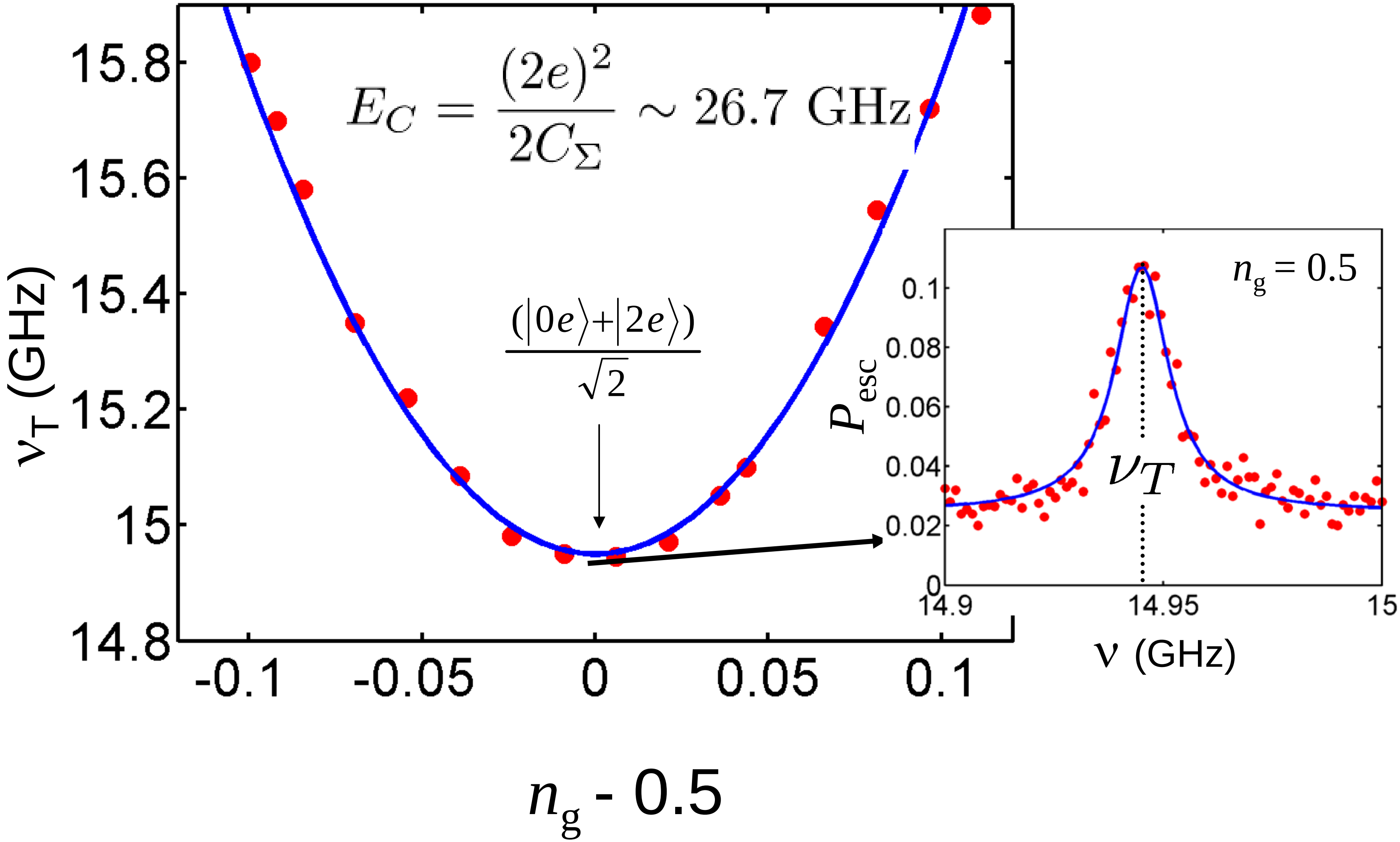
ACPT spectroscopy



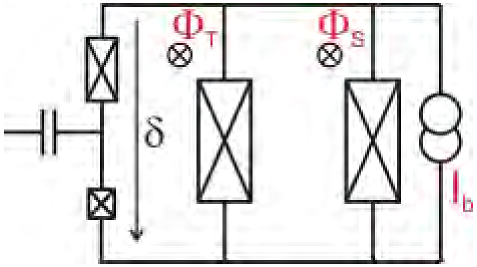




Transistor frequency versus *n*g

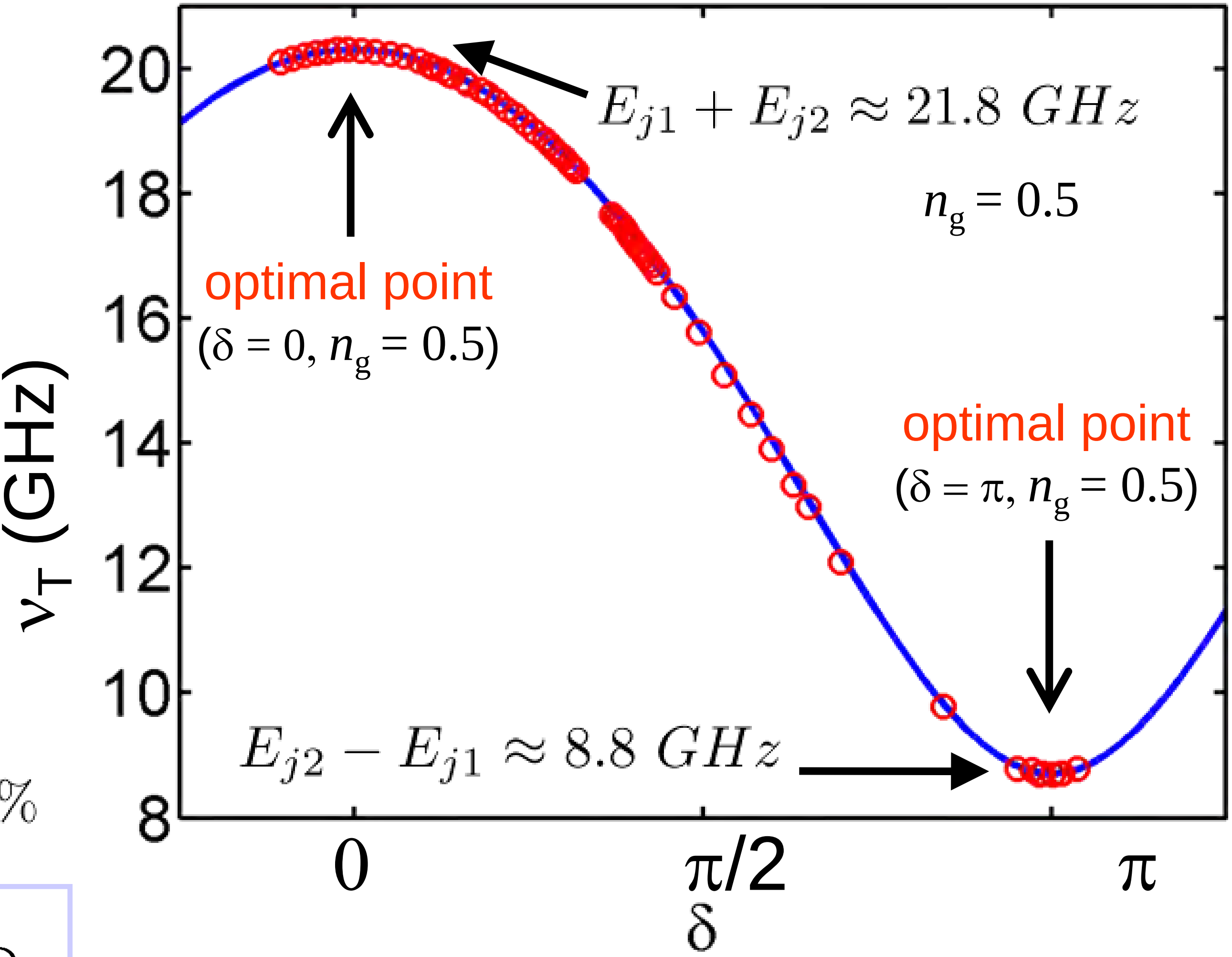


Transistor frequency versus δ



$$\mu = \frac{E_{j1} - E_{j2}}{E_{j1} + E_{j2}} = 41.9\%$$

$$\boxed{E_j / E_c \sim 0.8}$$



$E_{j1} + E_{j2} \approx 21.8\ GHz$

$n_g = 0.5$

optimal point

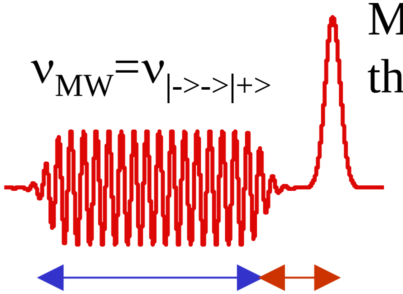
($\delta = 0$, $n_g = 0.5$)

optimal point

($\delta = \pi$, $n_g = 0.5$)

$E_{j2} - E_{j1} \approx 8.8\ GHz$

Properties at the new optimal point

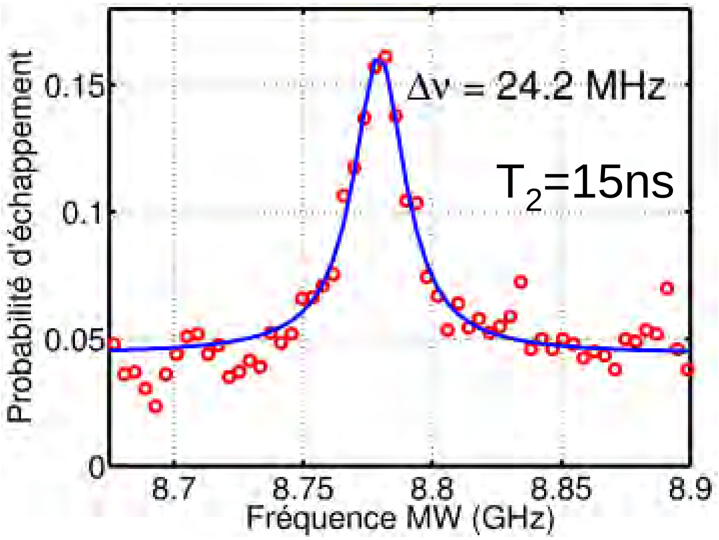


$\nu_{MW}=\nu_{|->->|+>}$

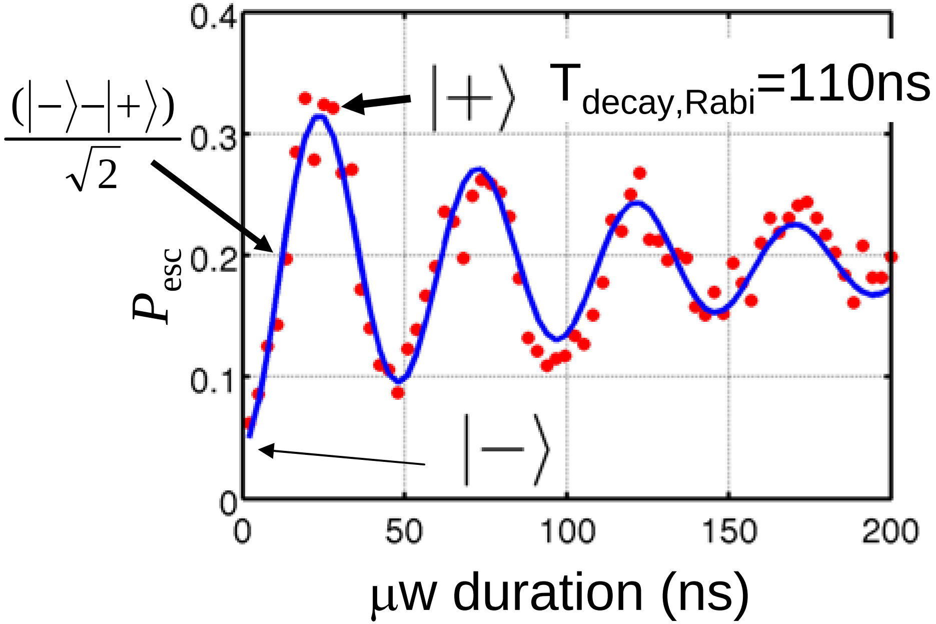
Measurement on the SQUID

T_{MW}: tunable

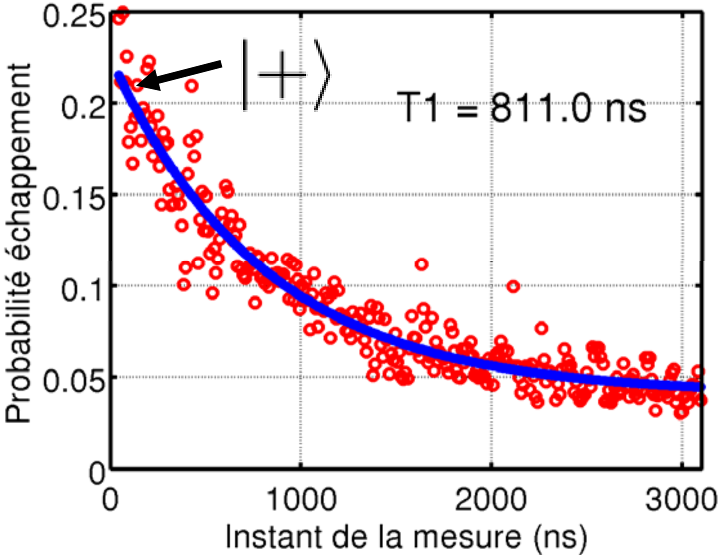
T_{delay}



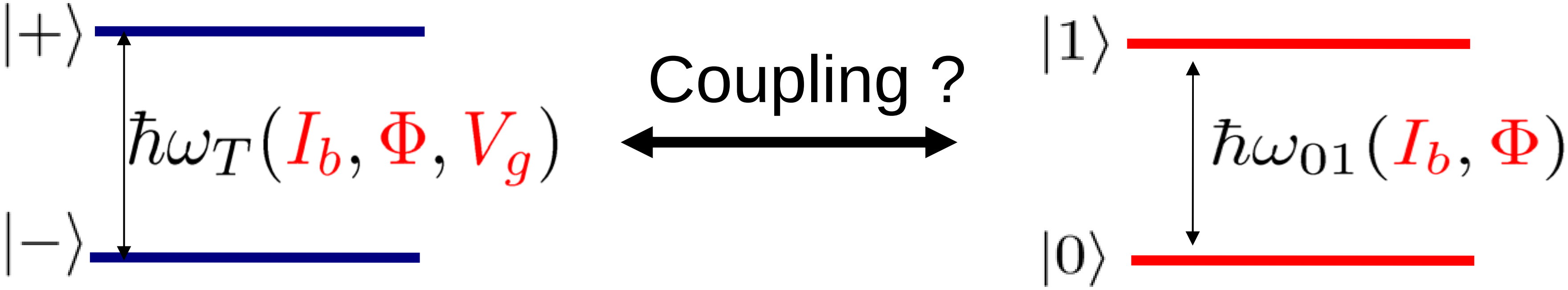
Rabi oscillations :



Energy relaxation



Coupling between the two qubits



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Driven anharmonic oscillator

- Introduction on Josephson junction
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- multilevel quantum system
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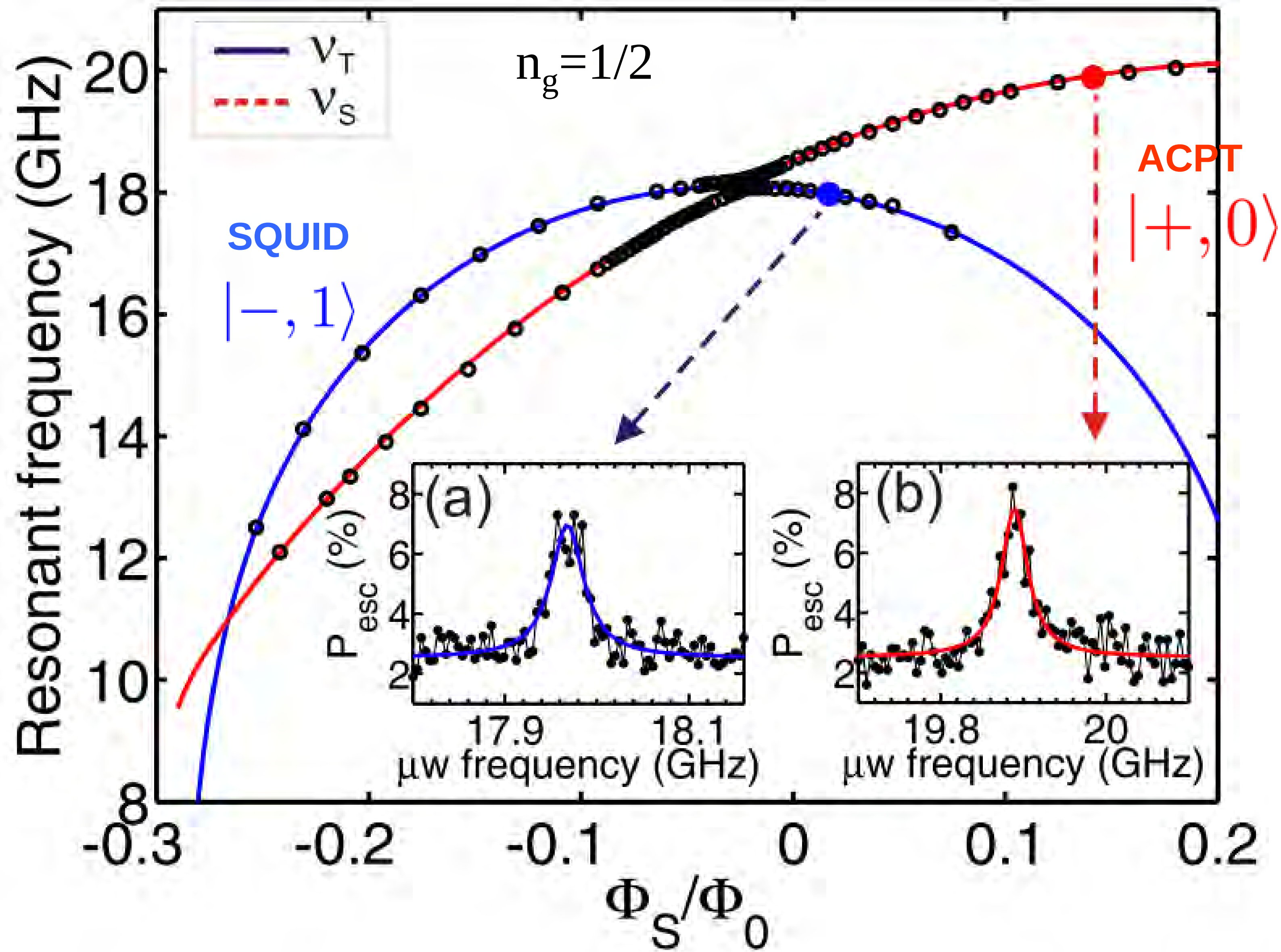
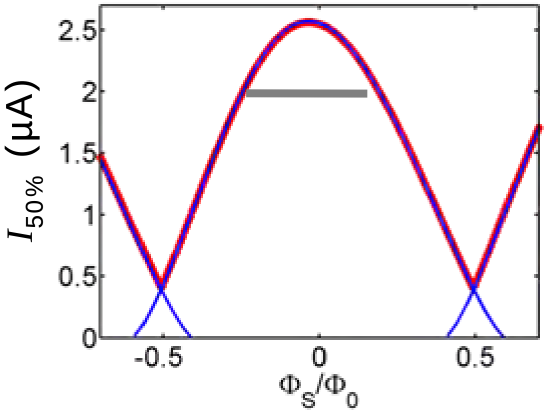
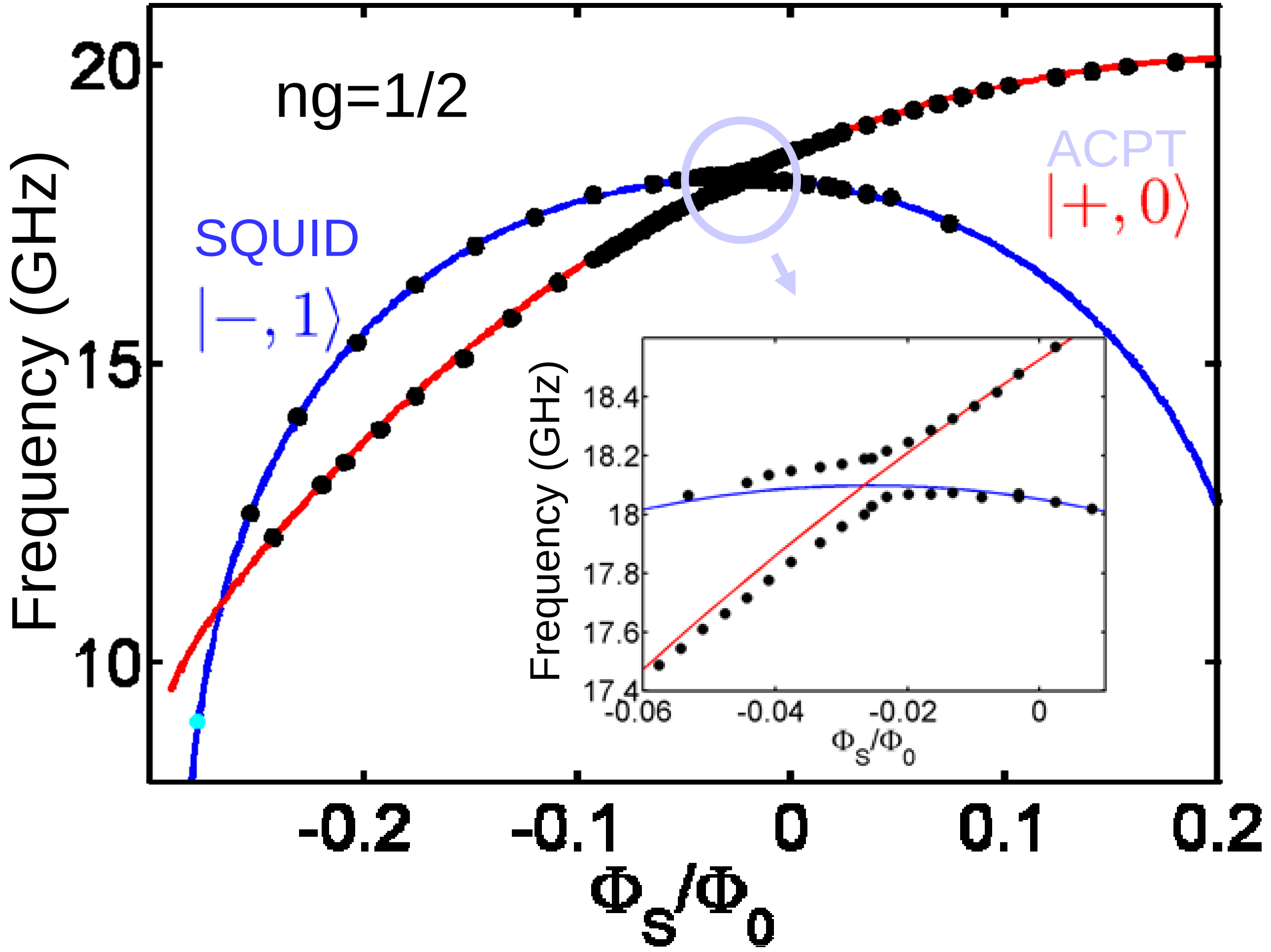
Spectroscopy measurement of the two quantum systems



Coupled qubits spectroscopy

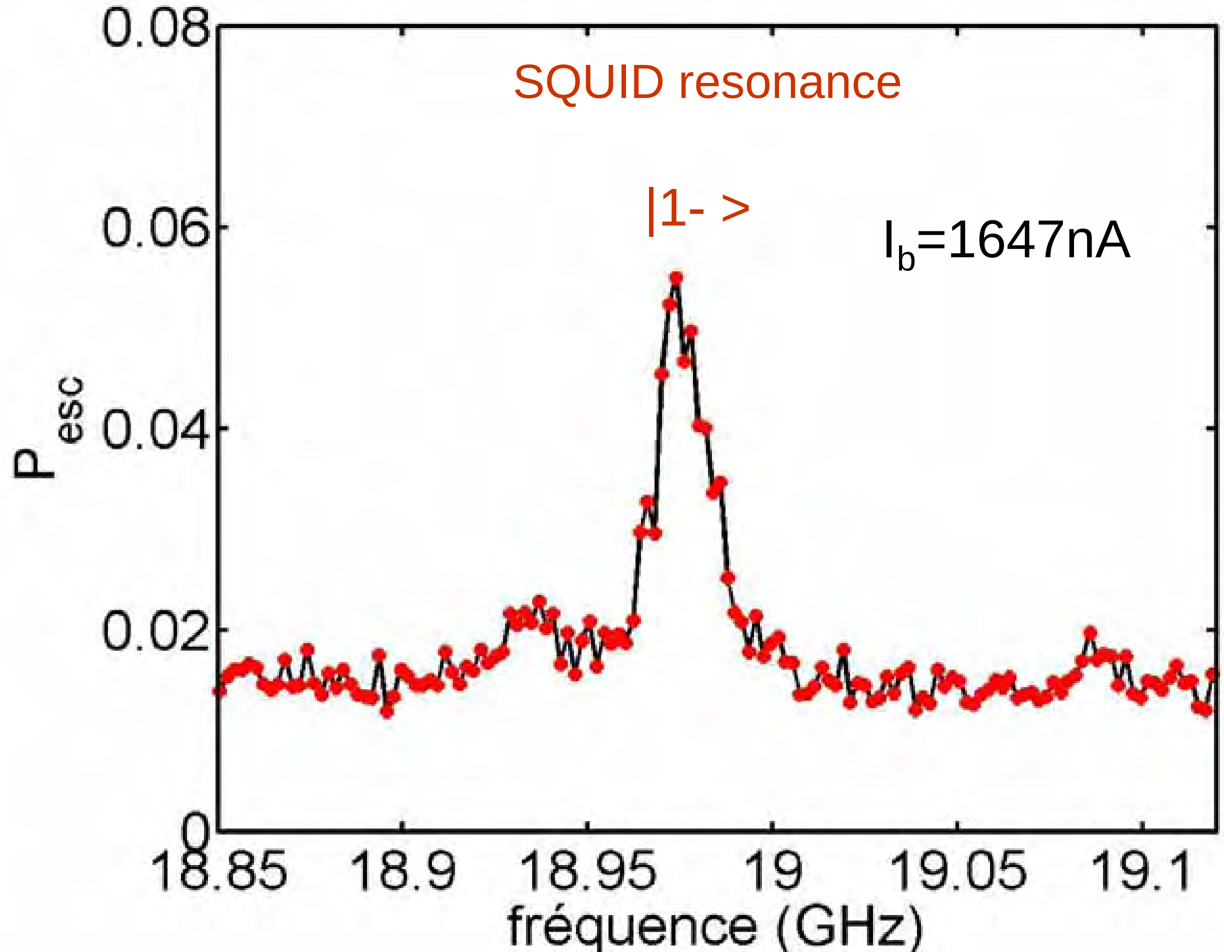
Spectroscopy at I_{bias} = 1890 nA

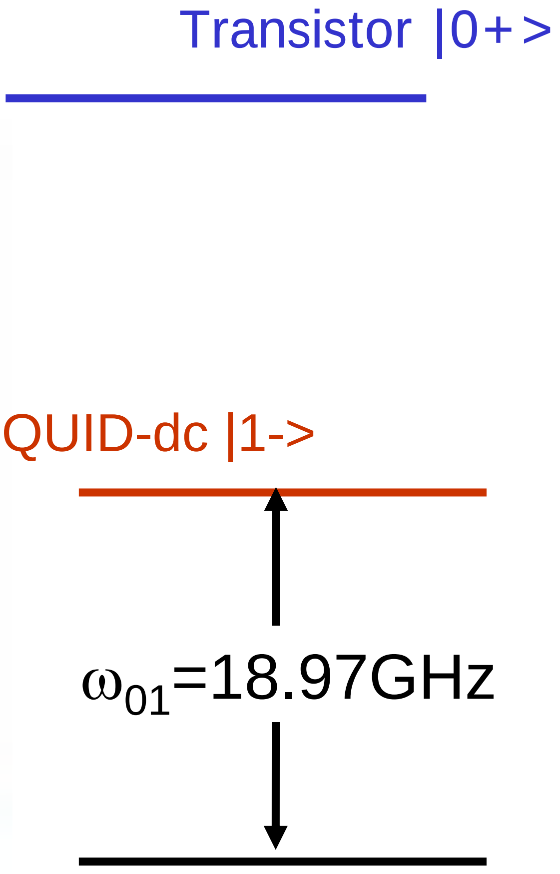




Entangled states between the two qubits

At $n_g \neq 1/2$



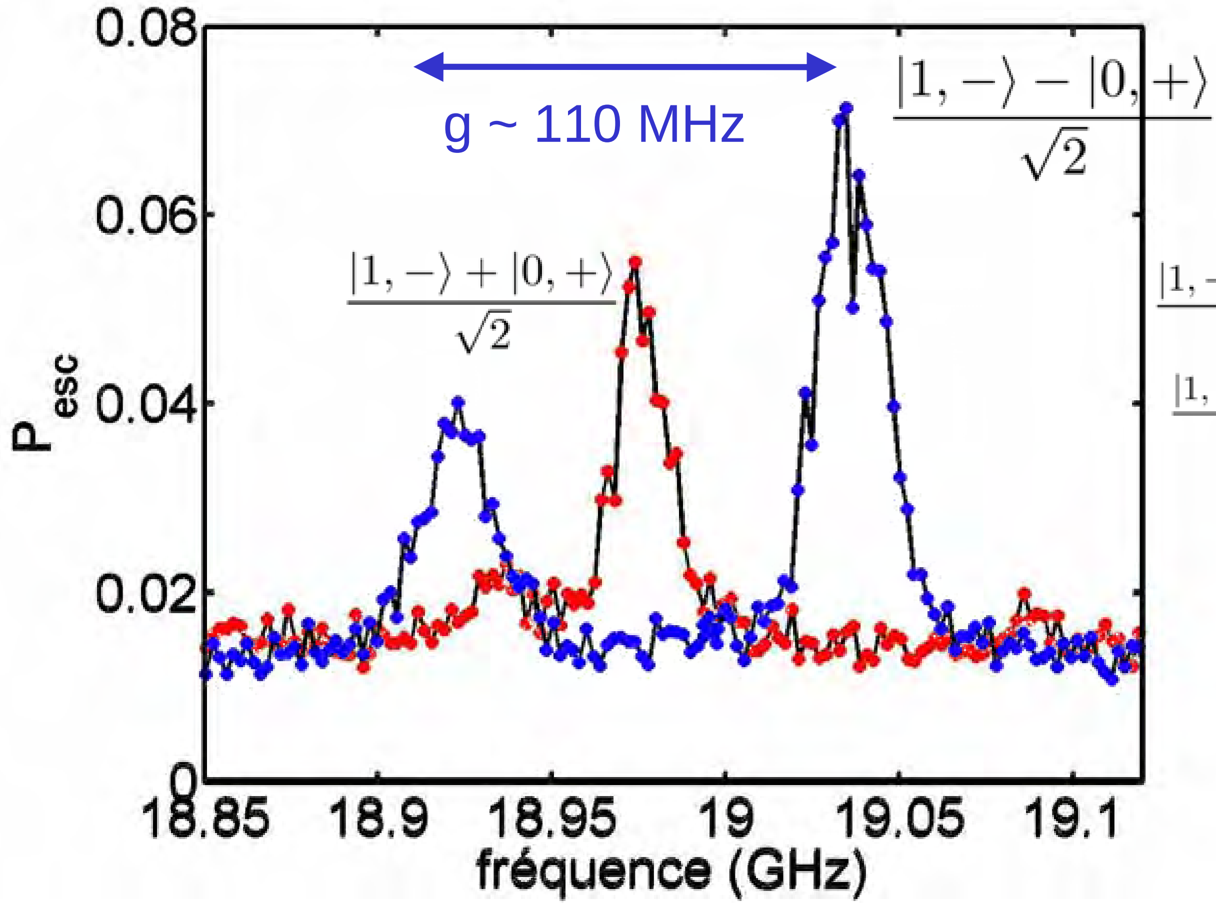

Transistor |0+>

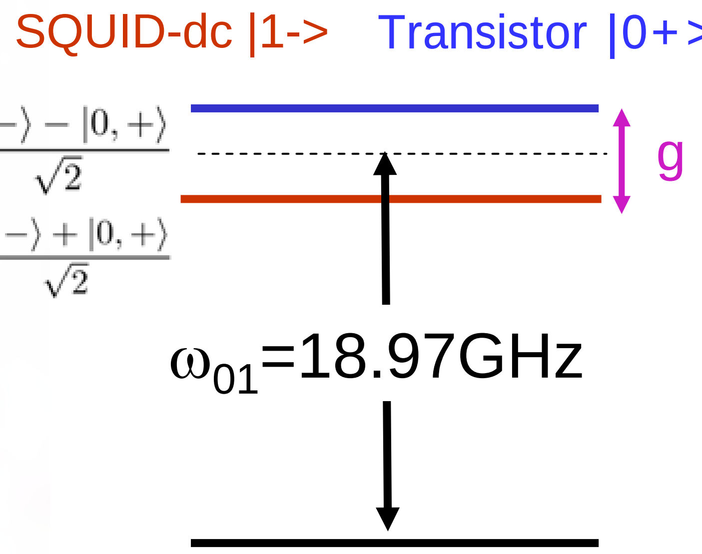
SQUID-dc |1->

ω_{01}=18.97GHz

Entangled states between the two qubits

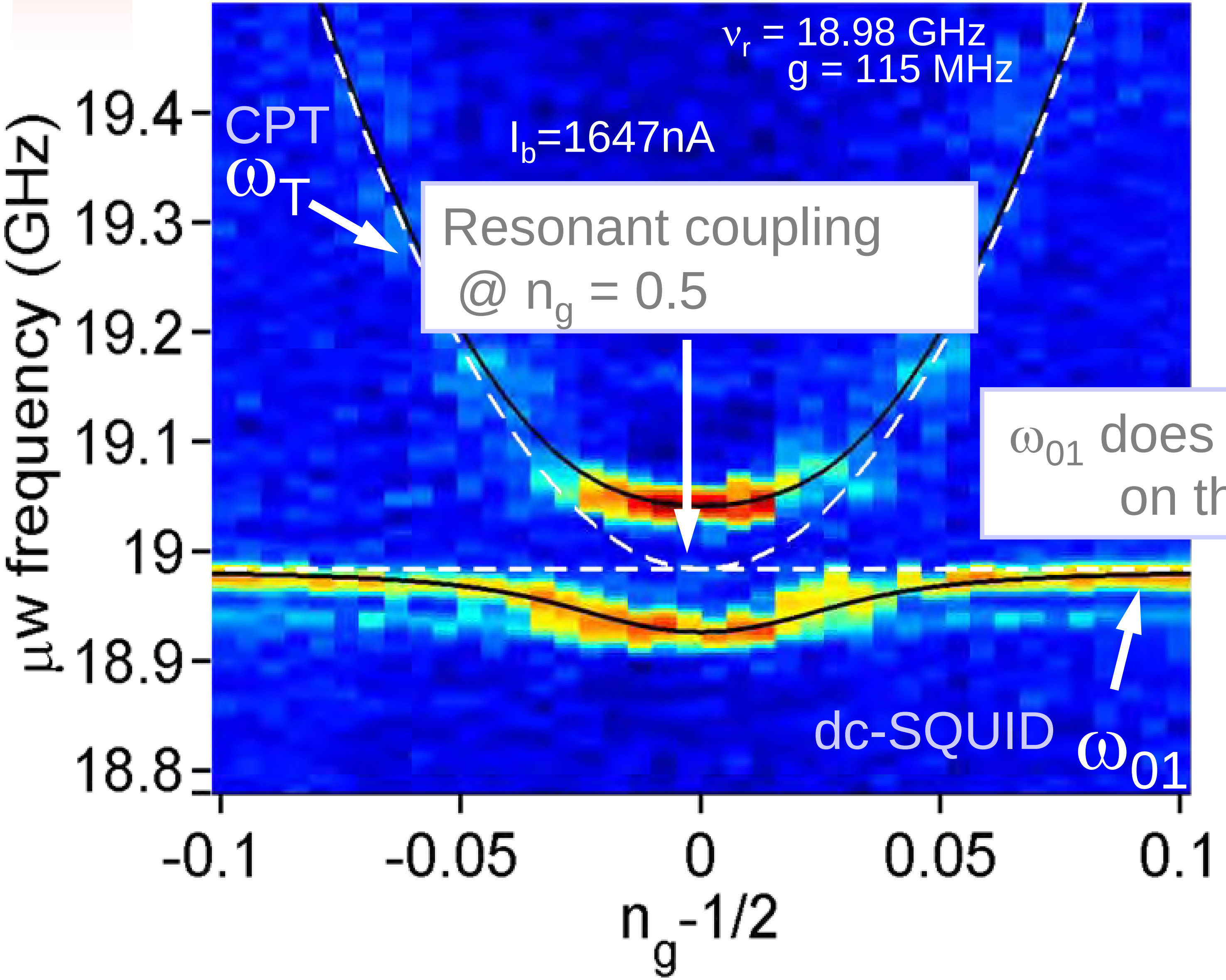
At n_g=1/2   resonant coupling at this working point





ω_{01}=18.97GHz

Spectroscopy versus V_g



Resonant coupling @ n_g = 0.5

ω_{01} does not depend on the gate

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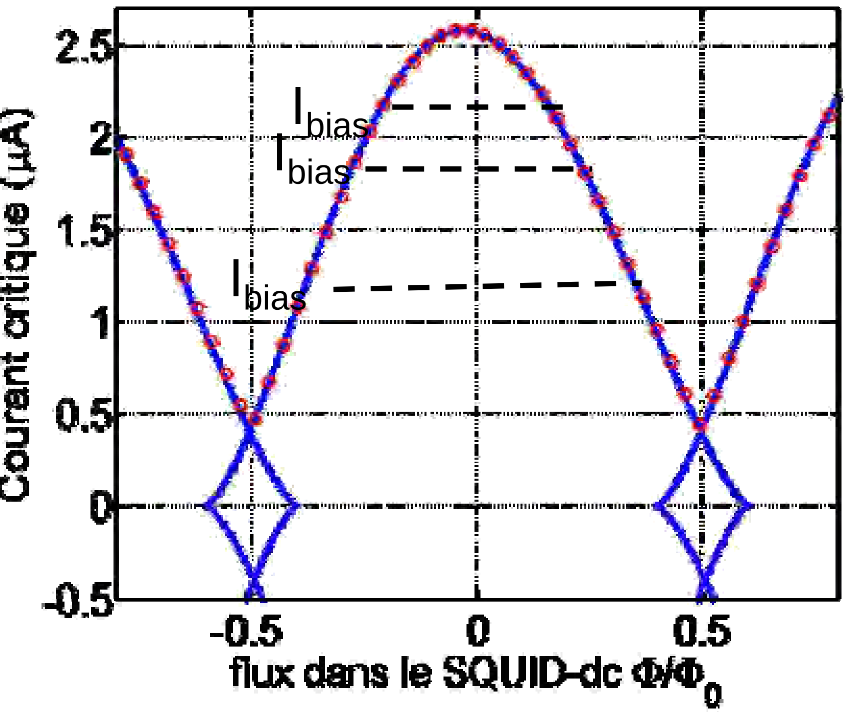
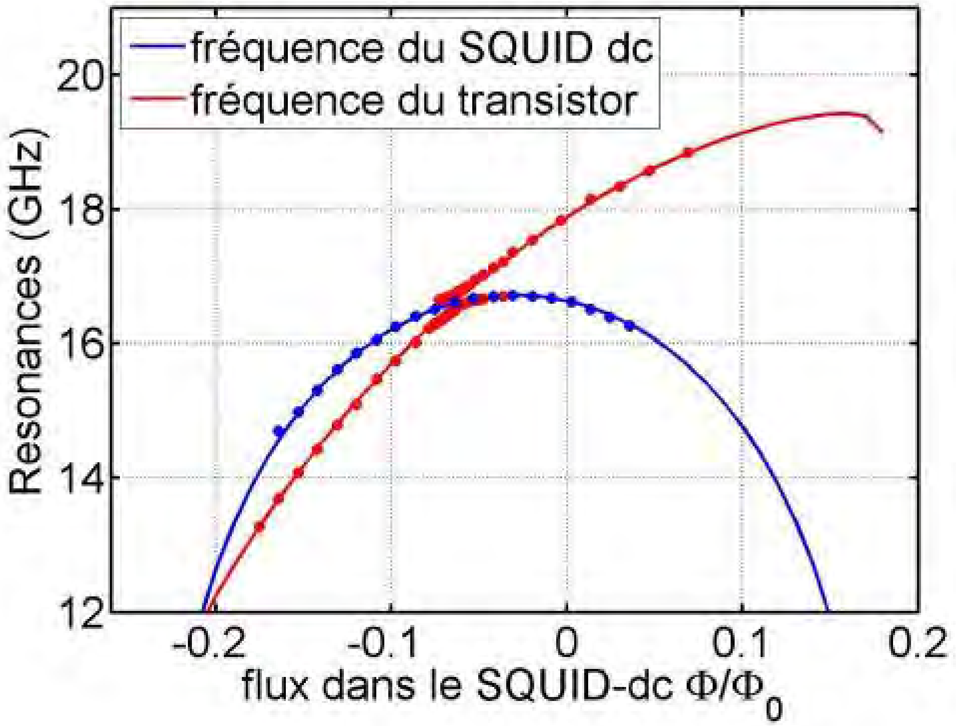
Spectroscopy of the coupled circuit

Spectroscopy @ I_{bias} = 2160 nA

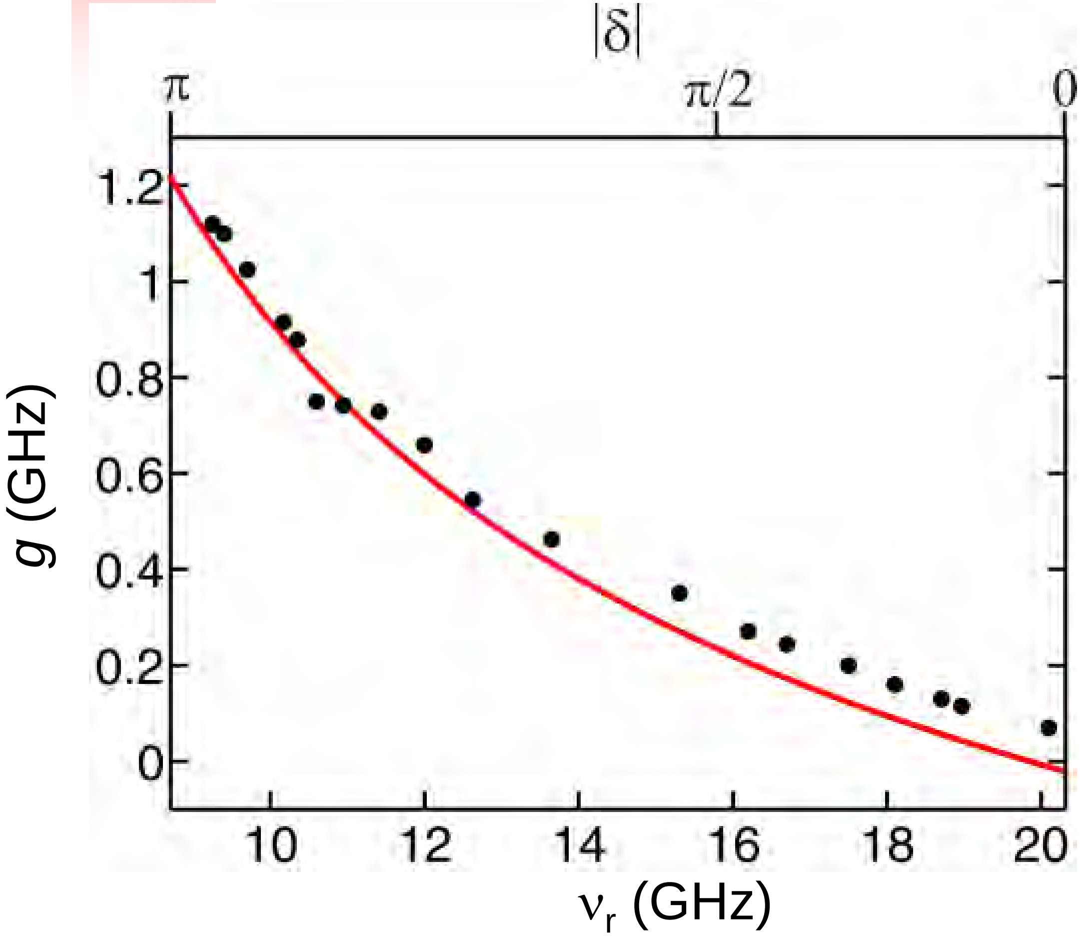




- SQUID and ACPT frequency are tunable over a large frequency range!
- The resonant frequency between the two qubits varies from 9 to 20.3 GHz !

Tunable coupling experience versus theory

(A. Fay, W. Guichard, E. Hoskinson, F. Hekking, L. Lévy, and OB, PRL07)



At the resonance:

$$H_{coupling} = 1/2\,hg\left(\sigma_S^+\sigma_T^- + \sigma_S^-\sigma_T^+\right)$$

Capacitive and Josephson coupling:

$$hg = \left(E_{c,c}/2 - E_{c,j}\cos(\delta/2-\chi)\right)$$

$$E_{c,c} = \sqrt{\frac{E_C^{\ S}}{\hbar\omega_p}}(1-\lambda)\hbar\omega_p$$

$$E_{c,j} = (1-\mu)\sqrt{\frac{E_C^{\ S}}{\hbar\omega_p}}E_j^{\ T}/2$$

$$\lambda = (C_1^T - C_2^T)/(C_1^T + C_2^T)$$

$$\mu = (e_{j,1}^T - e_{j,2}^T)/E_j^T$$

$$\tan(\chi) = \mu\tan(\delta/2)$$

We consider : λ=μ=41.9%

If symmetric transistor, λ=μ=0 No coupling!!

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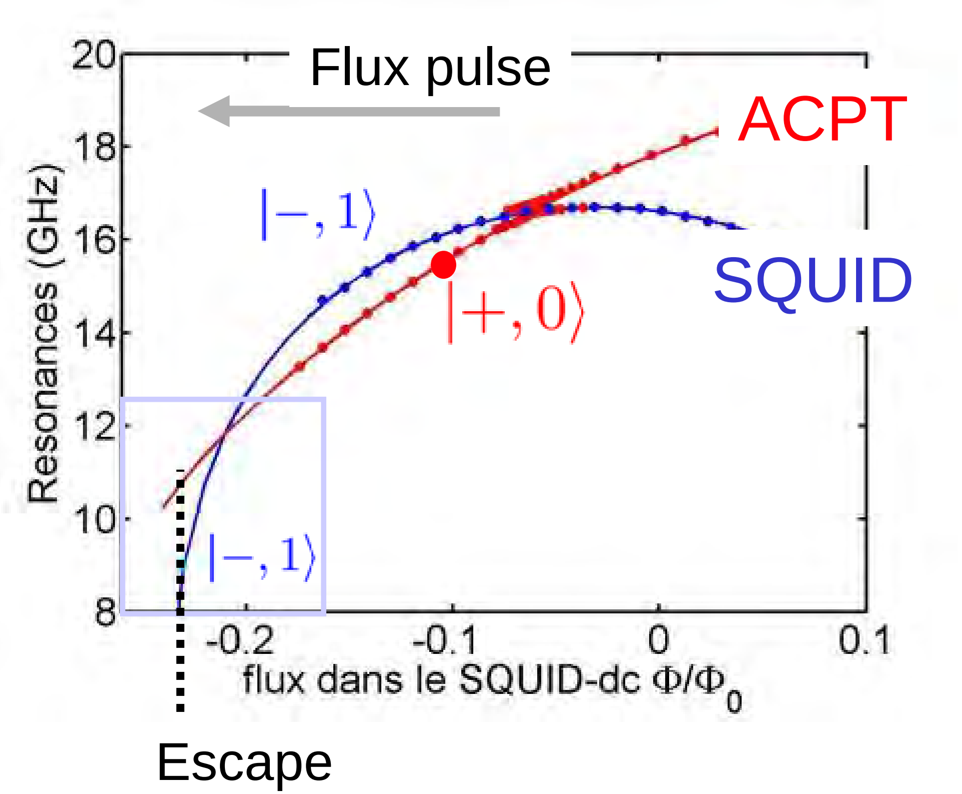
Charge qubit read-out

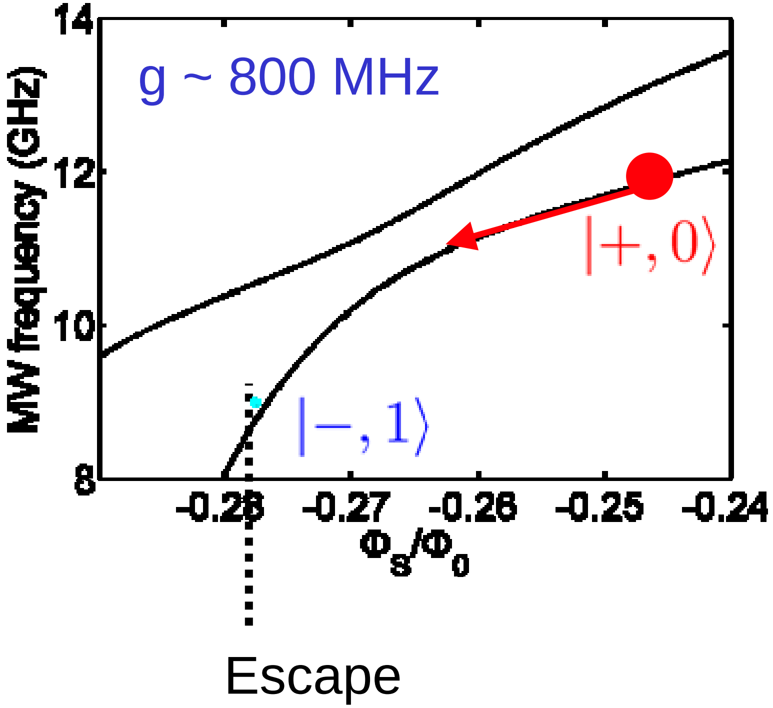
Aurelien Fay Thesis

Quantronium read-out : classical Josephson junction $\omega_{01}<<\omega_T$

In our case: $\omega_{01}\approx\omega_T$!!!



Adiabatic quantum transfer



The |0+> state is transfered to the |1-> state

Measured contrast > 30% (not optimized)

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