INTRODUCTION À LA PHYSIQUE MÉSOSCOPIQUE: ÉLECTRONS ET PHOTONS

INTRODUCTION TO MESOSCOPIC PHYSICS: ELECTRONS AND PHOTONS

Première Leçon / First Lecture
AIM OF THESE TWO LECTURES

Discuss a selection of basic concepts of mesoscopic physics and contrast their treatment with that of atomic physics

OUTLINE

1. General remarks on mesoscopic physics
   a) fundamental constants
   b) survival of quantum effects in macrosystems
2. What are "electrons"?
   a) example of mesoscopic resistor
   b) screening
   c) finite lifetime
3. What are "photons"?
   a) example of transmission line
   b) longitudinal and transverse modes
   c) 1-D Planck's law
4. Conclusion: quantum transport v.s. quantum optics
General remarks

Purpose: explain the novelty of mesoscopic phenomena by discussing status of variables and parameters
MACRO versus micro

MACRO SYSTEM

- classical mechanics
- many particles, strongly coupled to environment
  
\[ H = \frac{p_\theta^2}{2M} + \frac{k\theta^2}{2} + \ldots \]

- can have engineered structural parameters

micro system

- quantum mechanics
- few particles, almost isolated from environment
  
\[ \hat{H} = \frac{\hat{p}^2}{2m_e} - \frac{e^2}{4\pi\varepsilon_0\hat{r}} + \ldots \]

- always "God-given" structural parameters
MESOSCOPIC PHENOMENA:

Number of particles: macroscopic
Collective degrees of freedom: quantum

High-Q nanomechanical resonators @ low temperatures: clearly mesoscopic
Superconducting magnets for MRI: clearly not mesoscopic

Characteristic energies in mesoscopic phenomena involve not only fundamental constants, but also geometric dimensions adjusted by design and fabrication
# REVIEW OF UNIVERSAL CONSTANTS

**S. I. Units:** s, m, kg, A, K

## Classical constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light</td>
<td>$c$</td>
</tr>
<tr>
<td>impedance of vacuum</td>
<td>$Z_{\text{vac}} \approx 377\Omega$</td>
</tr>
<tr>
<td>electrical permittivity of vacuum</td>
<td>$\varepsilon_0 = \frac{1}{cZ_{\text{vac}}}$</td>
</tr>
<tr>
<td>magnetic permeability of vacuum</td>
<td>$\mu_0 = \frac{Z_{\text{vac}}}{c}$</td>
</tr>
<tr>
<td>Boltzmann's constant</td>
<td>$k_B$</td>
</tr>
<tr>
<td>gravitational constant</td>
<td>$G$</td>
</tr>
</tbody>
</table>

## Quantum constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>charge quantum</td>
<td>$e$</td>
</tr>
<tr>
<td>action quantum</td>
<td>$\hbar$</td>
</tr>
<tr>
<td>flux quantum</td>
<td>$\Phi_0 = \frac{\hbar}{2e}$</td>
</tr>
<tr>
<td>resistance quantum</td>
<td>$R_K = \frac{\hbar}{e^2} \approx 26k\Omega$</td>
</tr>
<tr>
<td>fine structure cst.</td>
<td>$\alpha = \frac{Z_{\text{vac}}}{2R_K} \approx \frac{1}{137}$</td>
</tr>
</tbody>
</table>
# Universal and Microscopic Quantum Constants

<table>
<thead>
<tr>
<th>Universal constants</th>
<th>Microscopic constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>charge of electron: $e$</td>
<td>mass of electron: $m_e$</td>
</tr>
<tr>
<td>Planck’s constant: $\hbar$</td>
<td>magn. mt. of electron: $\mu_B$</td>
</tr>
<tr>
<td>flux quantum: $\Phi_0 = \frac{\hbar}{2e}$</td>
<td>mass of proton: $m_p$</td>
</tr>
<tr>
<td>resistance quantum: $R_K = \frac{\hbar}{e^2} \approx 26\Omega$</td>
<td>mass of neutron $m_n$</td>
</tr>
<tr>
<td>fine structure cst.: $\alpha = \frac{Z_{vac}}{2R_K} \approx \frac{1}{137}$</td>
<td>magn. mt. of proton $\mu_p$</td>
</tr>
</tbody>
</table>

**07-1-7**
FORMS OF CHARACTERISTIC ENERGIES

\[ \hbar \omega = 20 \text{ GHz} \]

\[ eV = 80 \mu \text{V} \]

\[ k_B T = 1 \text{K} \]
USUAL ENERGY SCALES OF MICROSCOPIC QUANTUM EFFECTS

Atomic matter

\[ E_n = \frac{-Ry}{n^2} \]

\[ Ry = \frac{m_e c^2}{2} \alpha^2 = 13.6\text{eV} \]

\[ \Delta E \sim \text{eV} \]

Bulk condensed matter

\[ E_{\text{gap}} \sim \text{eV} \]

\[ E_{\text{Debye}} \sim 10\text{meV} \]

\[ \Delta_{sc} = E_{\text{Debye}} e^{-\frac{1}{N(0)V}} \sim \text{meV} \]

IN MESOSCOPIC QUANTUM EFFECTS, NEW ENERGY SCALES EMERGE!
MESOSCOPIC PHYSICS
CHALLENGES COMMON WISDOM
ABOUT THE SUPPRESSION OF COLLECTIVE
QUANTUM EFFECTS IN LARGE AND
DIRTY SYSTEMS

• THERMAL ENERGY CAN BE BIGGER THAN SINGLE PARTICLE
  ENERGY LEVEL SPACINGS

• DISORDER DOES NOT FULLY SUPPRESS INTERFERENCES,
  IT IS DECOHERENCE WHICH IS IMPORTANT

• A SINGLE ELECTRON CAN COUNT

• LOCAL CIRCUIT THEORY WILL UNEXPECTEDLY FAIL
MESOSCOPIC PHYSICS
CHALLENGES COMMON WISDOM
ABOUT THE SUPPRESSION OF COLLECTIVE
QUANTUM EFFECTS IN LARGE AND
DIRTY SYSTEMS

• THERMAL ENERGY CAN BE BIGGER THAN SINGLE PARTICLE
  ENERGY LEVEL SPACINGS

• DISORDER DOES NOT FULLY SUPPRESS INTERFERENCES,
  IT IS DECOHERENCE WHICH IS IMPORTANT

• A SINGLE ELECTRON CAN COUNT

• LOCAL CIRCUIT THEORY WILL UNEXPECTEDLY FAIL
EXAMPLE OF COMMON WISDOM AT WORK: THE CROSSOVER TO DULONG-PETIT LAW

\[ \frac{C_V}{Nk_B T} \]

\( k_B T \approx \hbar \omega_{\text{Debye}} \)

quantum \quad \text{classical}
2-DIMENSIONAL ELECTRON GAS SYSTEM

n-AlGaAs i-GaAs
gate electrode
contact electrode

before charge transfer
after charge transfer

a 2DEG forms here!
PINCHING THE 2DEG INTO A SMALL WIRE

Ballistic transport: $L \ll \ell_e$

what if $k_B T \ll \frac{\hbar v_F}{d}$?
CONDUCTANCE QUANTIZATION

\[
G / \frac{e^2}{h} \rightarrow d
\]

\[
G = \left( \frac{dI}{dV} \right)_{eV / k_B T \rightarrow 0}
\]

waveguide effect!

\[
k_B T \ll \frac{\hbar v_F}{d}
\]

van Wees et al. 1988:
UP TO WHAT SIZE CAN QUANTUM EFFECTS PERSIST IN PRESENCE OF DISORDER?

\[
\frac{\hbar}{\tau_\varphi} \ll \frac{\hbar \nu_F L}{L} \ll \frac{\hbar \nu_F k_F}{(Lk_F)^2} \ll \frac{\hbar \nu_F \ell_e}{L^2}
\]

Thouless energy

Can be much larger than level spacing in box of size \(L\)!
BOOKS ON MESOSCOPIC PHYSICS

S. Datta
Y. Imry
E. Akkermans and G. Montambaux
H. Grabert and M. Devoret

REQUISITES:
Quantum mechanics
Solid state physics
Statistical mechanics

Useful reference books on quantum statistical physics: Feynman; Schrieffer; Pines and Nozières
2. How mesoscopic physics models the "electron"

Purpose: provide groundwork for Landauer's approach of transport phenomena
THE MESOSCOPIC RESISTOR

Landauer reservoir: metallic electrode which injects into the quantum coherent region quasi-electron waves with well-defined chemical potential and temperature. It also accepts without reflection any quasi-electron waves and thoroughly recycles them. The Landauer reservoir is to Fermi waves what a black-body is to Bose waves.
Collection of independent channels

\[ \begin{align*}
E & \\
\mu_+ & \\
\mu_- & \\
f_+ & \\
f_- & \\
eV & 
\end{align*} \]
THE LANDAUER-BÜTTIKER FORMULA

\[ I = I_+ - I_- \]

\[ I_\pm = \frac{e}{\hbar} \sum_m \int_{-\infty}^{+\infty} f_\pm (E) \left| t_m (E) \right|^2 \, dE \]

\[ f_\pm (E) = \frac{1}{1 + \exp \left( \frac{E - \mu_\pm}{k_B T} \right)} \]

\[ \mu_+ - \mu_- = eV \]

schizophrenic! why does it work?
RESISTANCE DETERMINED BY ELASTIC TRANSMISSION COEFFICIENT!

WHAT ABOUT JOULE HEATING?
**QUESTION:** What are the differences between a quasi-electron (a.k.a. dressed electron) and the usual (bare) electron?

<table>
<thead>
<tr>
<th>PARTICLE IDENTIFICATION CARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Name: Electron</td>
</tr>
<tr>
<td>First name: Bare</td>
</tr>
<tr>
<td>Address: Vacuum</td>
</tr>
<tr>
<td>Genre: Fermion</td>
</tr>
<tr>
<td>Occupation: Wave packet</td>
</tr>
<tr>
<td>Lifetime: infinite</td>
</tr>
<tr>
<td>Average energy: $\hbar \omega$</td>
</tr>
<tr>
<td>Average momentum: $\hbar k$</td>
</tr>
<tr>
<td>Velocity: $v = \frac{d \omega}{dk}$</td>
</tr>
<tr>
<td>Mass: $\frac{\hbar k}{dv} = m_e$</td>
</tr>
<tr>
<td>Charge: $-e$</td>
</tr>
<tr>
<td>Spin: $S = 1/2$</td>
</tr>
<tr>
<td>Magnetic moment: $g_e S \mu_B$</td>
</tr>
</tbody>
</table>

$$g_e = 2 \left[ 1 + \frac{\alpha}{2\pi} + O(\alpha^2) \right]$$
An example of a Feynman diagram involving the usual electron and photon of atomic physics which propagate in vacuum.
THE "ELECTRON" OF MESOSCOPICS

PARTICLE IDENTIFICATION CARD

Last Name: Electron
Address: Metal
Occupation: Wave packet
Average energy: $\hbar \omega$
Velocity: $v = d\omega / dk$
Transverse charge: -e
Spin: $S = 1/2$
First name: Quasi
Genre: Fermion
Lifetime: finite, except @ $k_F$
Average momentum: $\hbar k$
Mass: $\hbar dk/dv = m_{\text{eff}}(k)$
Longitudinal charge: 0 ($q \rightarrow 0$)
Magnetic moment: $g_{\text{eff}} S \mu_B$
Definition of the longitudinal and transverse part of a field:

\[ \vec{F} = \vec{F}_l + \vec{F}_t \]

\[ \nabla \cdot \vec{F}_t = 0 \]

\[ \nabla \times \vec{F}_l = 0 \]

The longitudinal and transverse charges are the sources of the longitudinal and transverse parts of the electrical field, respectively.
SOLIDS

van der Waals

ionic

covalent

insulators

metal
\[ \langle v_x \rangle = \frac{(-e)E}{m} \tau \]

\[ j = n(-e)\langle v_x \rangle \]

\[ \sigma = \frac{j}{E} = \frac{ne^2}{m} \tau \]
PROBLEM: ELECTRONS ARE FERMIONIC WAVES!
REVIEW OF FREE FERMI GAS MODEL

Box volume $V$

Nb of fermions $N$

Total energy $E_K$

Length scale $a_0$

Energy scale $Ry$

$$a_0 = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}; \quad Ry = \frac{m_e e^4}{2\hbar^2 \cdot 4\pi\varepsilon_0}$$

$$\frac{N}{V} = \frac{n}{3} = \frac{4\pi}{3} \left(\frac{k_F^2}{2\pi}\right) = \frac{1}{4\pi a^3}; \quad r_s = \frac{a}{a_0}; \quad k_F = \frac{1.92}{r_s a_0}$$

$$\frac{E_K}{N} = \frac{3}{5} \left(\frac{\hbar k_F}{2m_e}\right)^2 = \frac{3}{5} E_F = \frac{2.22}{r_s^2} Ry$$

$$v_F = \frac{\hbar}{m_e} k_F = v_g \bigg|_{E_F}$$

$$\frac{E_C}{N} = \frac{1}{r_s} Ry$$
OTHER PROBLEM: ELECTRONS INTERACT STRONGLY!

... BUT TOO STRONG INTERACTION KILLS INTERACTION.....
Acknowledgements: D. Esteve, H. Pothier, D. Stone and C. Urbina