# Generation and measurement of multi-qubit entanglement in circuit quantum electrodynamics

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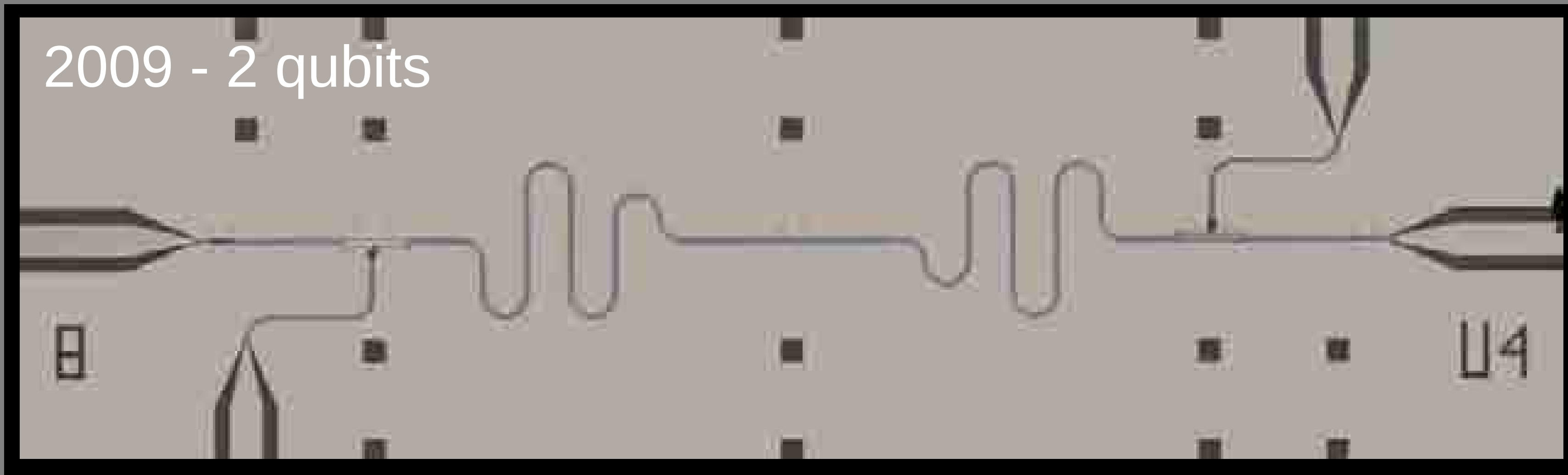


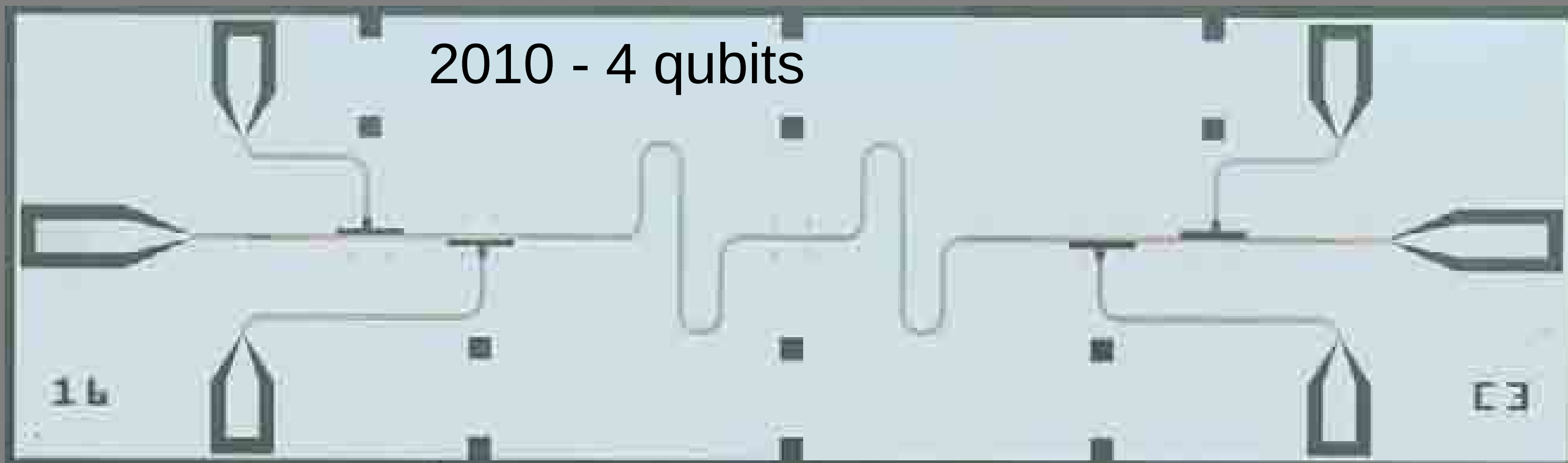


**PI's:** Robert Schoelkopf
Michel Devoret
Steven Girvin

# Collaborators

**Experiment:**
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**Theory:**
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Jens Koch
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Eran Ginossar

**PI's:** Robert Schoelkopf
Michel Devoret
Steven Girvin

# Outline

- What is quantum entanglement?

- How to detect it?
  the complete way: quantum state tomography
  the scalable way: entanglement witnesses

- Example: 2- & 3-qubit entanglement in cQED processors
  algorithmic generation using C-Phase gates
  detection by joint qubit readout

- Outlook

# What is entanglement?

*`Entanglement is simply Schrodinger's name for superposition in a multi-particle system.`*

*Greenberger, Horne & Zeilinger (GHZ), Physics Today 1993*

# Wavefunction description of pure two-qubit states

for $N$=2 qubits:

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$2^2$ complex numbers -
- normalization $\langle\psi|\psi\rangle = 1$
- irrelevant global phase

A pure *2*-qubit state is fully described by ***6*** real #s

# When are two qubits entangled?

Two qubits are entangled when their joint wavefunction cannot be split into a product of individual qubit wavefunctions

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \qquad \text{vs} \qquad |\psi\rangle = |\psi_1^a\rangle \otimes |\psi_2^a\rangle + |\psi_1^b\rangle \otimes |\psi_2^b\rangle$$

Some common terms:

Unentangled = separable = product state

Entangled = non-separable = non-product state

# Some separable & entangled states

| State | | Entangled? |
|---|---|---|
| $\lvert\psi\rangle = \lvert 1\rangle \otimes \lvert 1\rangle = \lvert 11\rangle$ | | no |
| $\lvert\psi\rangle = \frac{1}{\sqrt{2}}\left(\lvert 01\rangle - \lvert 10\rangle\right)$ | *The Bell singlet* | yes |
| $\lvert\psi\rangle = \frac{1}{2}\left(\lvert 00\rangle + \lvert 01\rangle + \lvert 10\rangle + \lvert 11\rangle\right) = \frac{1}{\sqrt{2}}\left(\lvert 0\rangle + \lvert 1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(\lvert 0\rangle + \lvert 1\rangle\right)$ | | no |
| $\lvert\psi\rangle = \frac{1}{2}\left(\lvert 00\rangle + \lvert 01\rangle - \lvert 10\rangle + \lvert 11\rangle\right)$ | | yes |

# Quantifying entanglement

Two qubits in a pure state

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

are entangled if they have nonzero *concurrence* $C$

$$C(\psi) = 2\left|c_{00}c_{11} - c_{01}c_{10}\right|$$

$$|\psi\rangle = |11\rangle \qquad C = 0$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \qquad C = 1$$

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) \qquad C = 0$$

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) \qquad C = 1$$

# Quantifying entanglement – pure states

The concurrence is an *entanglement monotone:*

$$0 \le C(\psi) \le 1$$

If $C(\psi_a) > C(\psi_b)$, we say state *a* is more entangled than state *b.*

If $C(\psi_a) = 1$, we say state *a* is maximally entangled.

Example:

The singlet $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$ is maximally entangled.

The state $|\psi\rangle = \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle$, with $C = 2/3$, is entangled, but less entangled than the singlet.

# Density-matrix description of mixed states

$$\rho = |\psi\rangle\langle\psi|$$

for a pure state

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

for a mixed state

$$p_i \in [0,1], \sum_i p_i = 1$$

Properties:

$$\mathrm{Dim}[\rho] = 2^N \times 2^N$$

$$\rho = \rho^\dagger$$ Hermitian

$$\mathrm{Tr}[\rho] = 1$$ Unity trace

Fully describing a *2*-qubit mixed state requires ***15*** real #s

# Quantifying entanglement – mixed states

The concurrence of a mixed state is given by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

The $\lambda_i$ are the eigenvalues of the matrix $\sqrt{\rho\tilde{\rho}}$ in decreasing order, and $\tilde{\rho} = YY\rho^* YY$

- Yes, it's non-intuitive!
- A very non-linear function of $\rho$
- Difficult to propagate experimental errors in tomography (systematics and noise) to error in $C$

Hill and Wootters, PRL (2007)
Wootters, PRL (2008)
Horodecki[4], RMP (2009)

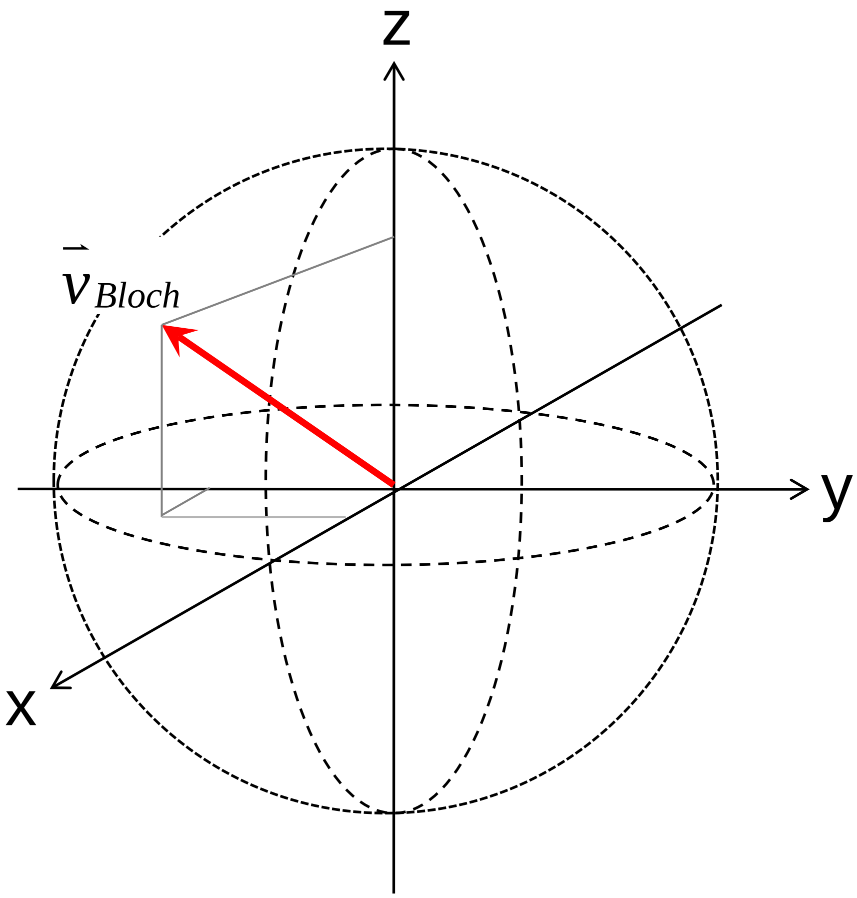
# Getting $\rho$ : quantum state tomography

# Geometric visualization for *N=1*: The Bloch sphere

$$\rho = \frac{1}{2} \sum_{j \in \{i,x,y,z\}} \langle \sigma_j \rangle \, \sigma_j$$

$$= \frac{I}{2} + \frac{1}{2} \underbrace{\left( \langle X \rangle, \langle Y \rangle, \langle Z \rangle \right)}_{\vec{v}_{Bloch}} \cdot \left( X, Y, Z \right)$$

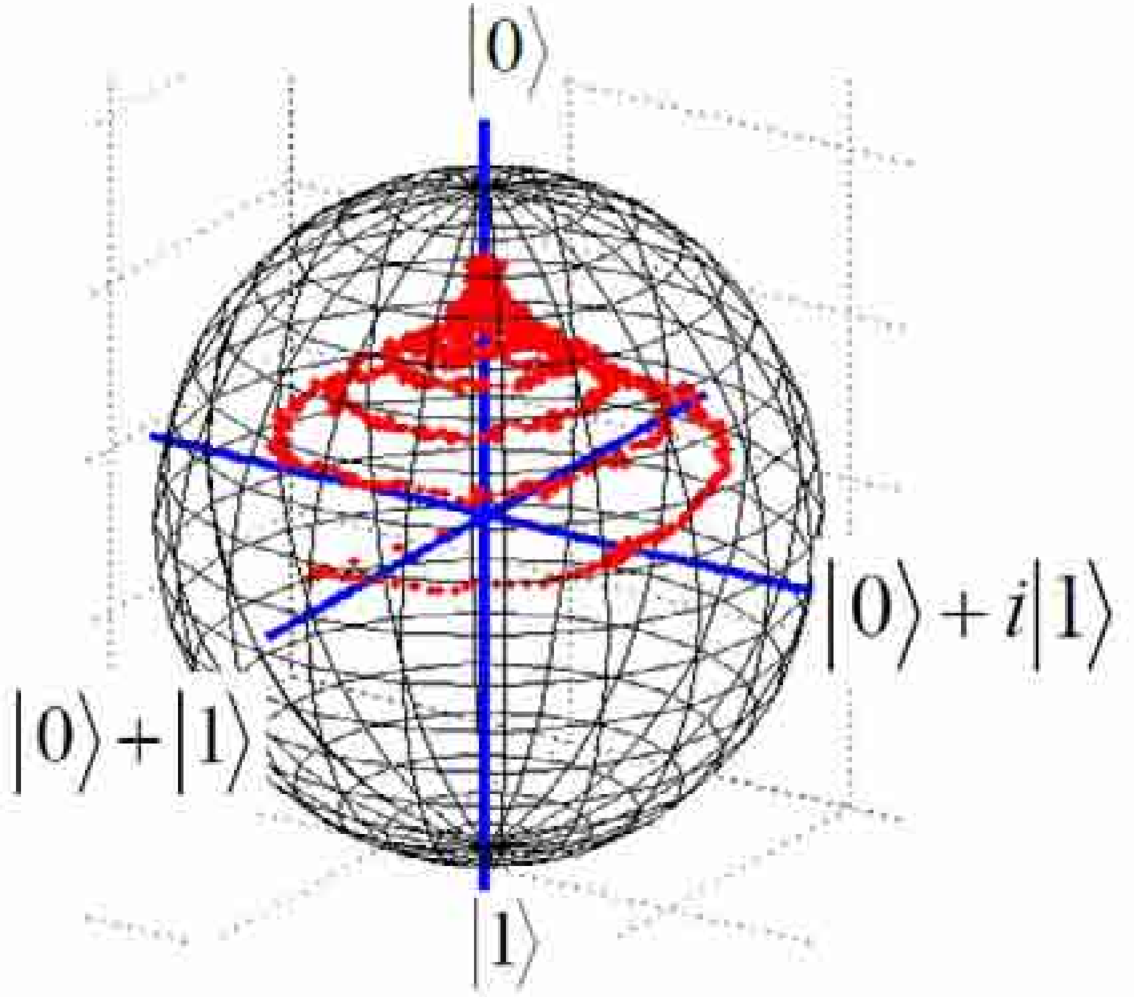
Bloch vector
(NMR)




$\left| \vec{v}_{Bloch} \right| = 1$ pure state

$\left| \vec{v}_{Bloch} \right| < 1$ mixed state

# State tomography of qubit decay




Is there a similarly practical description for *N=2* qubits?

Steffen *et al.*, PRL (2006)

# Generalizing the Bloch vector: The Pauli set

$$\rho = \frac{1}{4} \sum_{j,k \in \{i,x,y,z\}} \langle \sigma_j \sigma_k \rangle \, \sigma_j \sigma_k$$

The *Pauli set* $\boldsymbol{P}$ = the set of expectation values of the **16** 2-qubit Pauli operators.

- gives a full description of the 2-qubit state
- is the extension of the Bloch vector to 2 qubits
- generalizes to higher *N*

One of them, $\langle II \rangle = 1$ always

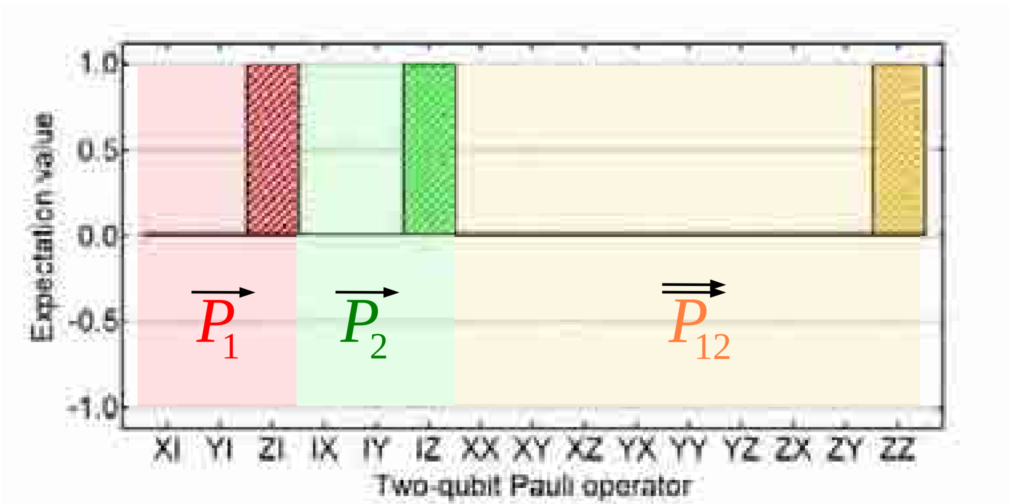
The two-qubit Pauli set can be divided into three sections:

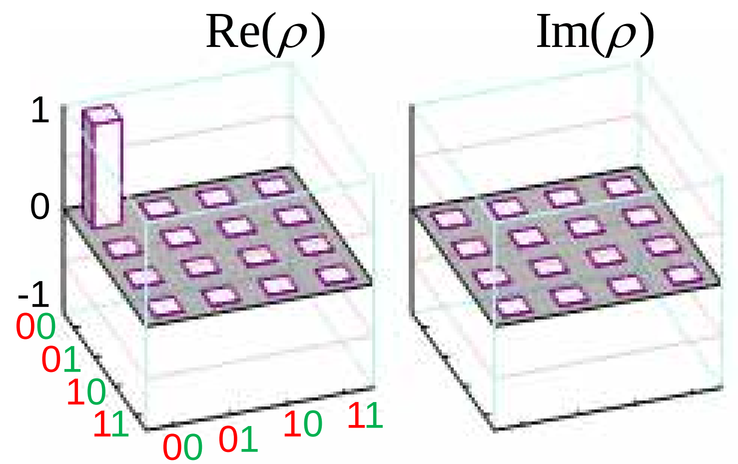
- Polarization of Qubit 1 $\qquad \overrightarrow{P_1} = \left( \langle XI \rangle, \langle YI \rangle, \langle ZI \rangle \right)$
- Polarization of Qubit 2 $\qquad \overrightarrow{P_2} = \left( \langle IX \rangle, \langle IY \rangle, \langle IZ \rangle \right)$
- Two-qubit correlations

$$\overrightarrow{\overrightarrow{P_{12}}} = \left( \langle XX \rangle, \langle XY \rangle, \langle XZ \rangle, \langle YX \rangle, \langle YY \rangle, \langle YZ \rangle, \langle ZX \rangle, \langle ZY \rangle, \langle ZZ \rangle \right)$$

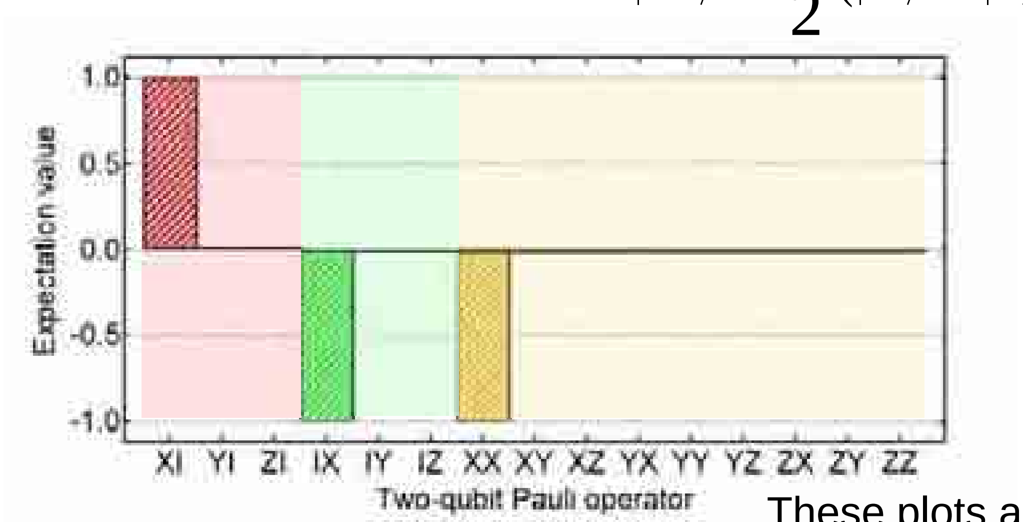
# Visualizing $N$=2 states: product states

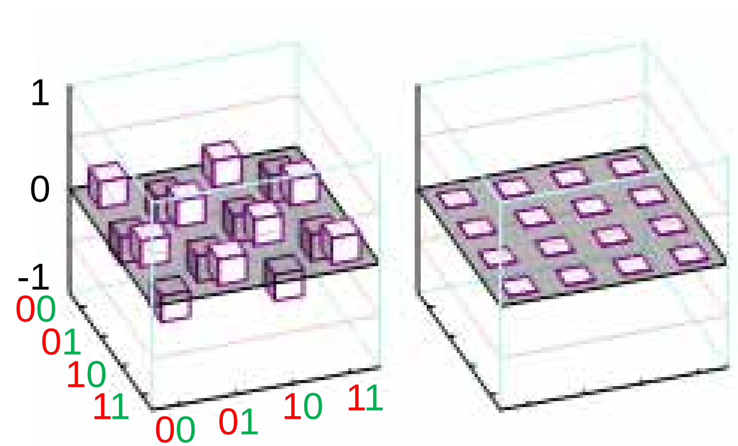
$$|\psi\rangle = |{\color{red}0}{\color{green}0}\rangle$$




$$|\psi\rangle = \frac{1}{2}\left(|{\color{red}0}\rangle + |{\color{red}1}\rangle\right) \otimes \left(|{\color{green}0}\rangle - |{\color{green}1}\rangle\right)$$

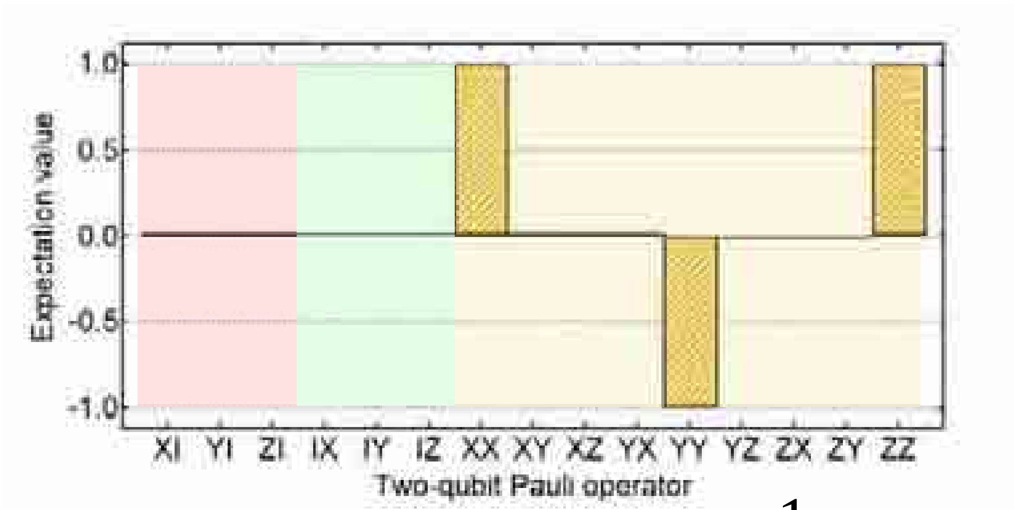


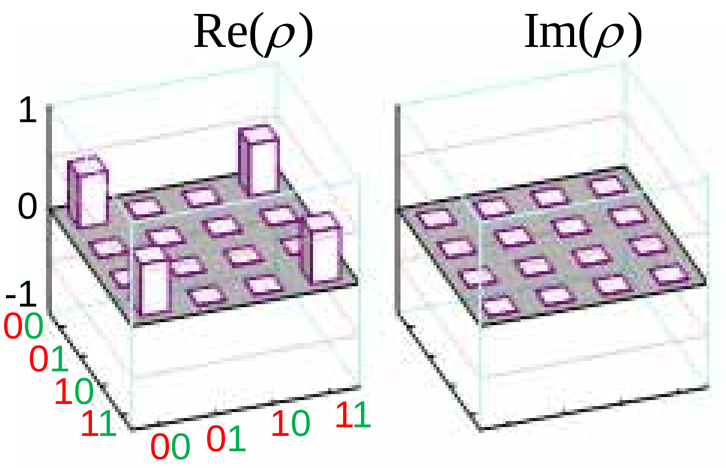




These plots are theory

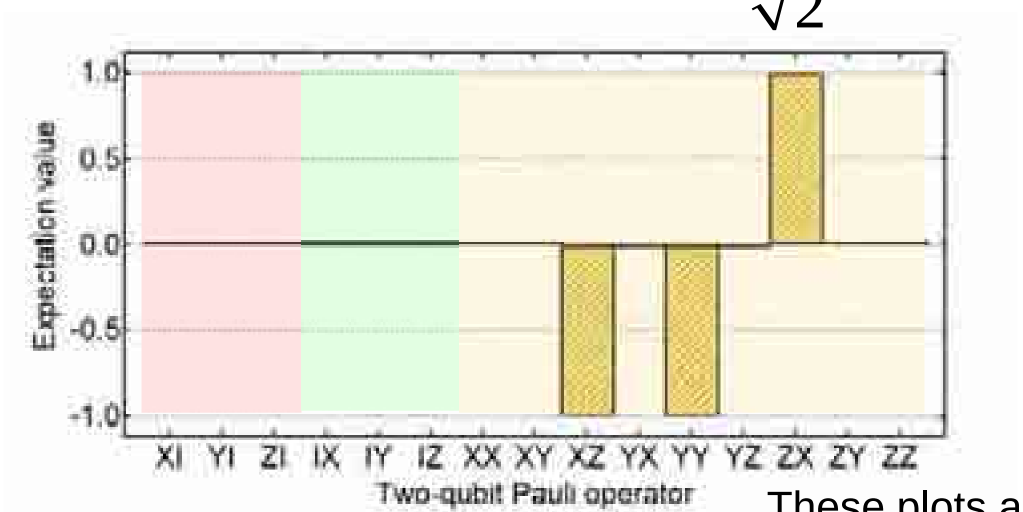
# Visualizing $N$=2 states: maximally entangled states

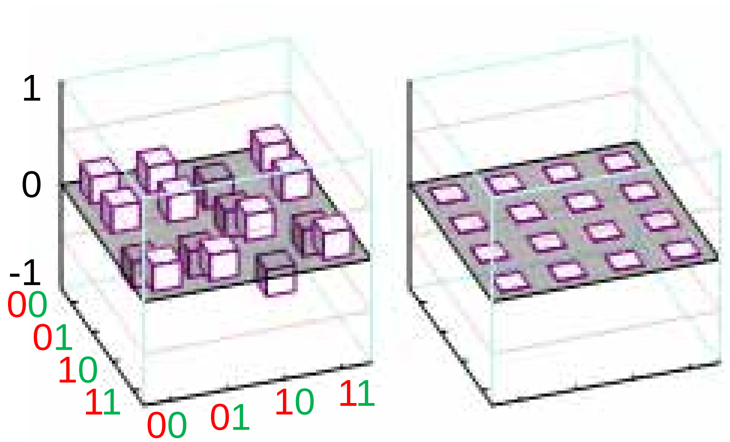
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$




$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$




These plots are theory

# Extracting useful metrics from the Pauli set

State purity:

$$\mathrm{Tr}[\rho^2] = \frac{1}{2^N}\boldsymbol{P}\bullet\boldsymbol{P}$$

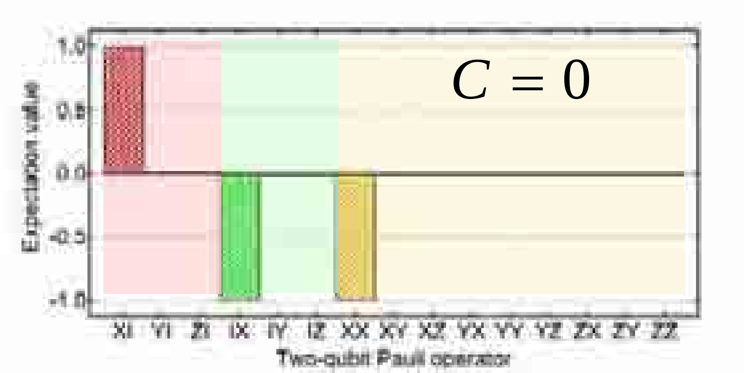
Fidelity to a target state $|\psi_{\mathrm{T}}\rangle$ :

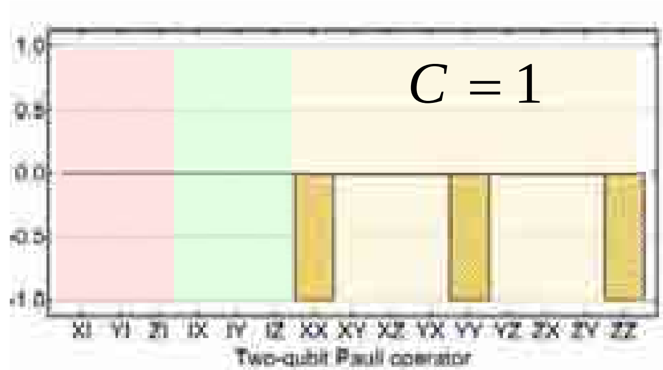
$$F = \langle\psi_{\mathrm{T}}|\rho|\psi_{\mathrm{T}}\rangle = \frac{1}{2^N}\boldsymbol{P}\bullet\boldsymbol{P}_T$$

Two-qubit Concurrence:

$$C(\psi) = \sqrt{\frac{\overrightarrow{P}_{12}\bullet\overrightarrow{P}_{12} - 1}{2}}$$

Warning:
for pure states only



# Short-circuiting Concurrence

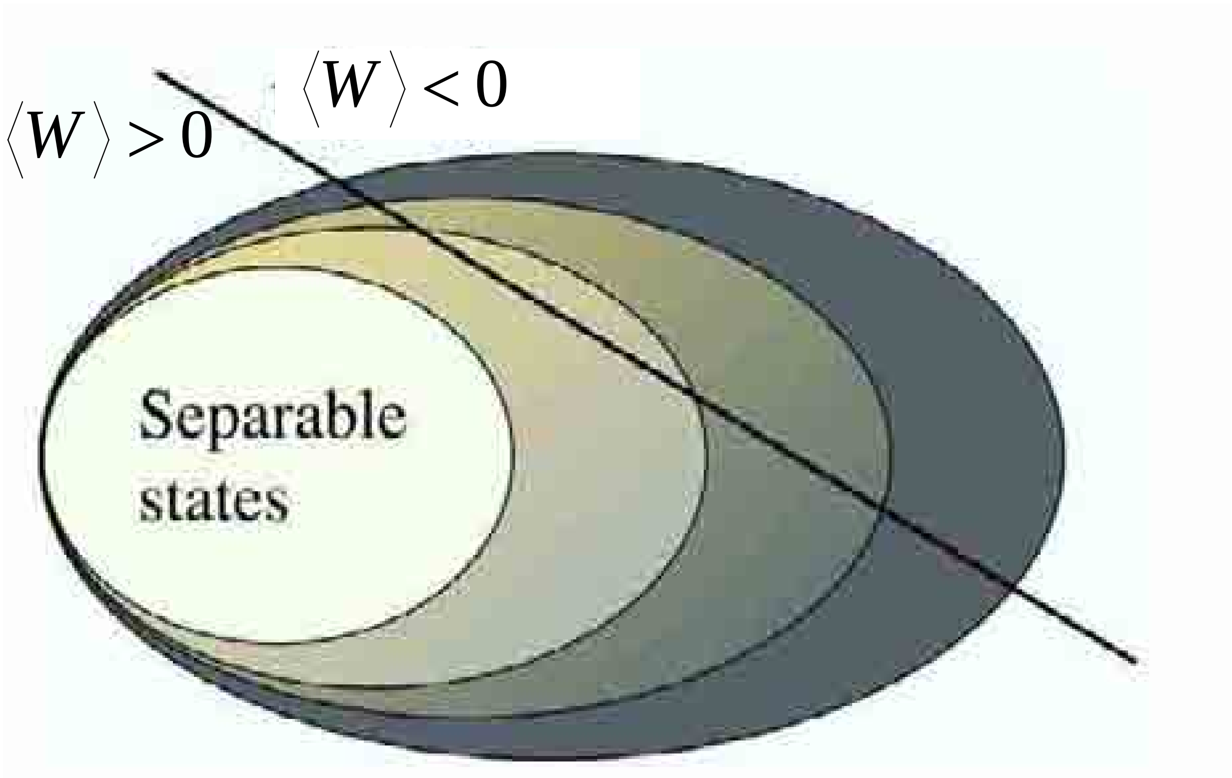
- Can we characterize (even possibly quantify) entanglement without reliance on $C$ ?
- Can we place lower bounds on $C$ without performing full state tomography?

# Witnessing entanglement with a subset of the Pauli set

An *entanglement witness* is an observable $W$ with a positive expectation value for all product states.

$\langle W \rangle < 0 \quad \Rightarrow$ state is entangled, *guaranteed*.

$\langle W \rangle \geq 0 \quad \Rightarrow$ witness simply doesn't know




Eisert *et al*., New J. Phys (2007)

# Witnessing entanglement with a subset of the Pauli set

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$\langle W \rangle < 0 \quad \Rightarrow$ state is entangled, *guaranteed*.

$\langle W \rangle \geq 0 \quad \Rightarrow$ witness simply doesn't know

- *Witnesses require only a subset of the Pauli set!*

- Example:

$$W = \frac{1}{4}\left( II + XX + YY + ZZ \right)$$

- This witness gives a lower bound on $C$: $\quad -2\langle W \rangle \leq C$

# Preparing and measuring entanglement in cQED

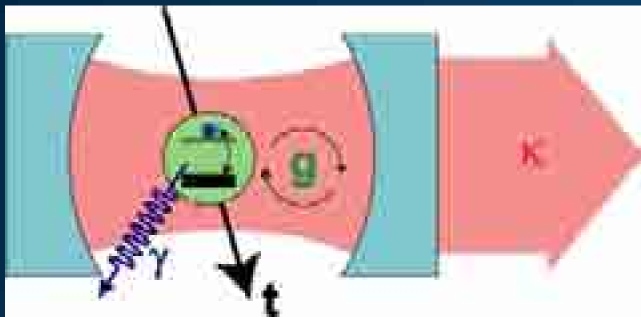
DiCarlo *et al*., Nature (2009)

Chow *et al*., arXiv 0908.1955

DiCarlo *et al*., arXiv 1004.4324

Reed *et al*., arXiv 1004.4323

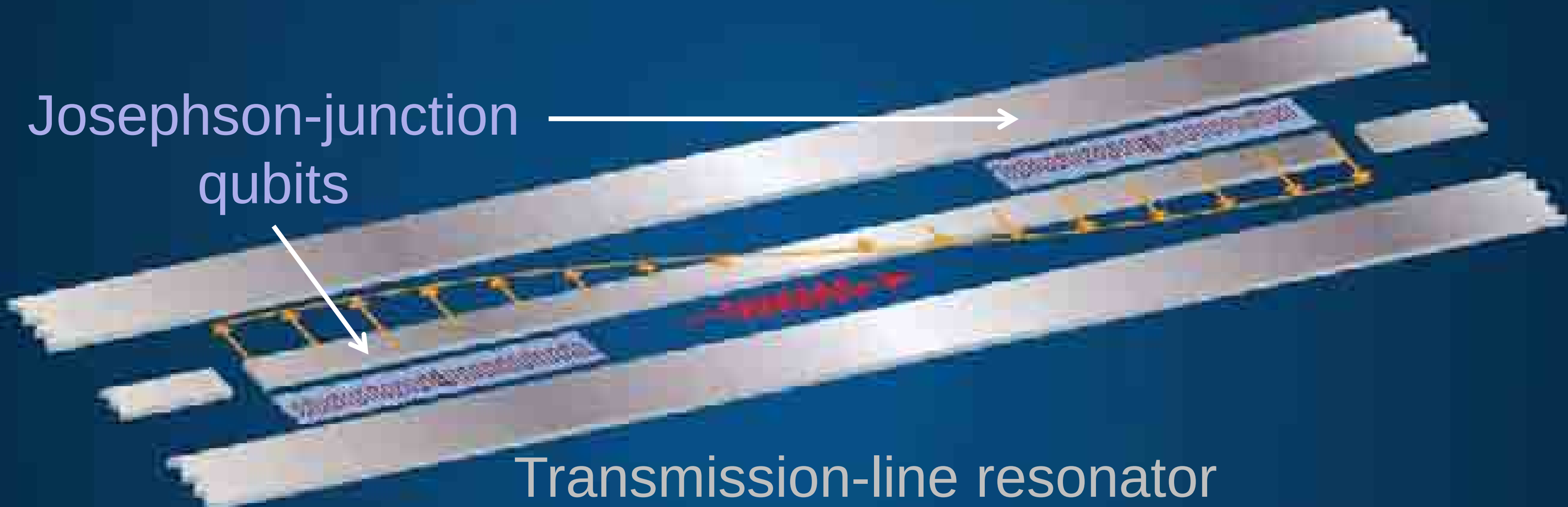
# Cavity QED with wires




**“Circuit QED”**

Blais *et al.*, Phys. Rev. A (2004)

Josephson-junction qubits

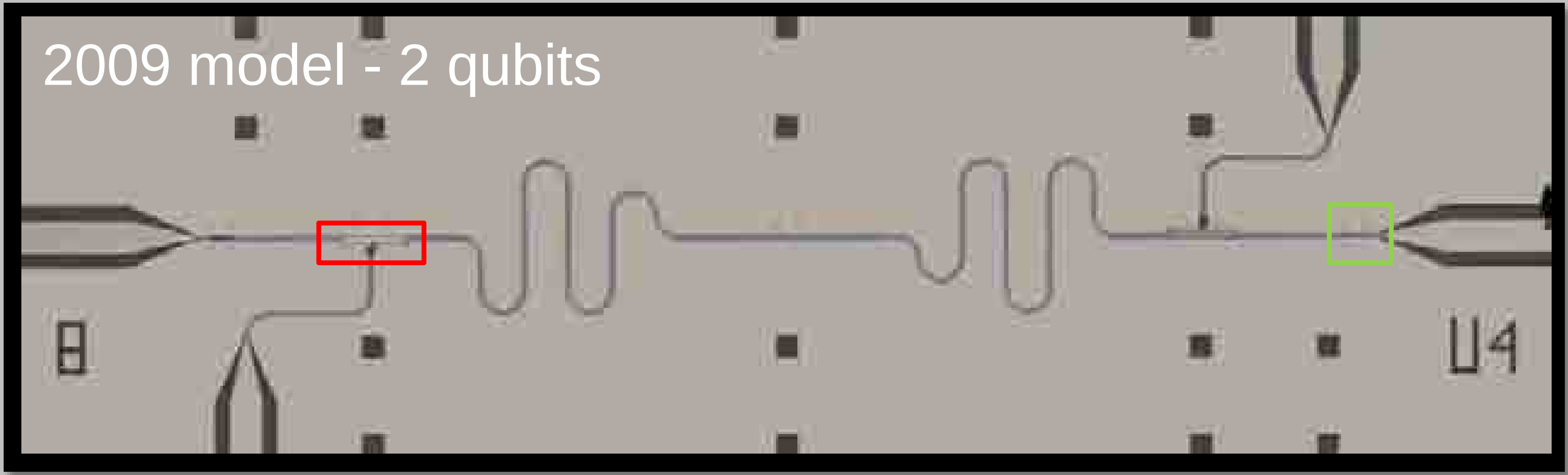


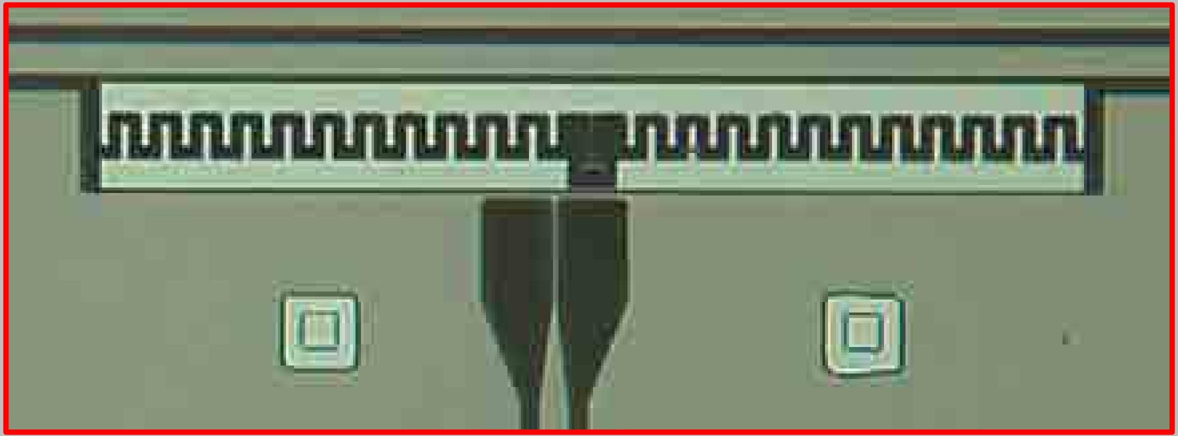
Transmission-line resonator

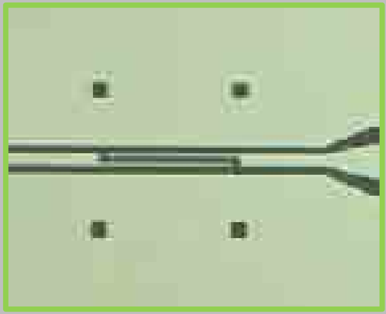
- mediates interaction between qubits
- protects qubits from continuum
- allows joint qubit readout

<u>Expts:</u> Majer, Chow *et al*., Nature (2007) (Charge qubits / Yale)
Sillanpää *et al*., Nature (2007) (Phase qubits / NIST)

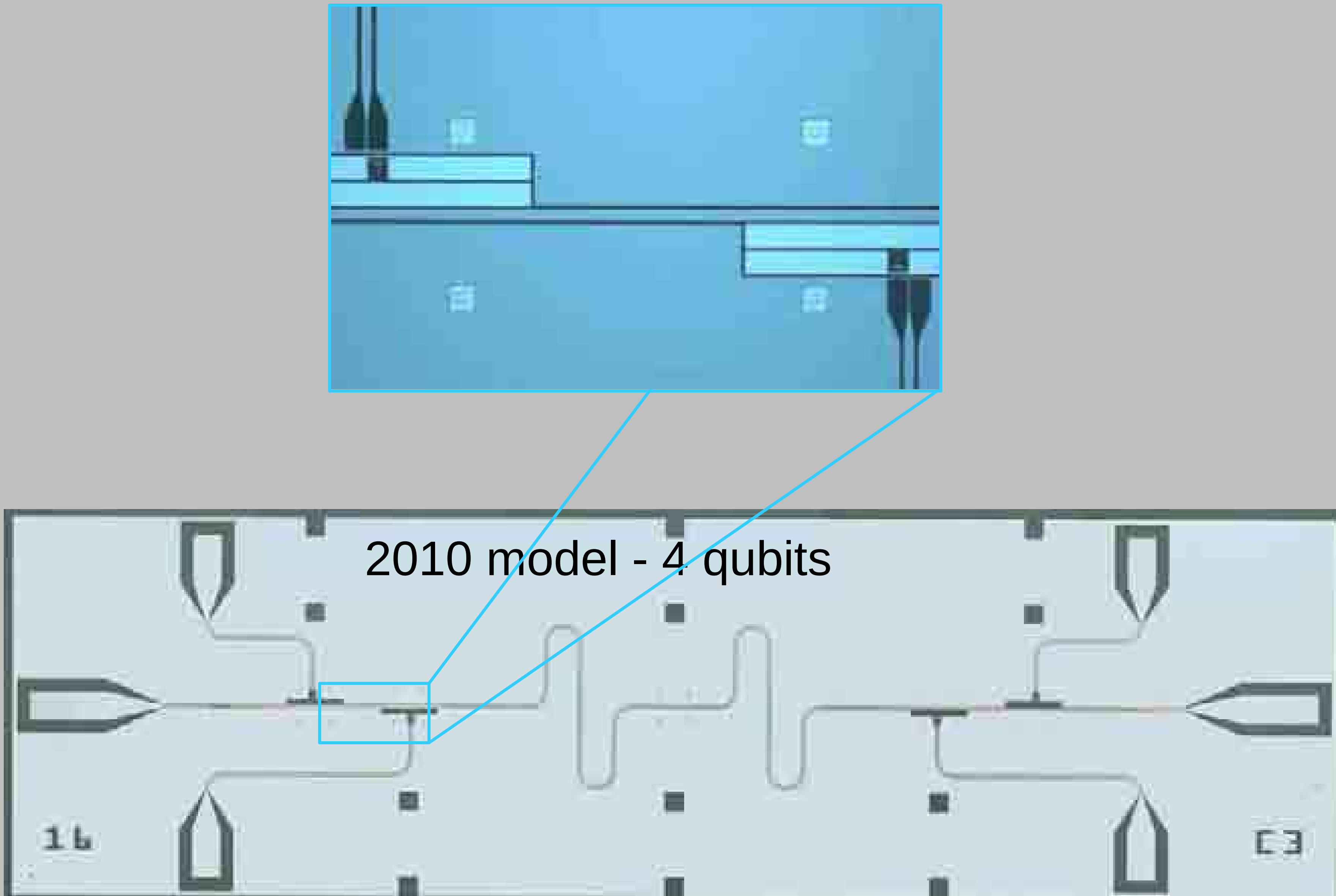
# Meet the quantum processors



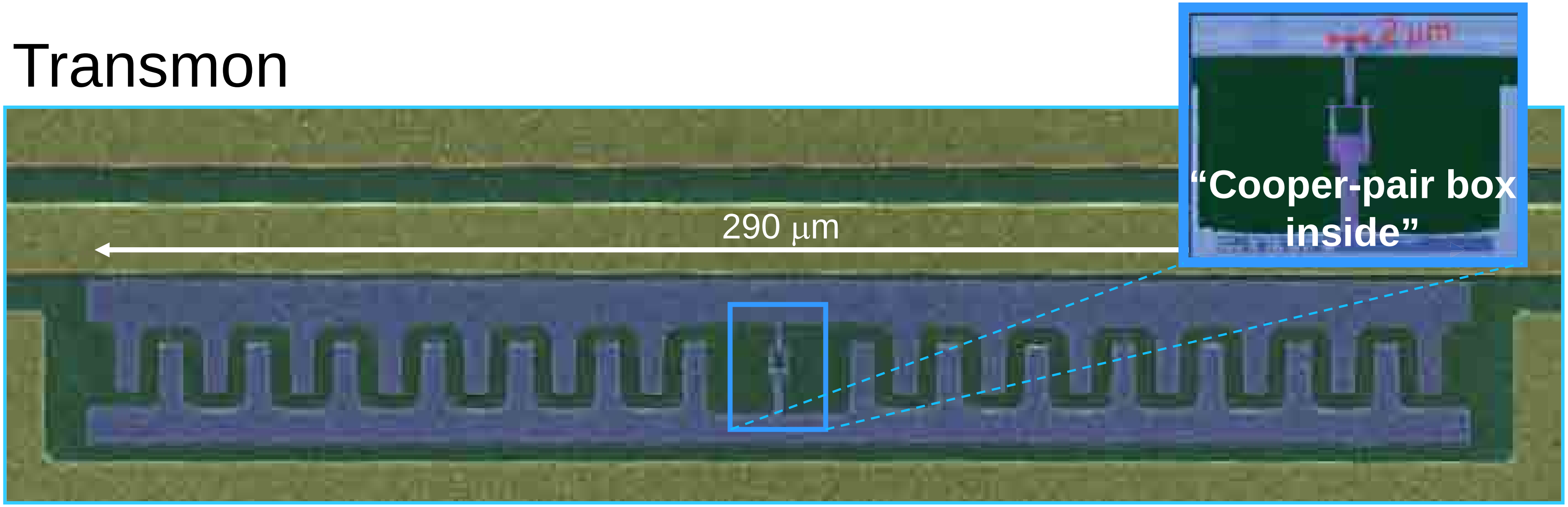


# Meet the quantum processors

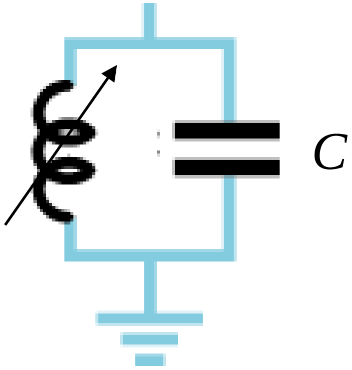


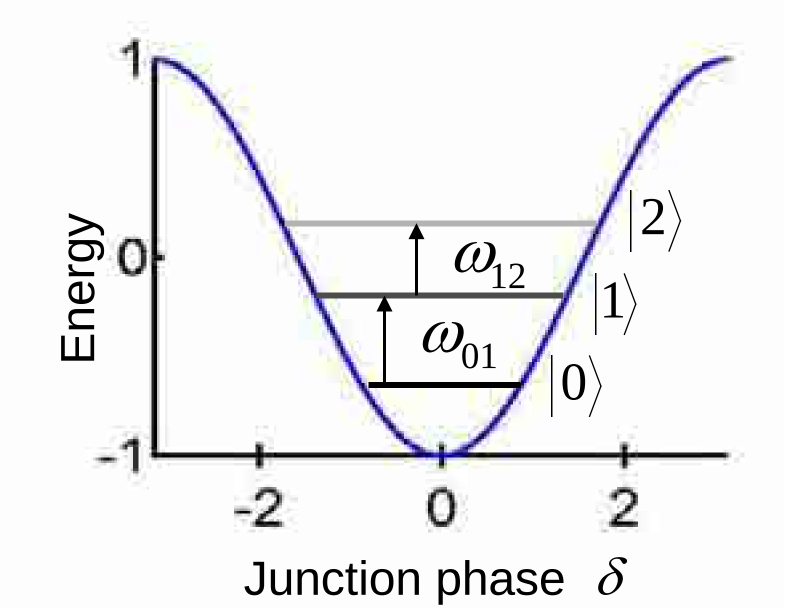
# Tunable artificial atoms

## Transmon



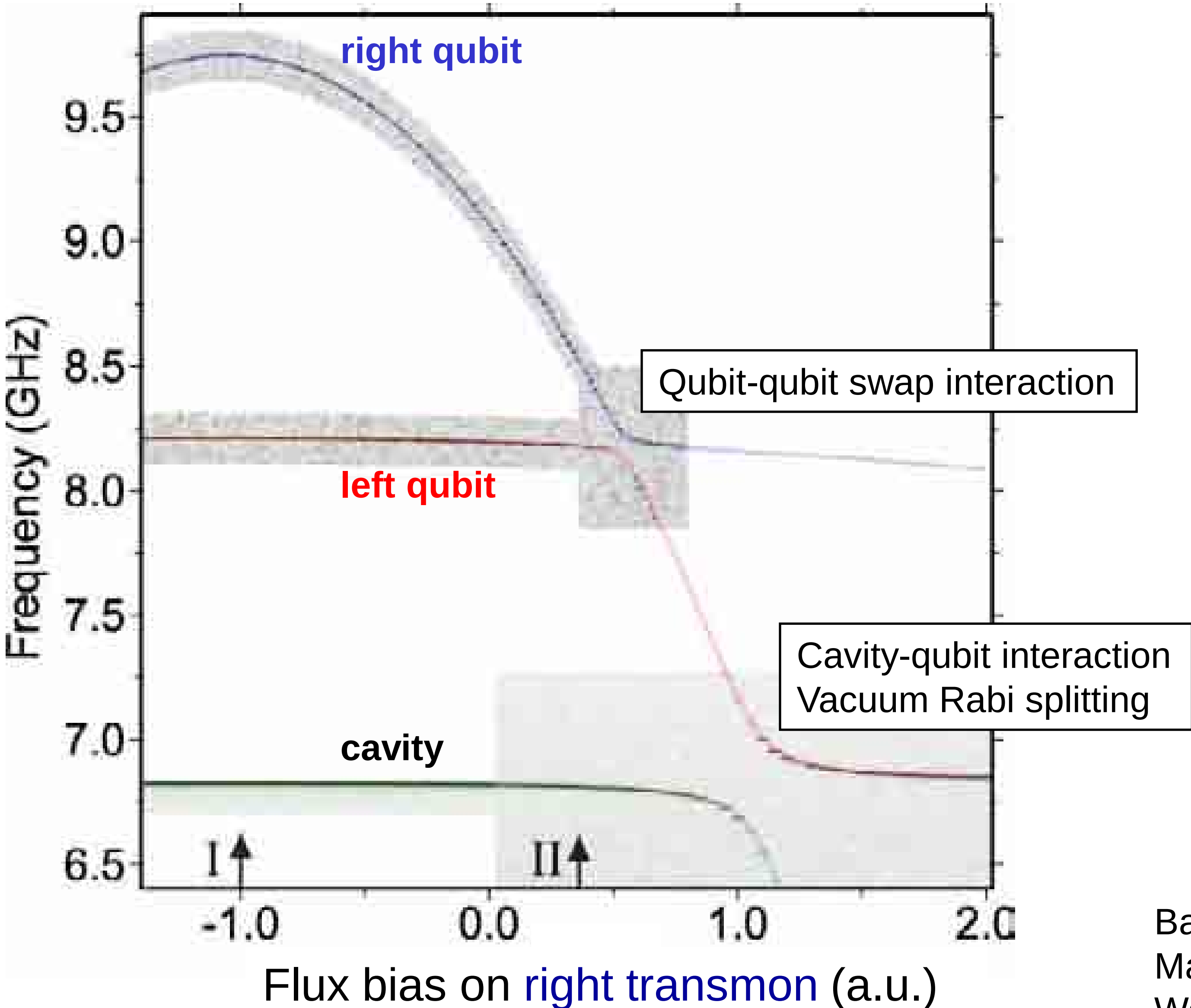

$$L = \frac{L_J}{\cos\delta}$$



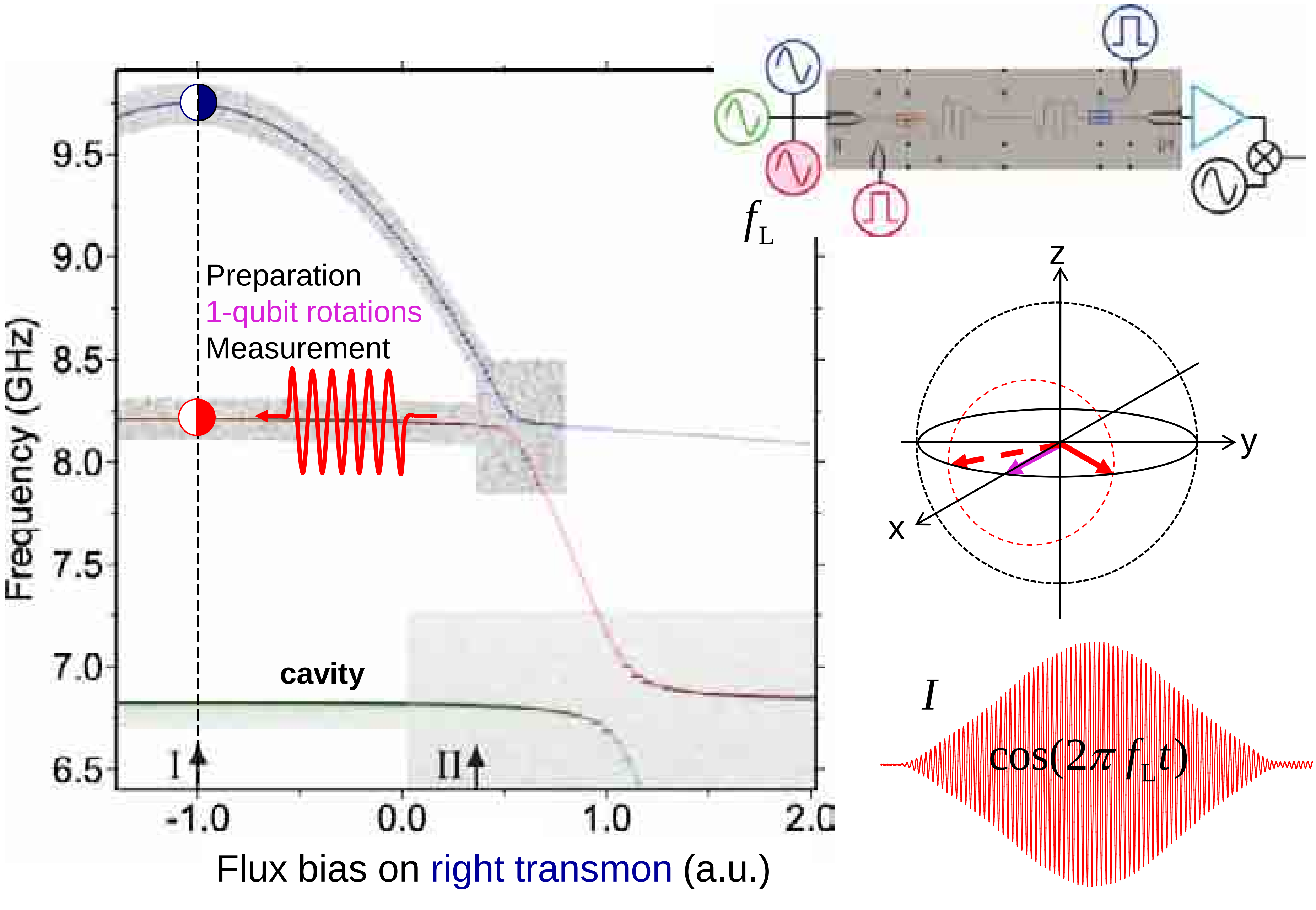

Theo: J. Koch *et al.*, PRA (2007)
Expt: J. Schreier *et al.*, PRB (2009)
Review: Houck *et al.*, Quant. Int. Proc. (2009)

# Spectroscopy of two qubits + cavity




Background:
Majer *et al.*, Nature (2007)
Wallraff *et al.*, Nature (2004)

# One-qubit gates: *X* and *Y* rotations



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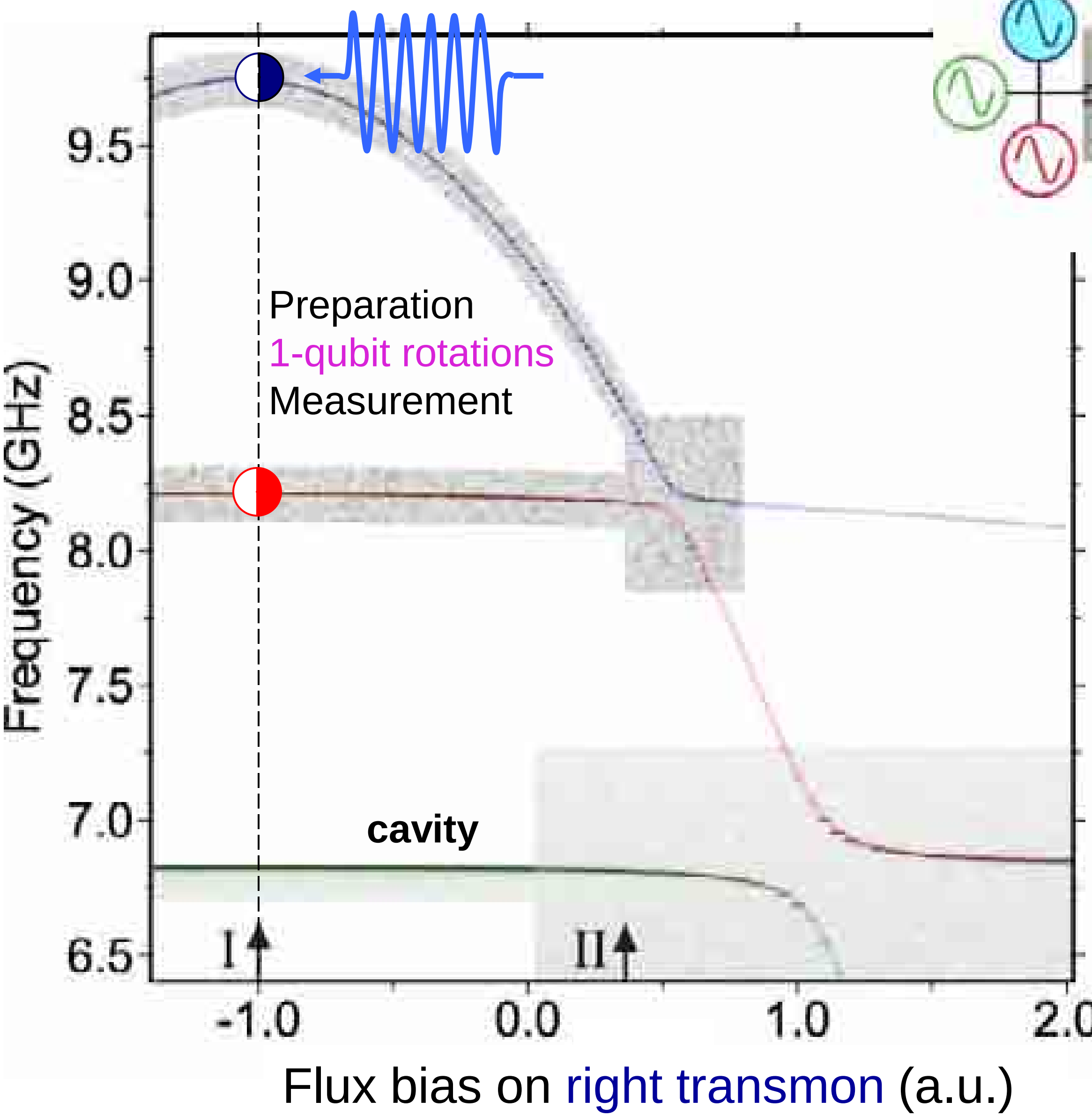


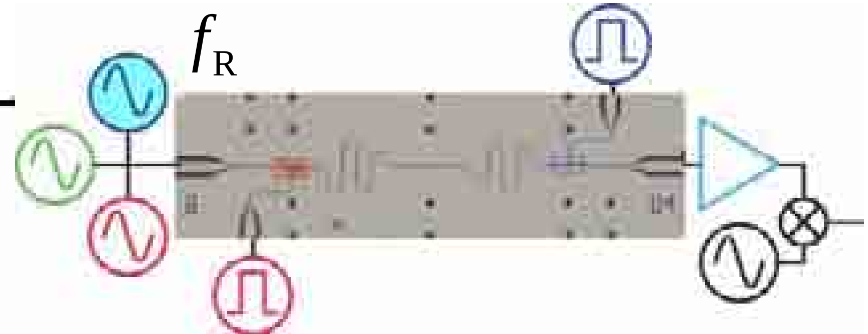


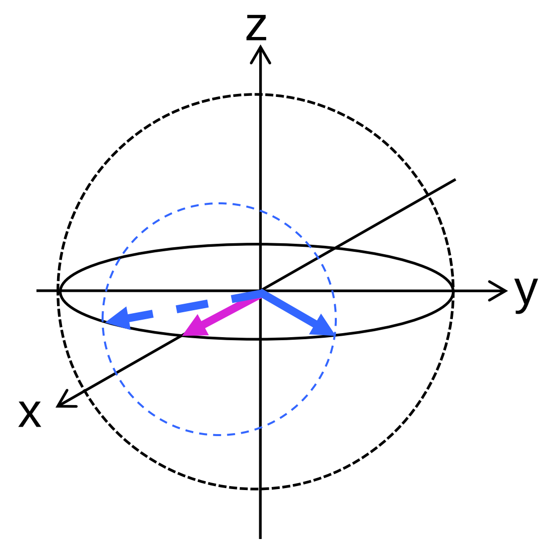


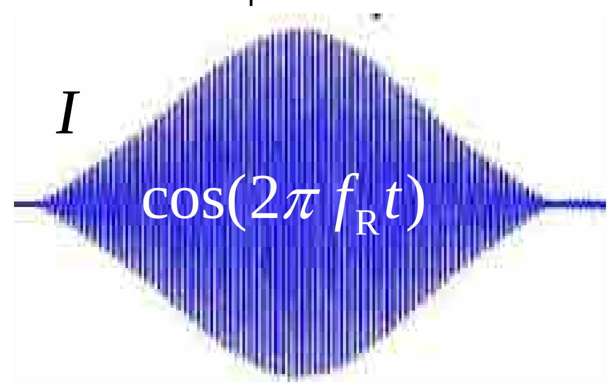

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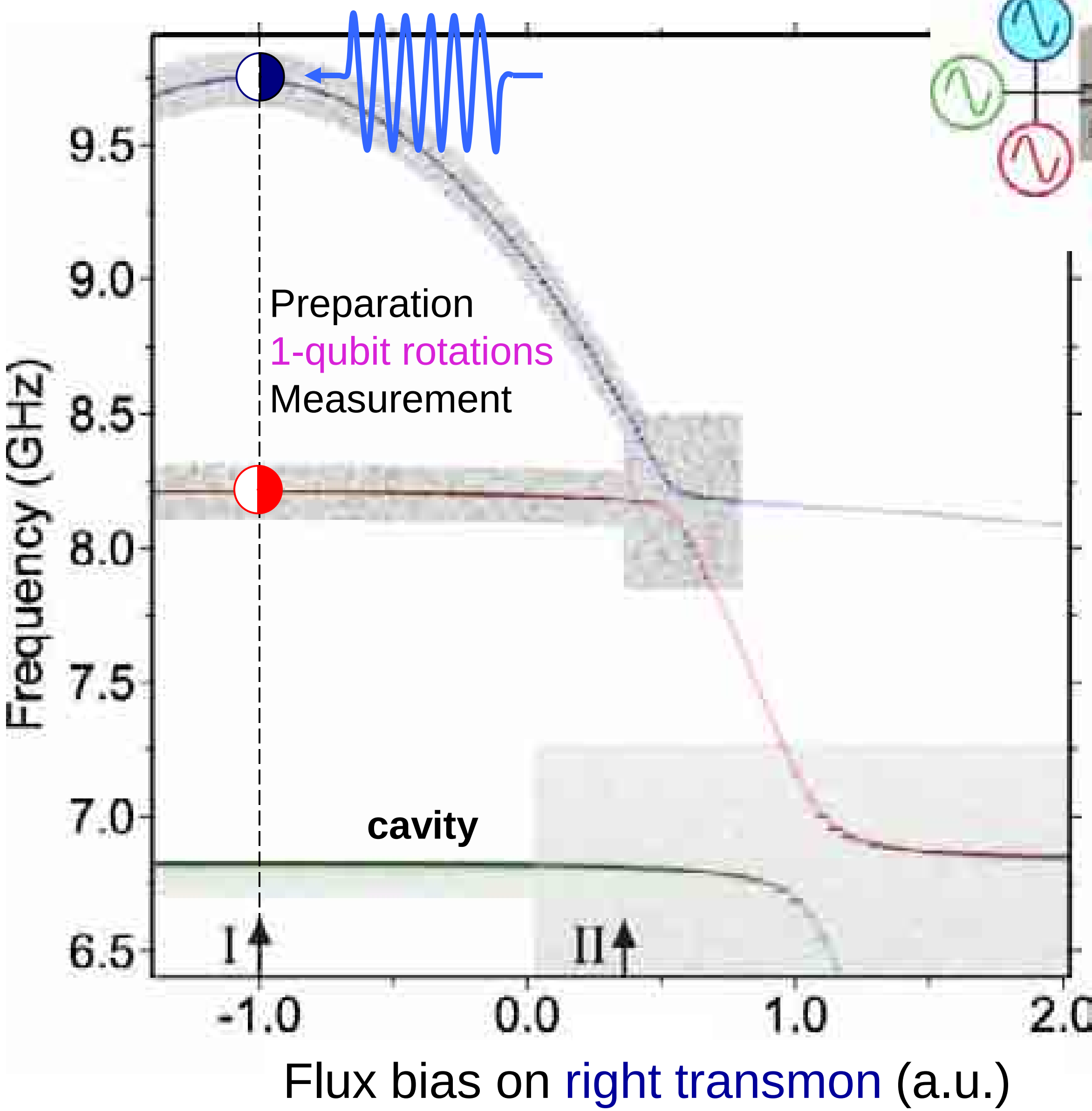
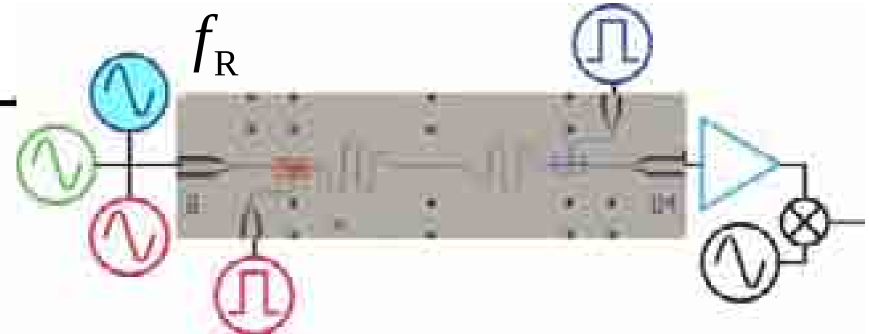
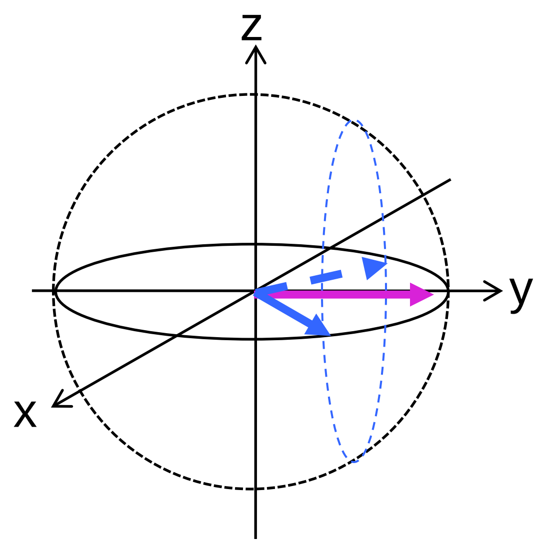
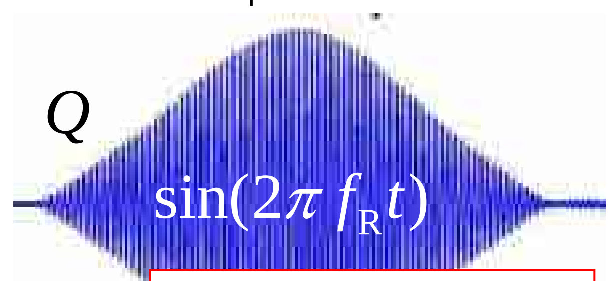


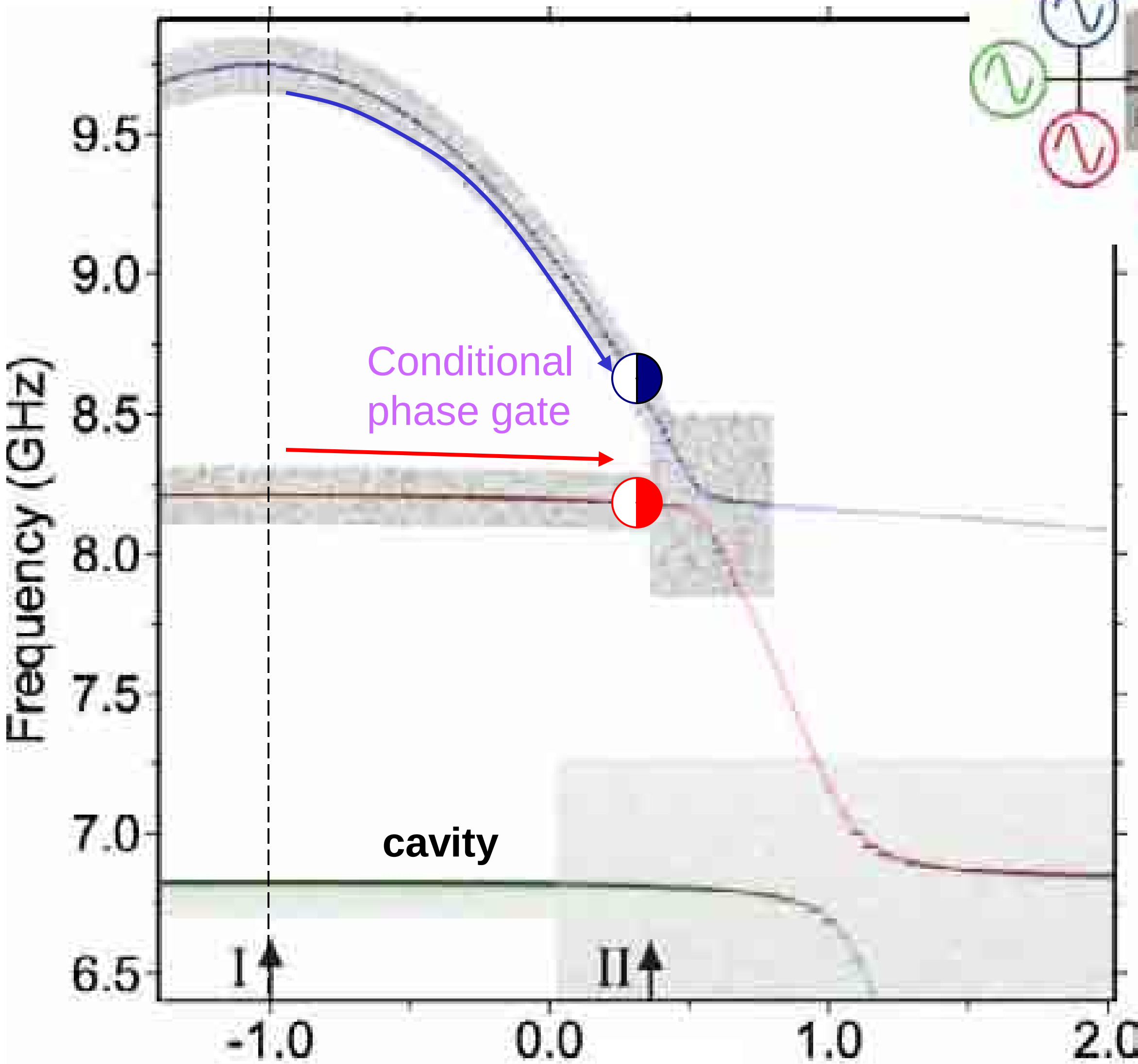



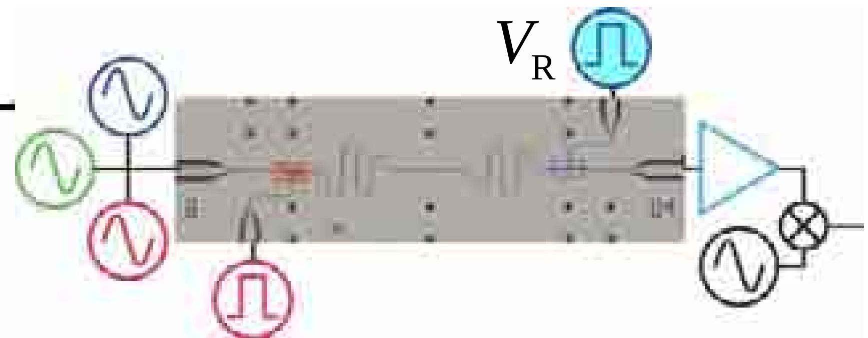

Fidelity = 99%

see J. Chow *et al.*, PRL (2009)

# Two-qubit gate: turn on interactions




Flux bias on right transmon (a.u.)



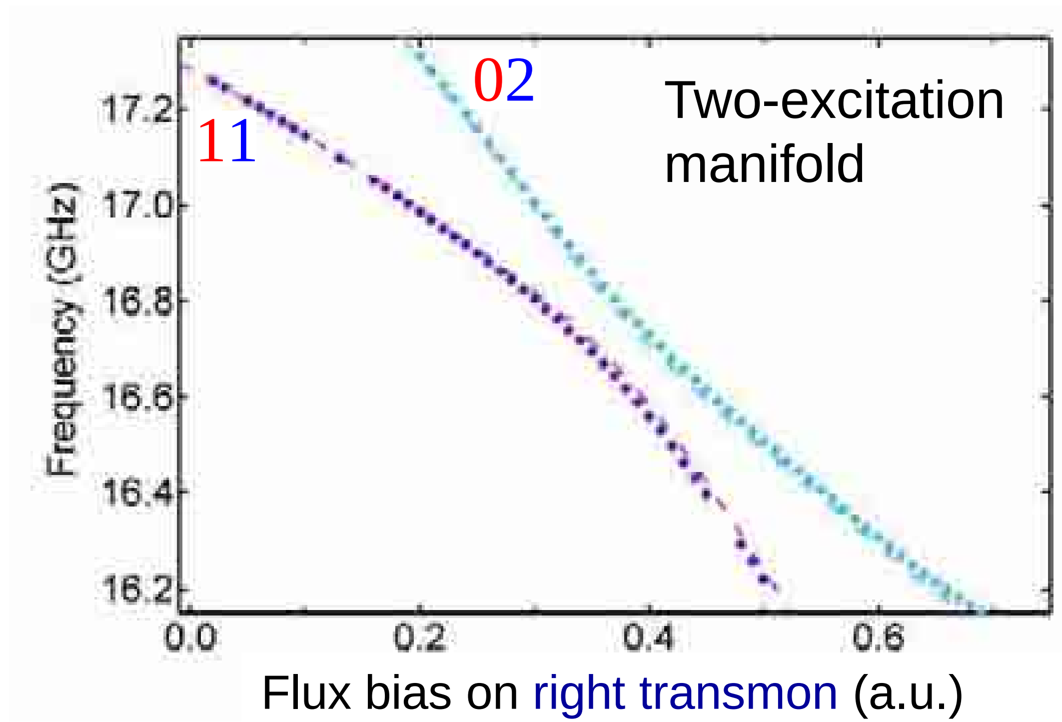

Use control lines to push qubits near a resonance

# Two-excitation manifold of system

- Transmon “qubits” have multiple levels…

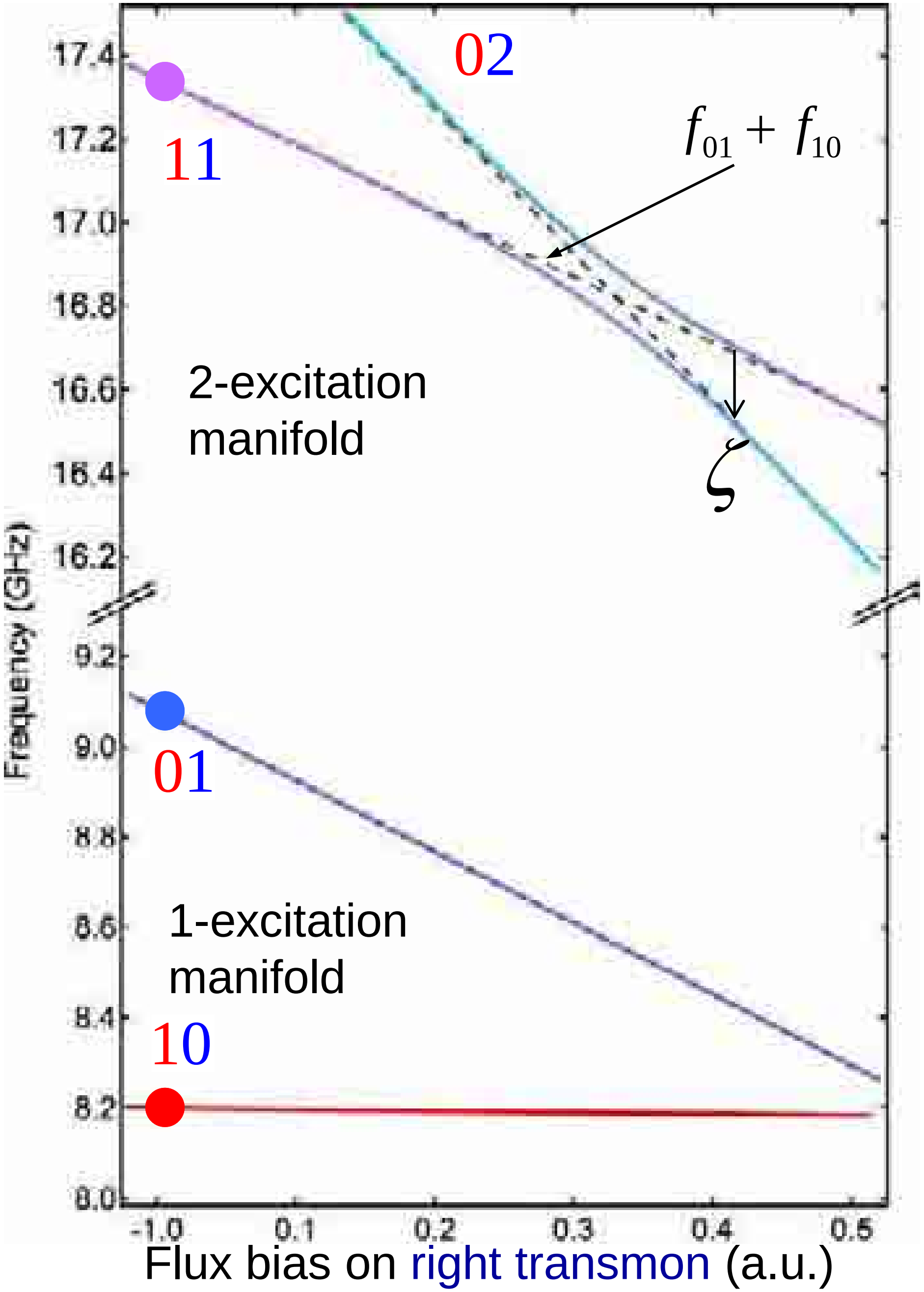
- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |02\rangle$$




Strauch *et al*. PRL (2003):
proposed using interactions with higher levels for computation in phase qubits

# Adiabatic conditional-phase gate




$$\varphi_a = -2\pi \int_{t_0}^{t_f} \delta f_a(t)\,dt$$

$$|11\rangle \rightarrow e^{i\varphi_{11}} |11\rangle$$

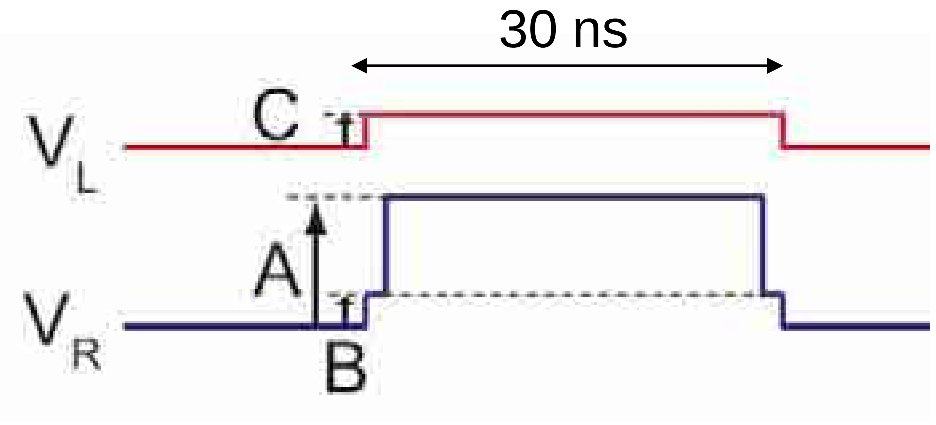
$$\varphi_{11} = \varphi_{10} + \varphi_{01} - 2\pi \int_{t_0}^{t_f} \zeta(t)\,dt$$

$$|01\rangle \rightarrow e^{i\varphi_{01}} |01\rangle$$

$$|10\rangle \rightarrow e^{i\varphi_{10}} |10\rangle$$

# Implementing C-Phase with 1 fancy pulse

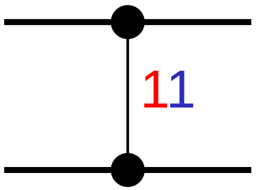
$$
\begin{array}{cc}
 & \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\
U = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\varphi_{10}} & 0 \\ 0 & 0 & 0 & e^{i\varphi_{11}} \end{pmatrix}
\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array}
\end{array}
$$



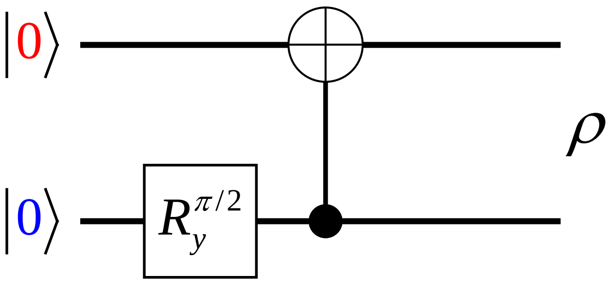

Adjust timing of flux pulse so that only quantum amplitude of $|11\rangle$ acquires a minus sign:

$$
\begin{array}{cc}
 & \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\
U = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array}
\end{array}
$$

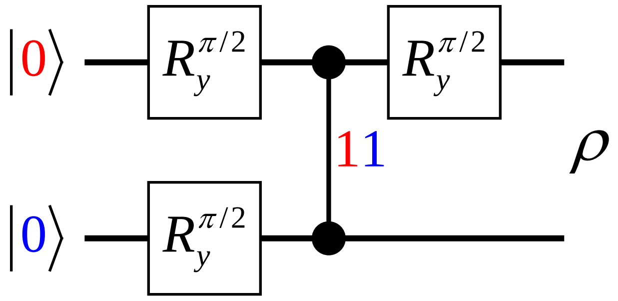
C-Phase$_{11}$



# Entanglement on demand



# Entanglement on demand



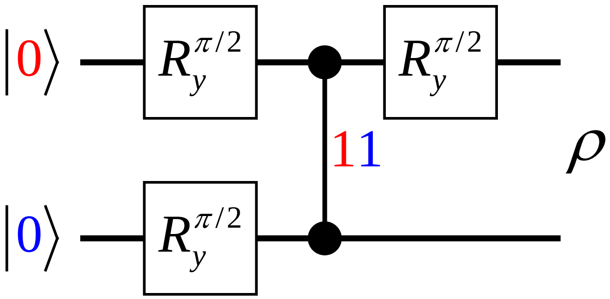

(1) Start in ground state:

$$|\Psi\rangle = |0\rangle \otimes |0\rangle$$

(2) $\pi / 2$ rotation on each qubit yields a maximal superposition:

$$|\Psi\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$
$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

# Entanglement on demand




(3) Apply ‘c-phase’ entangler:

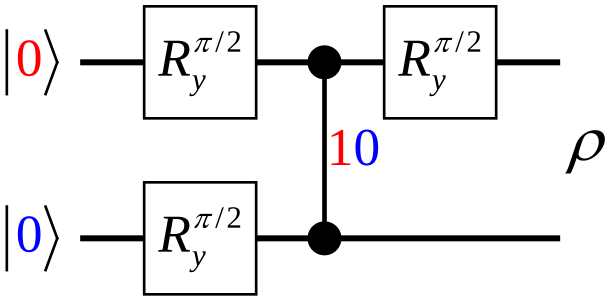
$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

No longer a product state!

(4) $\pi/2$ rotation on LEFT qubit:

$$\boxed{|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)}$$

# Entanglement on demand



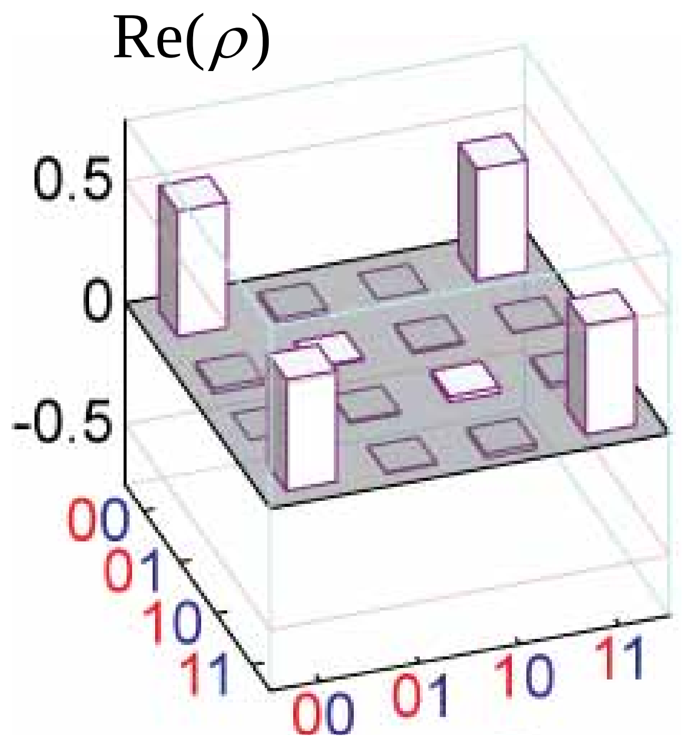

Ideally:

wavefunction $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

density matrix

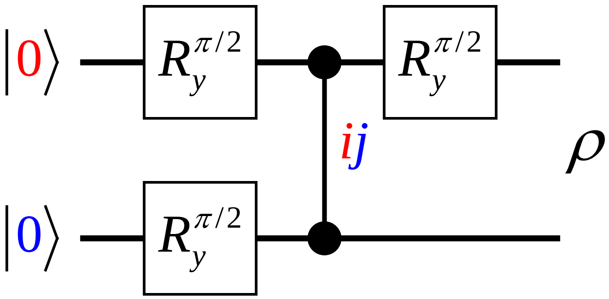
$$\rho = |\Psi\rangle\langle\Psi|$$

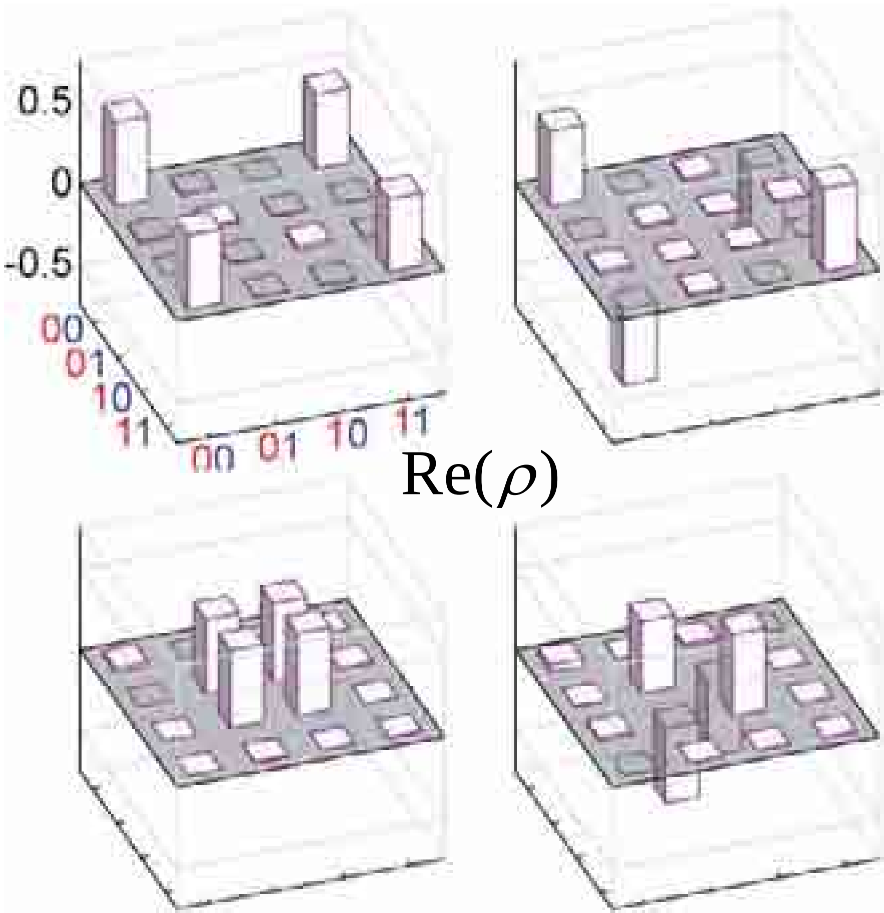
$$= \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$




Expt'l state tomography

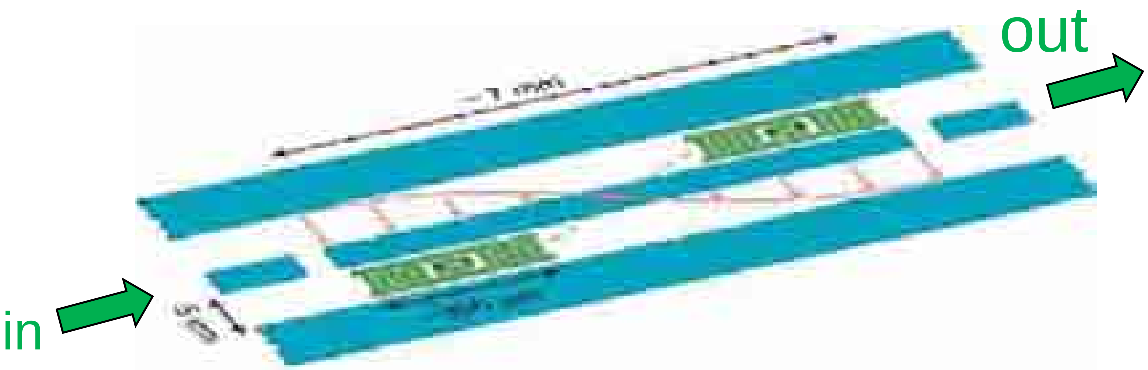
# Entanglement on demand

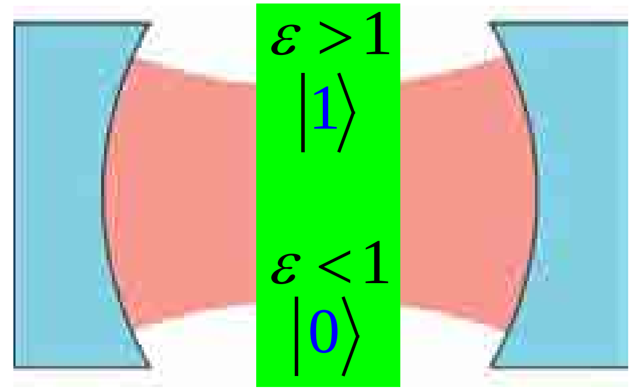


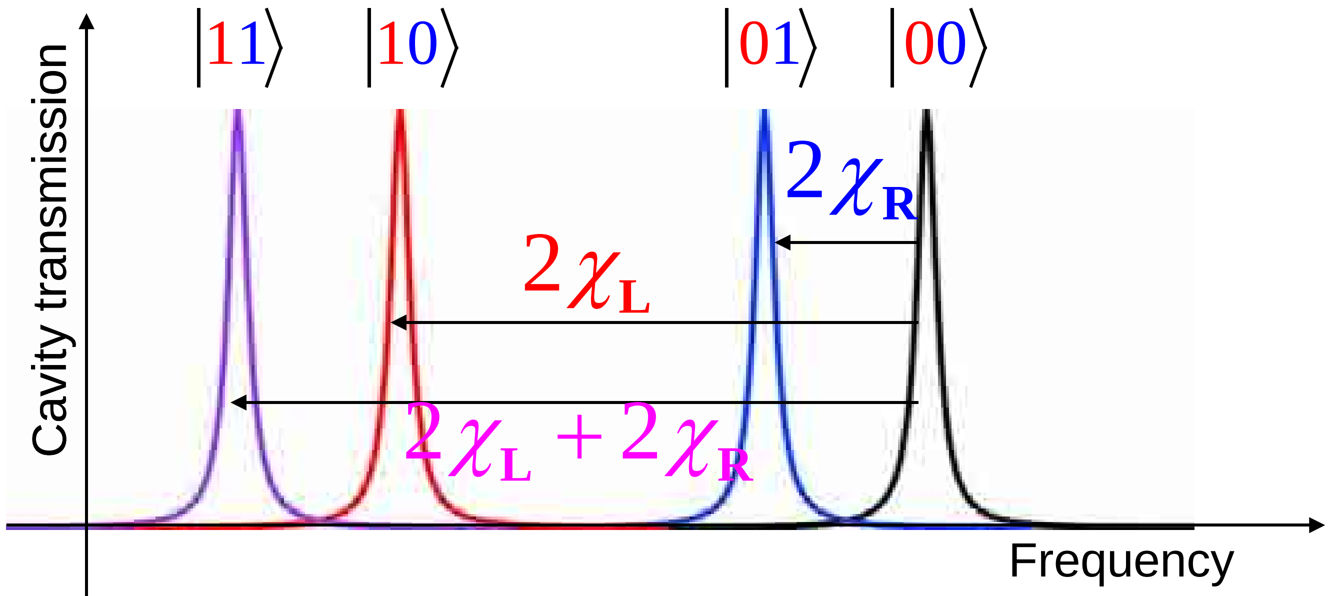




| Bell state | Fidelity | Hill-Wootters Concurrence |
|---|---|---|
| $\|00\rangle + \|11\rangle$ | 94% | 90% |
| $\|00\rangle - \|11\rangle$ | 94% | 94% |
| $\|01\rangle + \|10\rangle$ | 95% | 92% |
| $\|01\rangle - \|10\rangle$ | 93% | 90% |

# Joint qubit readout via cavity




$\chi_{\mathbf{R}} = 4$ **MHz**

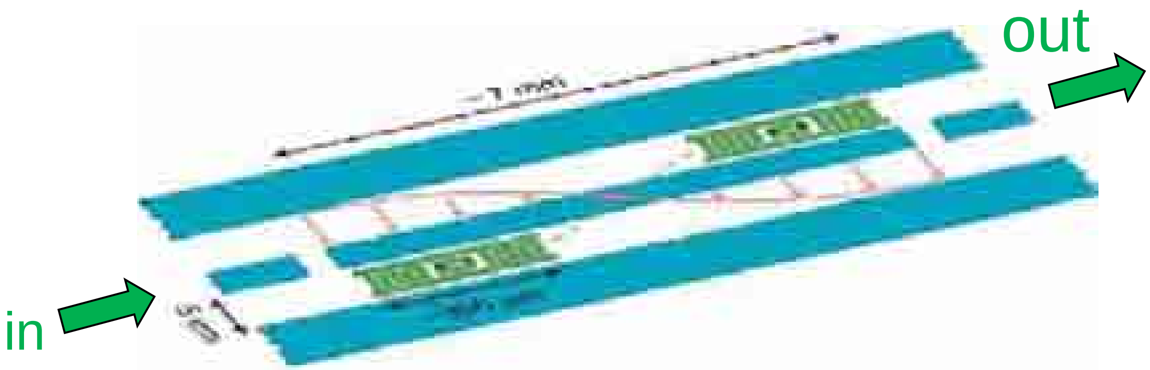
$\chi_{\mathbf{L}} = 13$ **MHz**

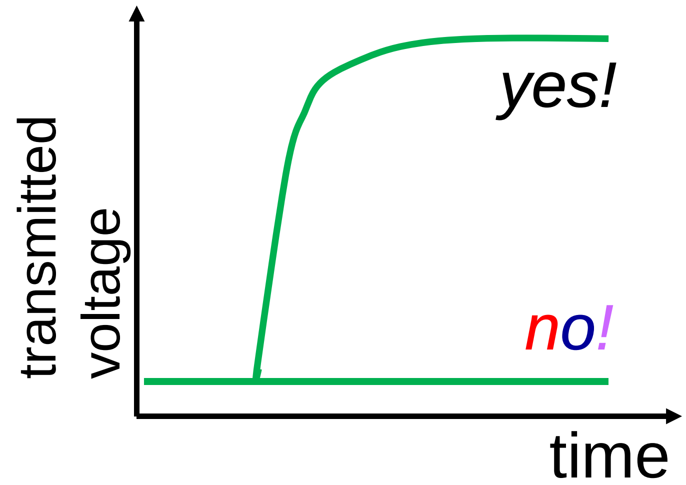
$\kappa = 1$ **MHz**

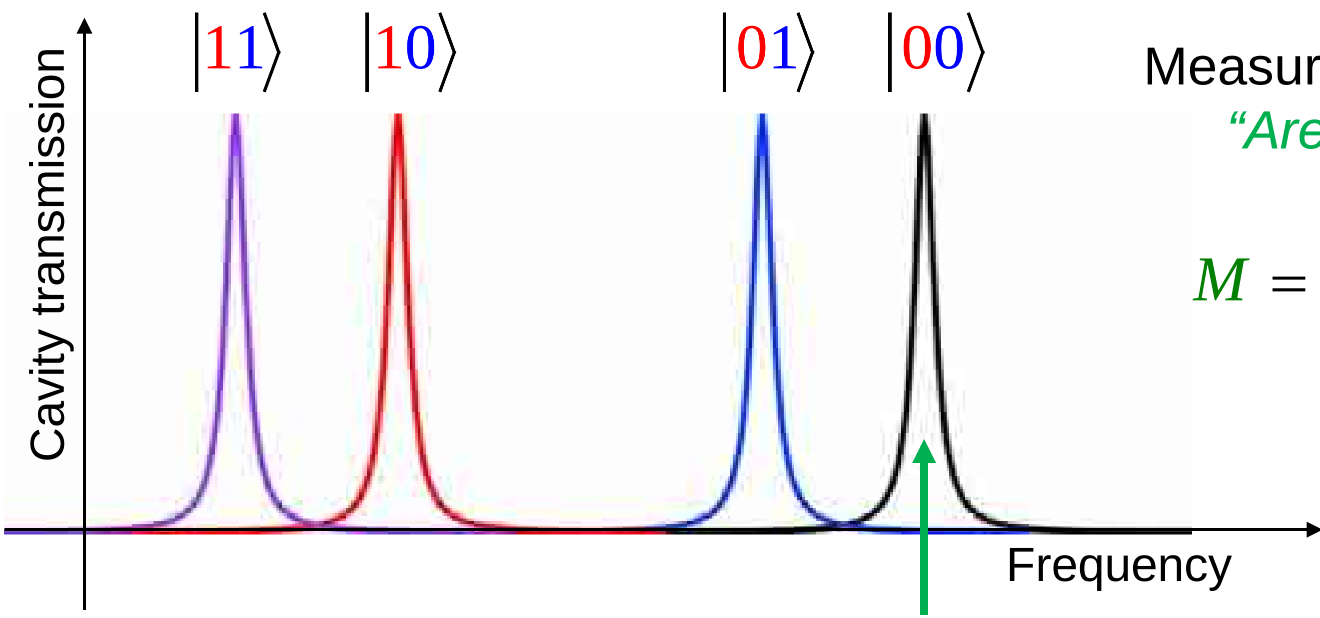
"Strong dispersive cQED"

Schuster, Houck *et al.*, Nature (2007)

# Joint qubit readout via cavity




Measure cavity transmission:

*"Are qubits **both** in their ground state?"*

$$M = |00\rangle\langle 00|$$

$$\propto ZI + IZ + ZZ$$

# Direct access to qubit correlations

**Problem:** How to extract $\rho$ from measurements of the form

$$\langle M \rangle = \langle ZI \rangle + \langle IZ \rangle + \langle ZZ \rangle \quad ?$$

**Answer:** Combine joint readout with one-qubit pre-rotations

**Example:** Extracting $\langle YZ \rangle$

$R_x^{\pm\pi/2}$

$R_x^{0,\pi}$

Joint Dispersive Readout

$$M = |00\rangle\langle 00| = \langle ZI \rangle + \langle IZ \rangle + \langle ZZ \rangle$$

Apply $R_x^{+\pi/2}$, then measure: $\langle YI \rangle + \langle IZ \rangle + \langle YZ \rangle$

+

Apply $R_x^{-\pi/2}$ & $R_x^{+\pi}$, then measure: $-\langle YI \rangle - \langle IZ \rangle + \langle YZ \rangle$

$$2\langle YZ \rangle$$

- It is possible to acquire correlation info. with one measurement channel!
- All Pauli set components are obtained by linear operations on raw data.

Filipp *et al.*, PRL (2009)

# Experimental $N$=2 Pauli sets

$$|\psi\rangle = |10\rangle$$

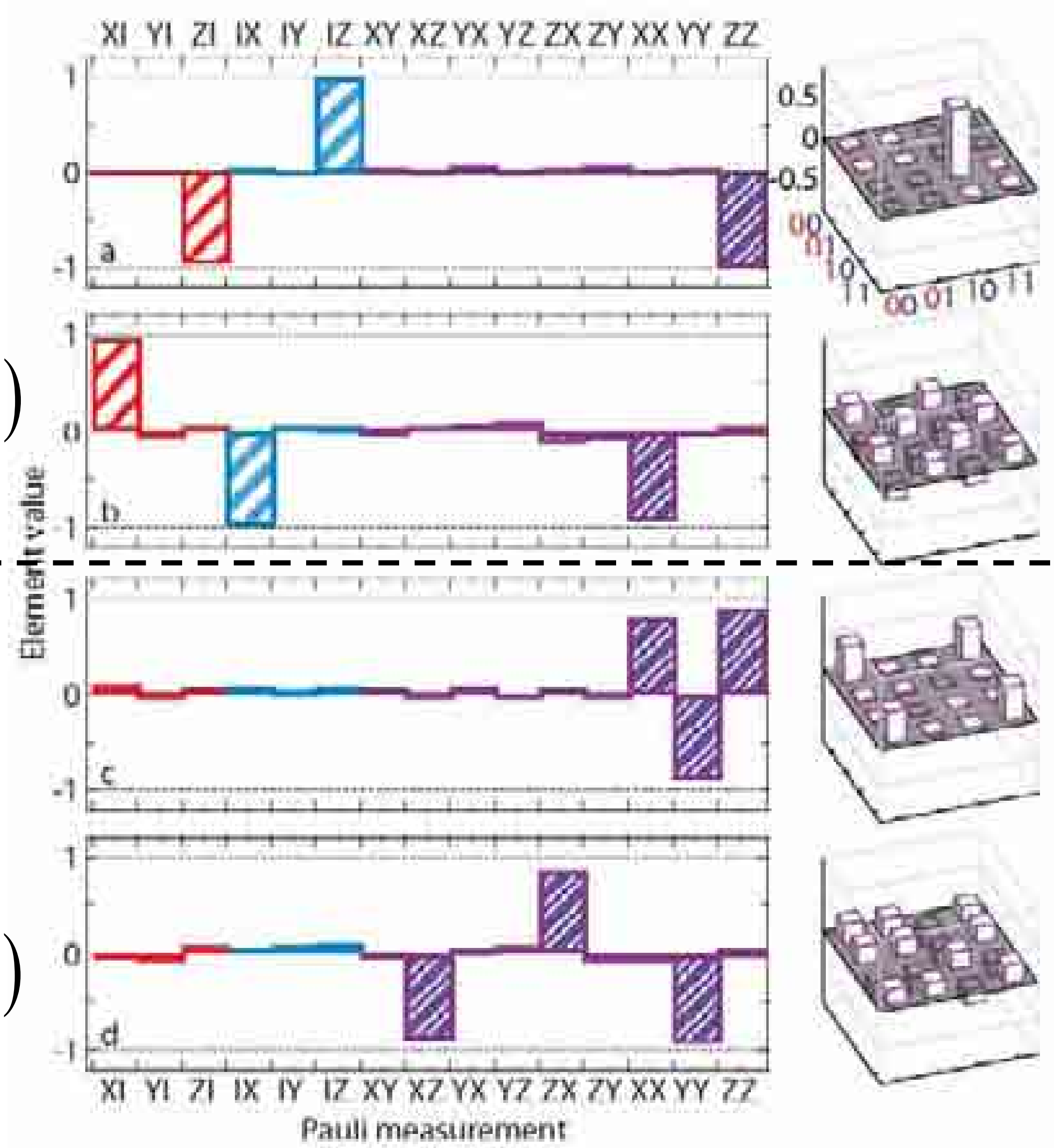
$$|\psi\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

separable

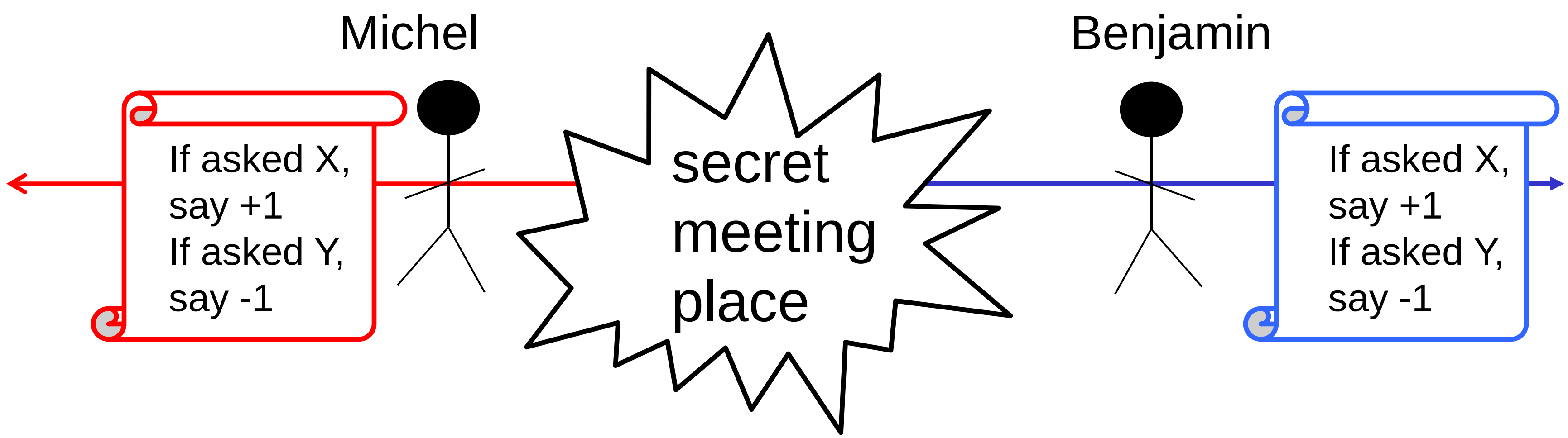
highly-entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$



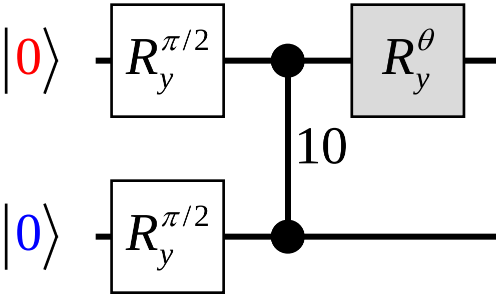
# How quantum is all this, really?



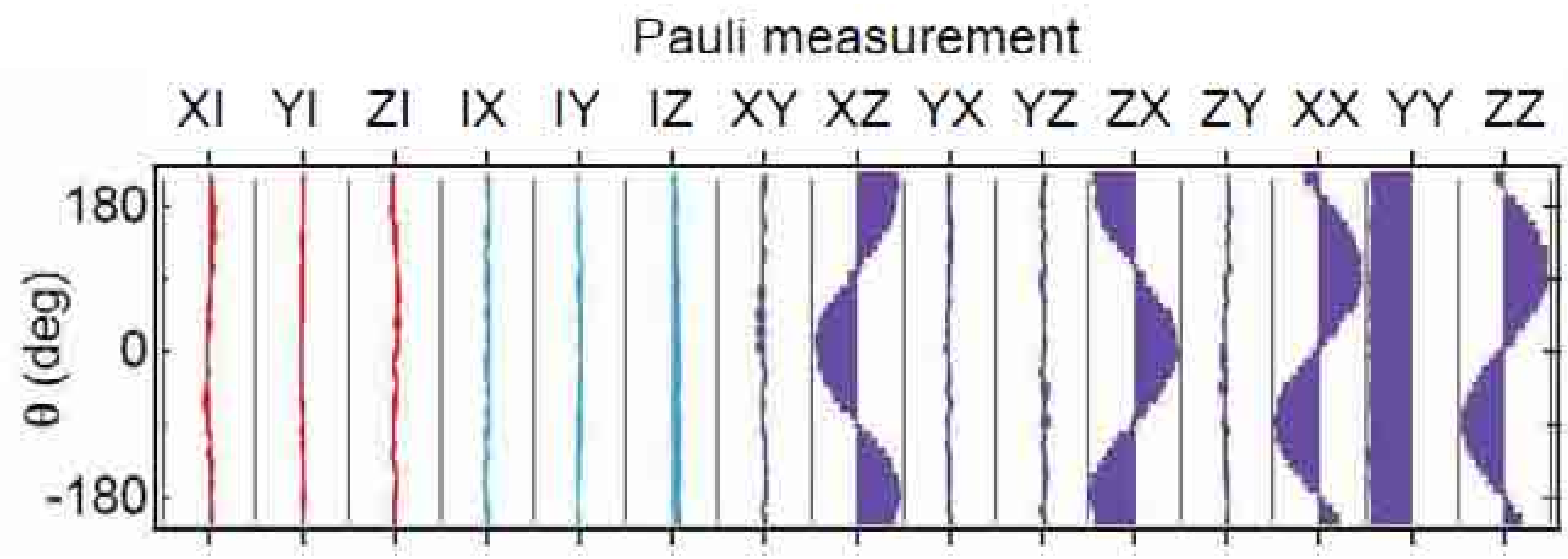

Instruction sets

| $X$ | $X$ | $Y$ | $Y$ | $Z$ | $Z$ | probability |
|---|---|---|---|---|---|---|
| $+1$ | $+1$ | $+1$ | $-1$ | $+1$ | $+1$ | $1/8$ |
| $+1$ | $+1$ | $+1$ | $-1$ | $-1$ | $-1$ | $1/8$ |
| $+1$ | $+1$ | $-1$ | $+1$ | $+1$ | $+1$ | $1/8$ |
| $+1$ | $+1$ | $-1$ | $+1$ | $-1$ | $-1$ | $1/8$ |
| $-1$ | $-1$ | $+1$ | $-1$ | $+1$ | $+1$ | $1/8$ |
| $-1$ | $-1$ | $+1$ | $-1$ | $-1$ | $-1$ | $1/8$ |
| $-1$ | $-1$ | $-1$ | $+1$ | $+1$ | $+1$ | $1/8$ |
| $-1$ | $-1$ | $-1$ | $+1$ | $-1$ | $-1$ | $1/8$ |

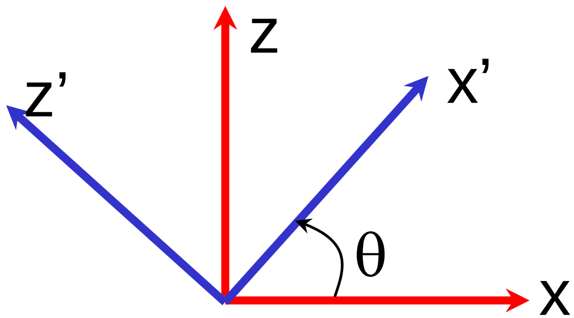
# Entangled-state movie




Prepare a Bell state and
Rotate left qubit about
*y*-axis by $\theta$



# Bell inequality violation




Clauser, Horne,
Shimony & Holt (1969)

LHV bound:

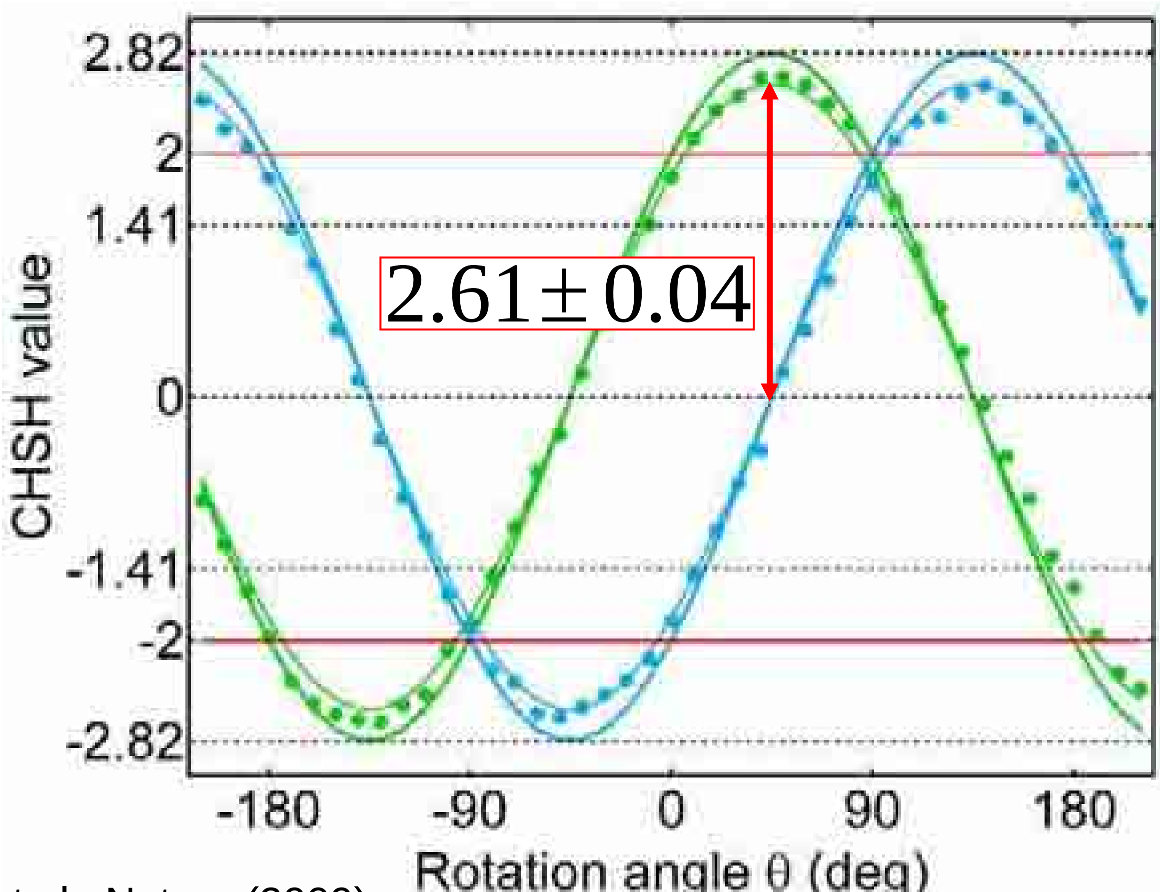
$$\left|\langle CHSH \rangle\right| \le 2$$

***not*** a foolproof
test of
hidden variables…
(system has loopholes)

CHSH operator = entanglement witness

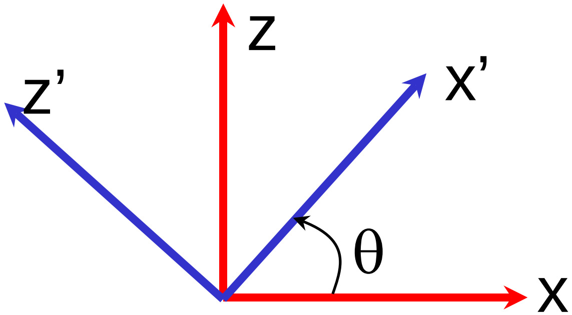
$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

$$\langle CHSH \rangle = \langle XX' \rangle + \langle XZ' \rangle - \langle ZX' \rangle + \langle ZZ' \rangle$$




Also UCSB group,
closing detection loophole, Ansmann et al., Nature (2009)

# Bell inequality as an entanglement witness

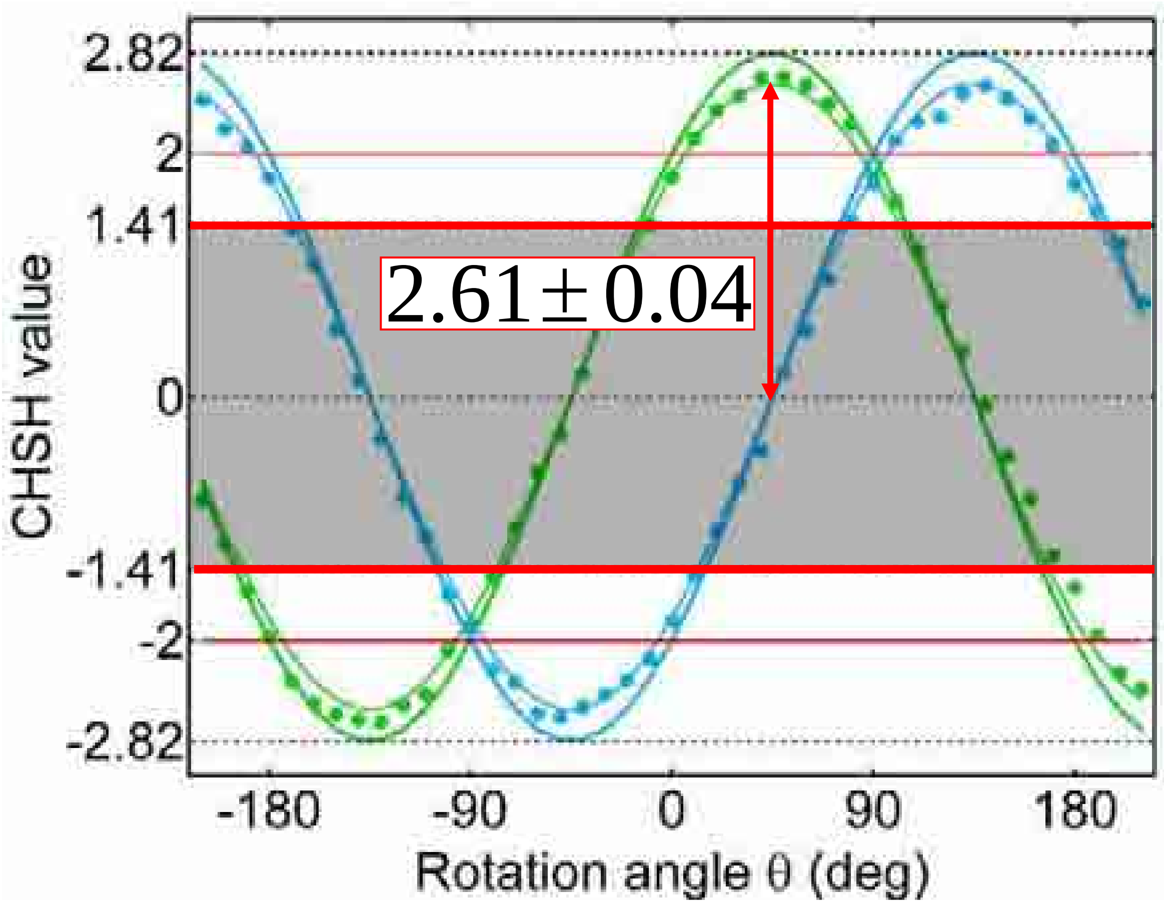



Clauser, Horne, Shimony & Holt (1969)

CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

$$\langle CHSH \rangle = \langle XX' \rangle + \langle XZ' \rangle - \langle ZX' \rangle + \langle ZZ' \rangle$$

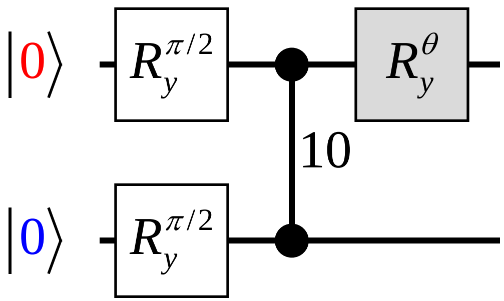



Separable bound:

$$\left|\langle CHSH \rangle\right| \leq \sqrt{2}$$

state is clearly highly entangled!

# Witnessing entanglement

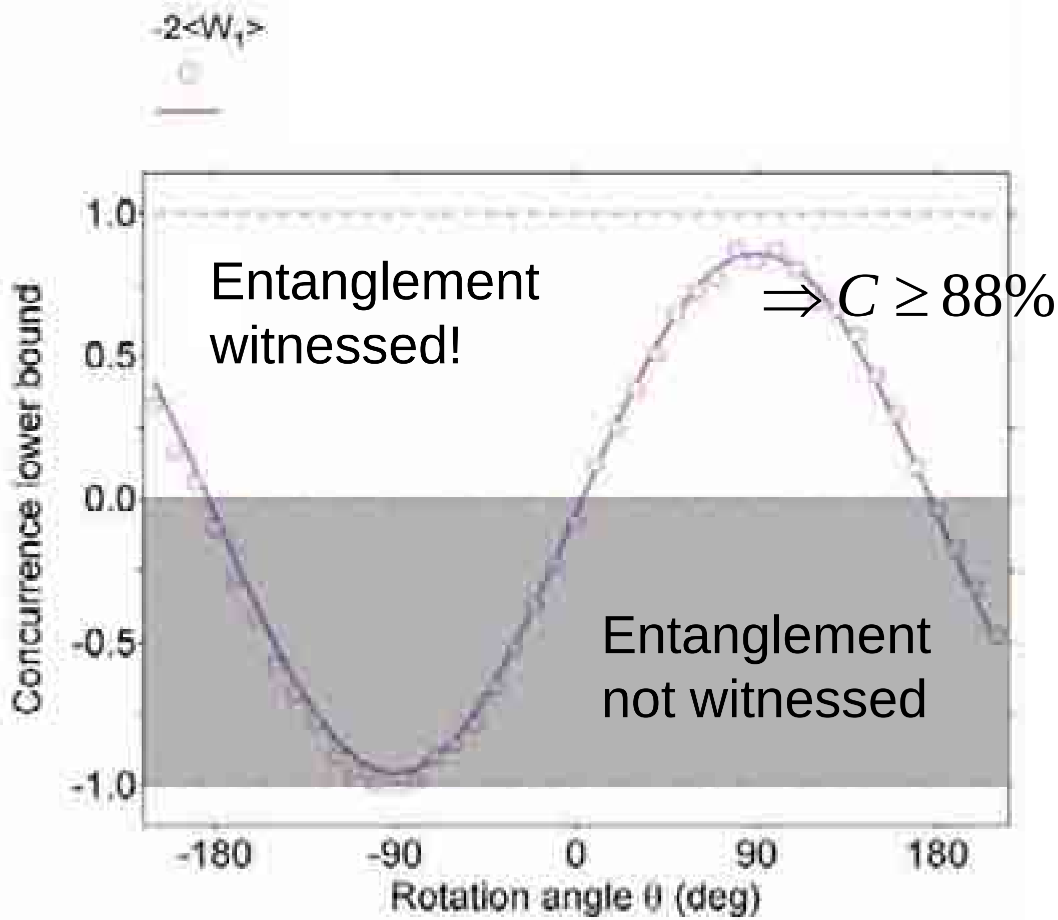



$$\langle W_1 \rangle = \frac{1}{4}\left(1 - \langle XX \rangle + \langle YY \rangle - \langle ZZ \rangle\right)$$

$$\langle W_2 \rangle = \frac{1}{4}\left(1 + \langle XX \rangle - \langle YY \rangle - \langle ZZ \rangle\right)$$

$$\langle W_3 \rangle = \frac{1}{4}\left(1 - \langle XX \rangle - \langle YY \rangle + \langle ZZ \rangle\right)$$

$$\langle W_4 \rangle = \frac{1}{4}\left(1 + \langle XX \rangle + \langle YY \rangle + \langle ZZ \rangle\right)$$




- Entanglement is witnessed (by either $W_1$ or $W_4$) at all angles!
- Two of the witnesses were looking the other way!

# Beyond two qubits

Exploration of quantum error correction starts with three-qubit entanglement

DiCarlo *et al*., arXiv 1004.4324 (2010)
Reed et al., arXiv 1004.4323 (2010)

# The density matrix for $N$=3

$$\rho = \frac{1}{8} \sum_{j,k,l \in \{i,x,y,z\}} \langle \sigma_j \sigma_k \sigma_l \rangle \; \sigma_j \sigma_k \sigma_l$$

Knowing the three-qubit state = expectation values of **63** Pauli operators

$\vec{P}_1 = (\langle XII \rangle, \langle YII \rangle, \langle ZII \rangle)$ Polarization of qubit 1

$\vec{P}_2 = (\langle IXI \rangle, \langle IYI \rangle, \langle IZI \rangle)$ Polarization of qubit 2

$\vec{P}_3 = (\langle IIX \rangle, \langle IIY \rangle, \langle IIZ \rangle)$ Polarization of qubit 3

2-qubit correlations

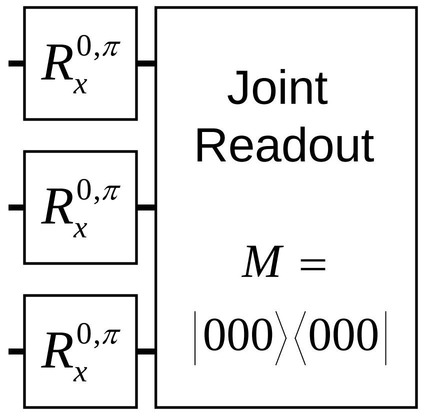
$$\vec{\vec{P}}_{12} = (\langle XXI \rangle, \langle XYI \rangle, \langle XZI \rangle, \langle YXI \rangle, \langle YYI \rangle, \langle YZI \rangle, \langle ZXI \rangle, \langle ZYI \rangle, \langle ZZI \rangle)$$
$$\vec{\vec{P}}_{13} = (\langle XIX \rangle, \langle XIY \rangle, \langle XIZ \rangle, \langle YIX \rangle, \langle YIY \rangle, \langle YIZ \rangle, \langle ZIX \rangle, \langle ZIY \rangle, \langle ZIZ \rangle)$$
$$\vec{\vec{P}}_{23} = (\langle IXX \rangle, \langle IXY \rangle, \langle IXZ \rangle, \langle IYX \rangle, \langle IYY \rangle, \langle IYZ \rangle, \langle IZX \rangle, \langle IZY \rangle, \langle IZZ \rangle)$$

3-qubit correlations

$$\vec{\vec{\vec{P}}}_{123} = (\langle XXX \rangle, \langle XXY \rangle, \langle XXZ \rangle, \quad \ldots \quad, \langle ZZX \rangle, \langle ZZY \rangle, \langle ZZZ \rangle)$$

# 3-Qubit state tomography

The trick still works!
Combining joint readout with one-qubit "analysis" gives access to all 3-qubit Pauli operators, only more rotations are necessary.




$$M = |000\rangle\langle 000|$$

$$\propto ZII + IZI + IIZ + ZZI + ZIZ + IZZ + ZZZ$$

Example: extract $\langle ZZZ \rangle$

no pre-rotation: $+\langle ZII\rangle + \langle IZI\rangle + \langle IIZ\rangle + \langle ZZI\rangle + \langle ZIZ\rangle + \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q1 and Q2: $-\langle ZII\rangle - \langle IZI\rangle + \langle IIZ\rangle + \langle ZZI\rangle - \langle ZIZ\rangle - \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q1 and Q3: $-\langle ZII\rangle + \langle IZI\rangle - \langle IIZ\rangle - \langle ZZI\rangle + \langle ZIZ\rangle - \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q2 and Q3: $+\langle ZII\rangle - \langle IZI\rangle - \langle IIZ\rangle - \langle ZZI\rangle - \langle ZIZ\rangle + \langle IZZ\rangle + \langle ZZZ\rangle$

---

$$4\langle ZZZ\rangle$$

# Experiment: Doing nothing… but very well!

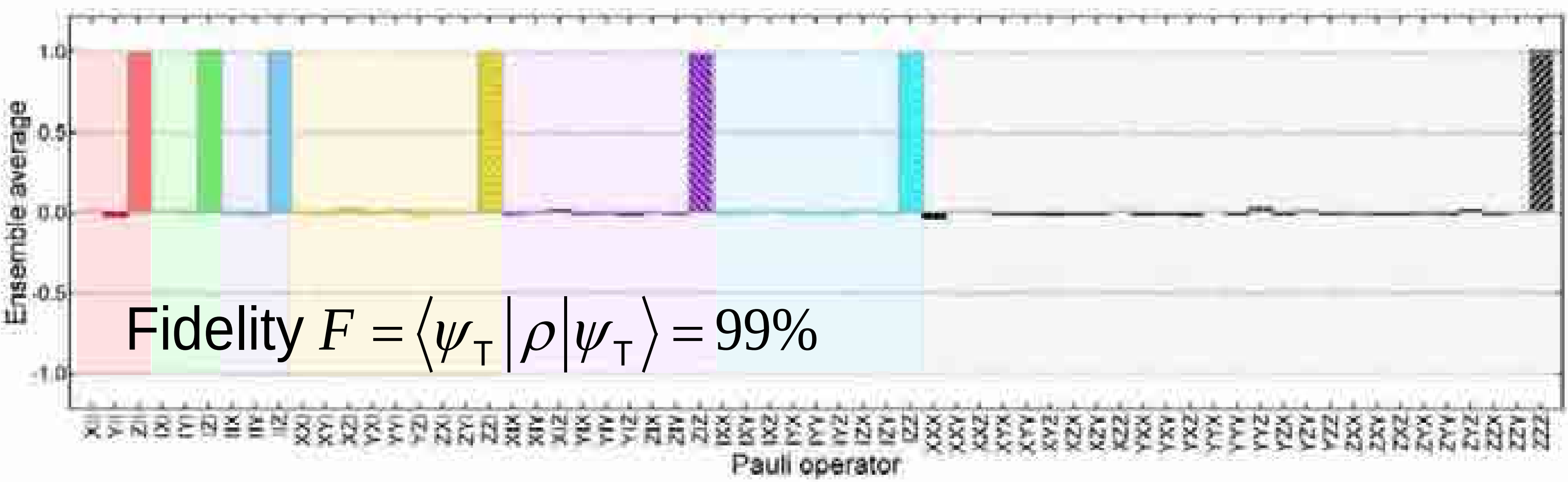
$|0\rangle$

$|0\rangle$

$|0\rangle$

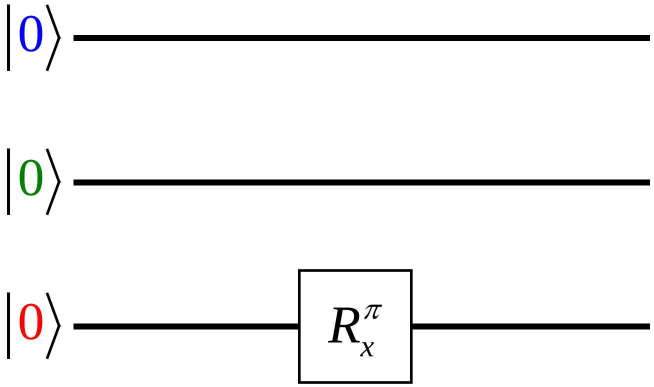
$$|\psi_{\mathrm{T}}\rangle = |000\rangle$$

$\vec{P}_1$ $\vec{P}_2$ $\vec{P}_3$ $\vec{\vec{P}}_{12}$ $\vec{\vec{P}}_{13}$ $\vec{\vec{P}}_{23}$ $\vec{\vec{\vec{P}}}_{123}$

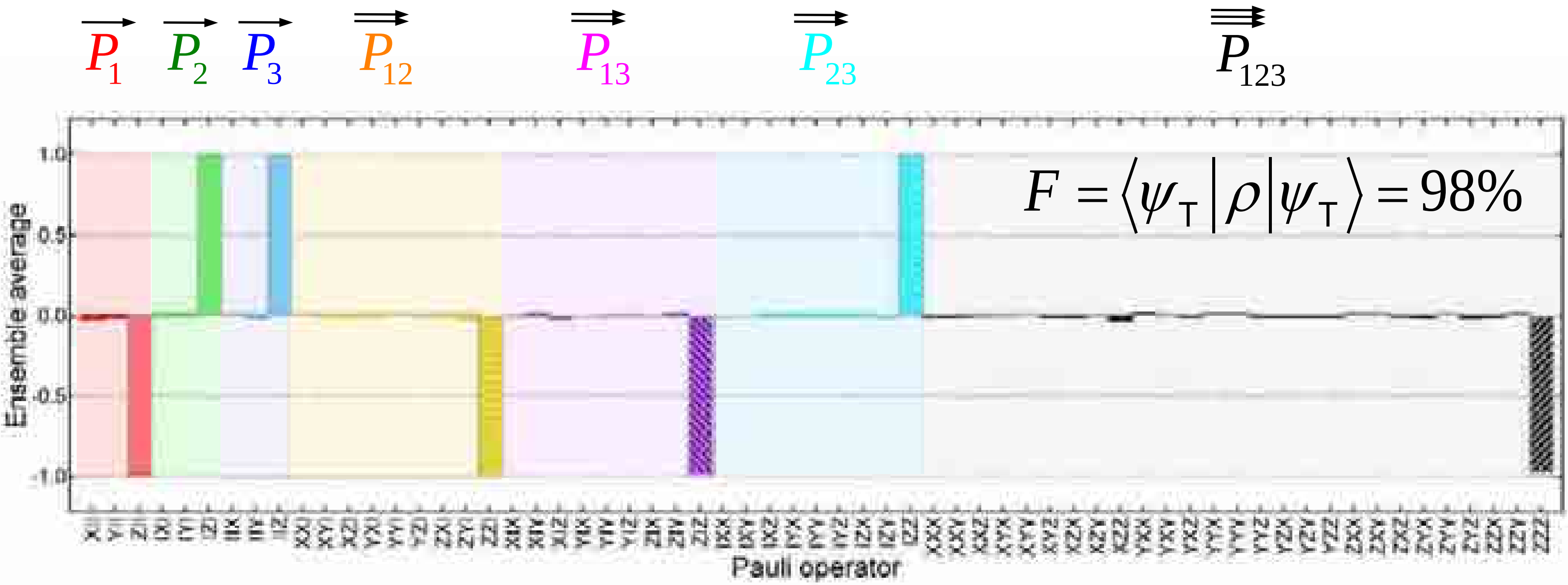



Fidelity $F = \langle \psi_{\mathrm{T}} | \rho | \psi_{\mathrm{T}} \rangle = 99\%$

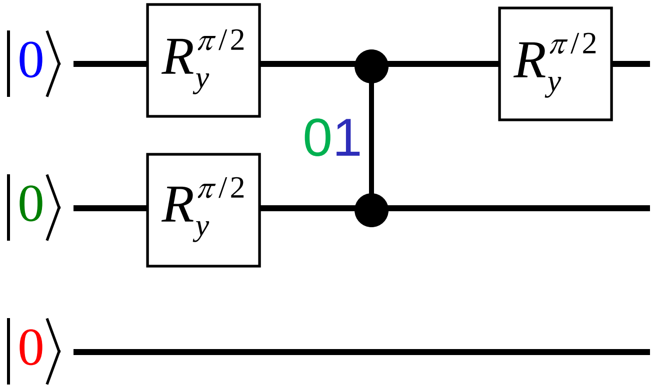
# A less trivial separable state



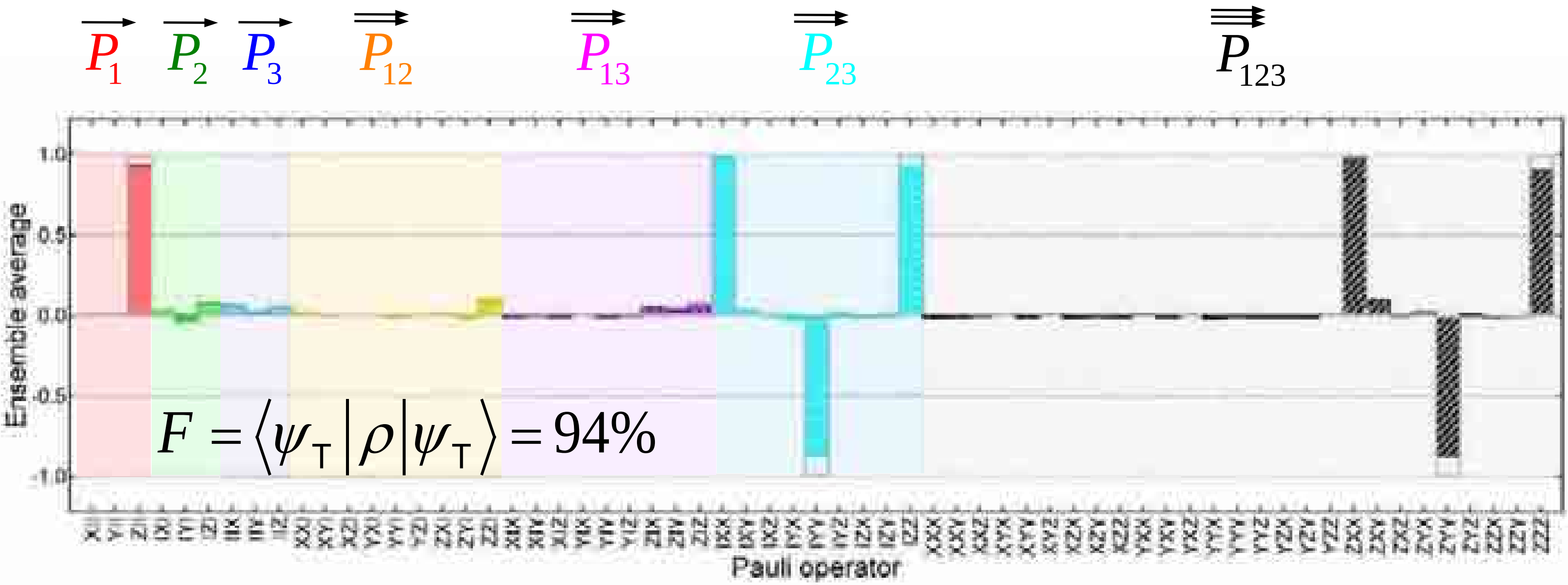

$$|\psi_{\mathrm{T}}\rangle = |100\rangle$$



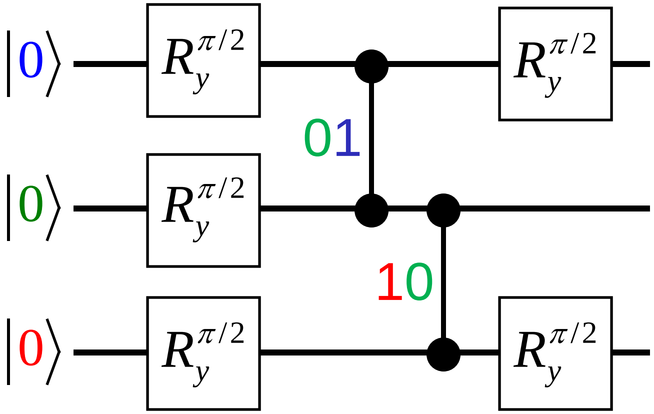
# Two-qubit entanglement in a 3-qubit register




$$|\psi_T\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

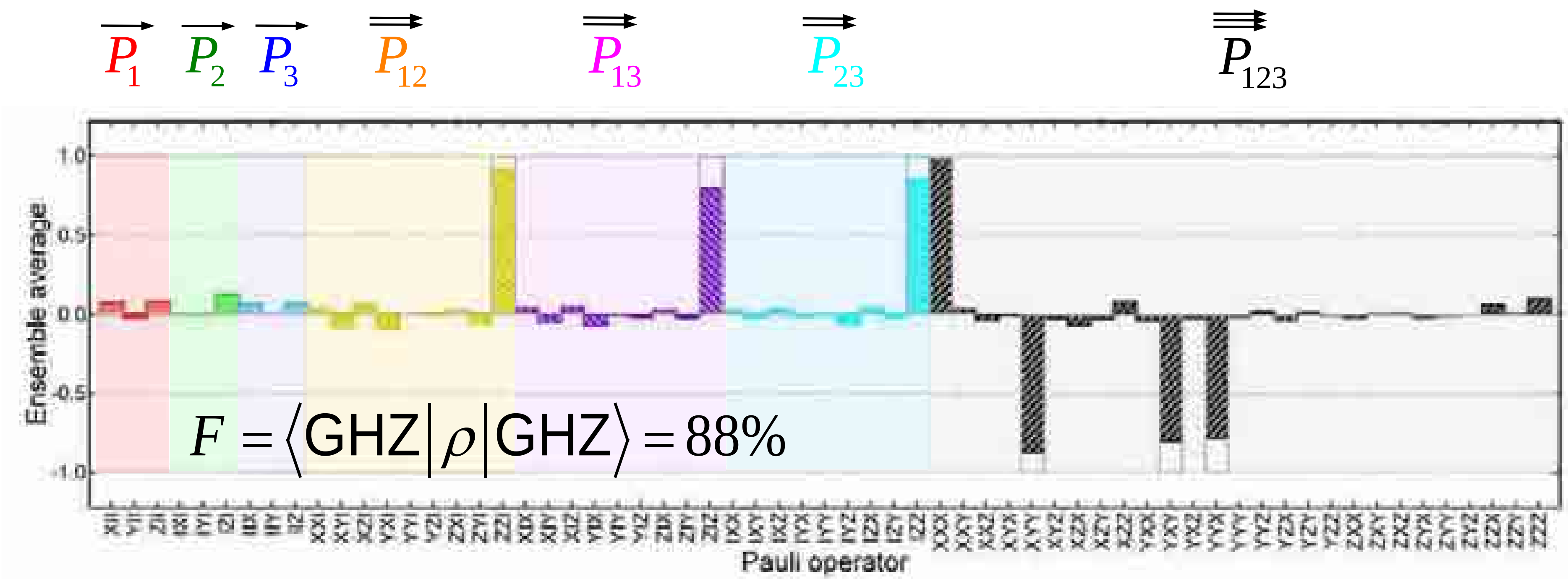


# Three-qubit entanglement




Greenberger-Horne-Zeilinger state

$$|\psi_{\mathrm{T}}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

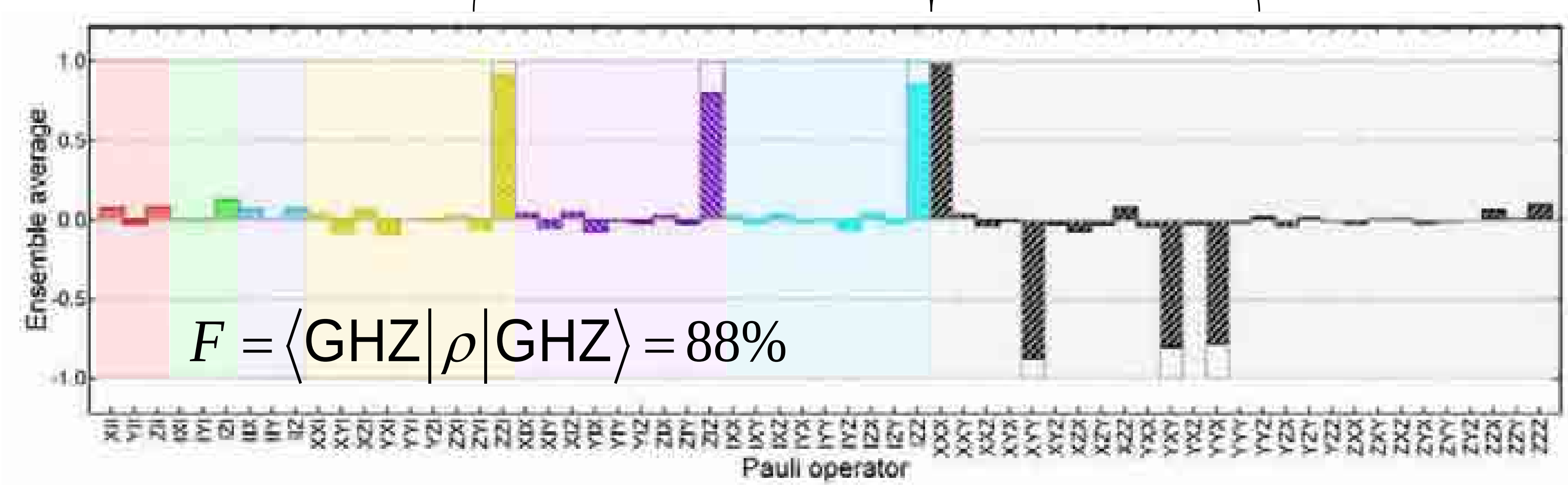



$$F = \langle \mathrm{GHZ} | \rho | \mathrm{GHZ} \rangle = 88\%$$

# What is special about GHZ?

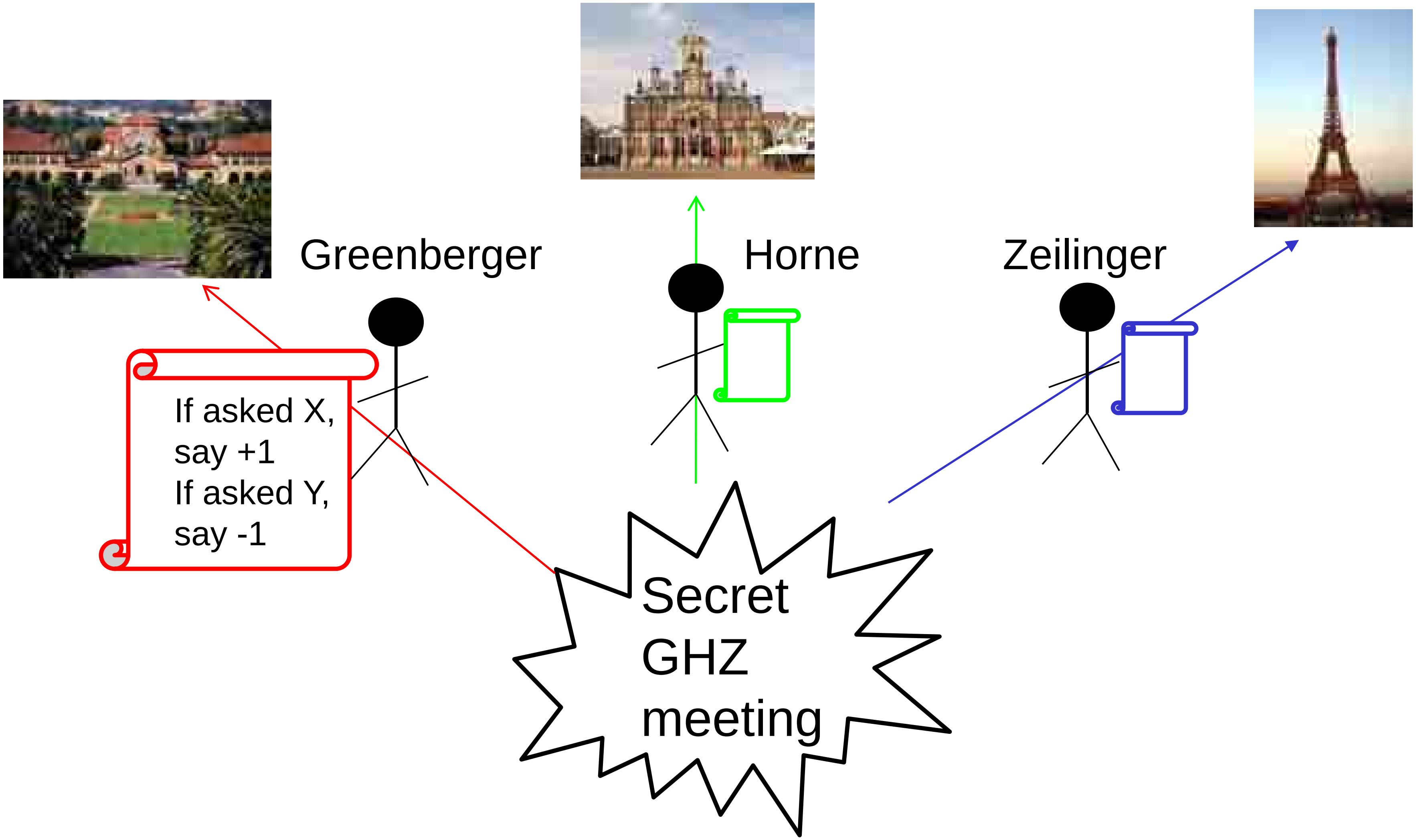
- Useful for one-shot (non-statistical) tests of QM.
  Four maximal 3-qubit correlators
  $$\langle XXX \rangle = -\langle XYY \rangle = -\langle YXY \rangle = -\langle YYX \rangle = 1$$
  are incompatible with local realism.

- States $\alpha|000\rangle + \beta|111\rangle$ are parity eigenstates
  useful for basic quantum error correction:
  $$\langle ZZI \rangle = \langle ZIZ \rangle = \langle IZZ \rangle = 1$$




$$F = \langle \mathrm{GHZ} | \rho | \mathrm{GHZ} \rangle = 88\%$$

# 3QE is qualitatively different: the Mermin-GHZ test



# 3QE is qualitatively different: the Mermin-GHZ test

- 64 possible instruction sets

| G | | H | | Z | | G | | H | | Z | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *X* | *Y* | *X* | *Y* | *X* | *Y* | *X* | *Y* | *X* | *Y* | *X* | *Y* |
| +1 | +1 | +1 | +1 | +1 | +1 | −1 | +1 | +1 | +1 | +1 | +1 |
| +1 | +1 | +1 | +1 | +1 | -1 | −1 | +1 | +1 | +1 | +1 | -1 |
| +1 | +1 | +1 | +1 | -1 | +1 | −1 | +1 | +1 | +1 | -1 | +1 |
| +1 | +1 | +1 | +1 | -1 | -1 | −1 | +1 | +1 | +1 | -1 | -1 |
| +1 | +1 | +1 | -1 | +1 | +1 | −1 | +1 | +1 | -1 | +1 | +1 |
| +1 | +1 | +1 | -1 | +1 | -1 | −1 | +1 | +1 | -1 | +1 | -1 |
| +1 | +1 | +1 | -1 | -1 | +1 | −1 | +1 | +1 | -1 | -1 | +1 |
| +1 | +1 | +1 | -1 | -1 | -1 | −1 | +1 | +1 | -1 | -1 | -1 |
| +1 | +1 | -1 | +1 | +1 | +1 | −1 | +1 | -1 | +1 | +1 | +1 |
| +1 | +1 | -1 | +1 | +1 | -1 | −1 | +1 | -1 | +1 | +1 | -1 |
| +1 | +1 | -1 | +1 | -1 | +1 | −1 | +1 | -1 | +1 | -1 | +1 |
| +1 | +1 | -1 | +1 | -1 | -1 | −1 | +1 | -1 | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 | +1 | +1 | −1 | +1 | -1 | -1 | +1 | +1 |
| +1 | +1 | -1 | -1 | +1 | -1 | −1 | +1 | -1 | -1 | +1 | -1 |
| +1 | +1 | -1 | -1 | -1 | +1 | −1 | +1 | -1 | -1 | -1 | +1 |
| +1 | +1 | -1 | -1 | -1 | -1 | −1 | +1 | -1 | -1 | -1 | -1 |
| +1 | -1 | +1 | +1 | +1 | +1 | −1 | -1 | +1 | +1 | +1 | +1 |
| +1 | -1 | +1 | +1 | +1 | -1 | −1 | -1 | +1 | +1 | +1 | -1 |
| +1 | -1 | +1 | +1 | -1 | +1 | −1 | -1 | +1 | +1 | -1 | +1 |
| +1 | -1 | +1 | +1 | -1 | -1 | −1 | -1 | +1 | +1 | -1 | -1 |
| +1 | -1 | +1 | -1 | +1 | +1 | −1 | -1 | +1 | -1 | +1 | +1 |
| +1 | -1 | +1 | -1 | +1 | -1 | −1 | -1 | +1 | -1 | +1 | -1 |
| +1 | -1 | +1 | -1 | -1 | +1 | −1 | -1 | +1 | -1 | -1 | +1 |
| +1 | -1 | +1 | -1 | -1 | -1 | −1 | -1 | +1 | -1 | -1 | -1 |
| +1 | -1 | +1 | +1 | +1 | +1 | −1 | -1 | +1 | +1 | +1 | +1 |
| +1 | -1 | -1 | +1 | +1 | -1 | −1 | -1 | -1 | +1 | +1 | -1 |
| +1 | -1 | -1 | +1 | -1 | +1 | −1 | -1 | -1 | +1 | -1 | +1 |
| +1 | -1 | -1 | +1 | -1 | -1 | −1 | -1 | -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | -1 | +1 | +1 | −1 | -1 | -1 | -1 | +1 | +1 |
| +1 | -1 | -1 | -1 | +1 | -1 | −1 | -1 | -1 | -1 | +1 | -1 |
| +1 | -1 | -1 | -1 | -1 | +1 | −1 | -1 | -1 | -1 | -1 | +1 |
| +1 | -1 | -1 | -1 | -1 | -1 | −1 | -1 | -1 | -1 | -1 | -1 |

# 3QE is qualitatively different: the Mermin-GHZ test

- 64 possible instruction sets

- Imposing $XYY = -1$
reduces the instruction sets to 32

| G | | H | | Z | |
|---|---|---|---|---|---|
| $X$ | $Y$ | $X$ | $Y$ | $X$ | $Y$ |
| +1 | +1 | +1 | +1 | +1 | -1 |
| +1 | +1 | +1 | +1 | -1 | -1 |
| +1 | +1 | +1 | -1 | +1 | +1 |
| +1 | +1 | +1 | -1 | -1 | +1 |
| +1 | +1 | -1 | +1 | +1 | -1 |
| +1 | +1 | -1 | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 | +1 | +1 |
| +1 | +1 | -1 | -1 | -1 | +1 |
| +1 | -1 | +1 | +1 | +1 | -1 |
| +1 | -1 | +1 | +1 | -1 | -1 |
| +1 | -1 | +1 | -1 | +1 | +1 |
| +1 | -1 | +1 | -1 | -1 | +1 |
| +1 | -1 | -1 | +1 | +1 | -1 |
| +1 | -1 | -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | -1 | +1 | +1 |
| +1 | -1 | -1 | -1 | -1 | +1 |
| −1 | +1 | +1 | +1 | +1 | +1 |
| −1 | +1 | +1 | +1 | -1 | +1 |
| −1 | +1 | +1 | -1 | +1 | -1 |
| −1 | +1 | +1 | -1 | -1 | -1 |
| −1 | +1 | -1 | +1 | +1 | +1 |
| −1 | +1 | -1 | +1 | -1 | +1 |
| −1 | +1 | -1 | -1 | +1 | -1 |
| −1 | +1 | -1 | -1 | -1 | -1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | +1 | +1 | -1 | +1 |
| −1 | -1 | +1 | -1 | +1 | -1 |
| −1 | -1 | +1 | -1 | -1 | -1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | -1 | +1 | -1 | +1 |
| −1 | -1 | -1 | -1 | +1 | -1 |
| −1 | -1 | -1 | -1 | -1 | -1 |

# 3QE is qualitatively different: the Mermin-GHZ test

- 64 possible instruction sets

- Imposing $XYY = -1$
reduces the instruction sets to 32

- Imposing $YXY = -1$
reduces the instruction sets to 16

| G | | H | | Z | |
|---|---|---|---|---|---|
| $X$ | $Y$ | $X$ | $Y$ | $X$ | $Y$ |
| +1 | +1 | +1 | +1 | +1 | -1 |
| +1 | +1 | +1 | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 | +1 | +1 |
| +1 | +1 | -1 | -1 | -1 | +1 |
| +1 | -1 | +1 | -1 | +1 | +1 |
| +1 | -1 | -1 | +1 | +1 | -1 |
| +1 | -1 | -1 | +1 | -1 | -1 |
| −1 | +1 | +1 | -1 | +1 | -1 |
| −1 | +1 | +1 | -1 | -1 | -1 |
| −1 | +1 | -1 | +1 | +1 | +1 |
| −1 | +1 | -1 | +1 | -1 | +1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | +1 | +1 | -1 | +1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | -1 | -1 | +1 | -1 |
| −1 | -1 | -1 | -1 | -1 | -1 |

# 3QE is qualitatively different: the Mermin-GHZ test

- 64 possible instruction sets

- Imposing $XYY = -1$
reduces the instruction sets to 32

- Imposing $YXY = -1$
reduces the instruction sets to 16

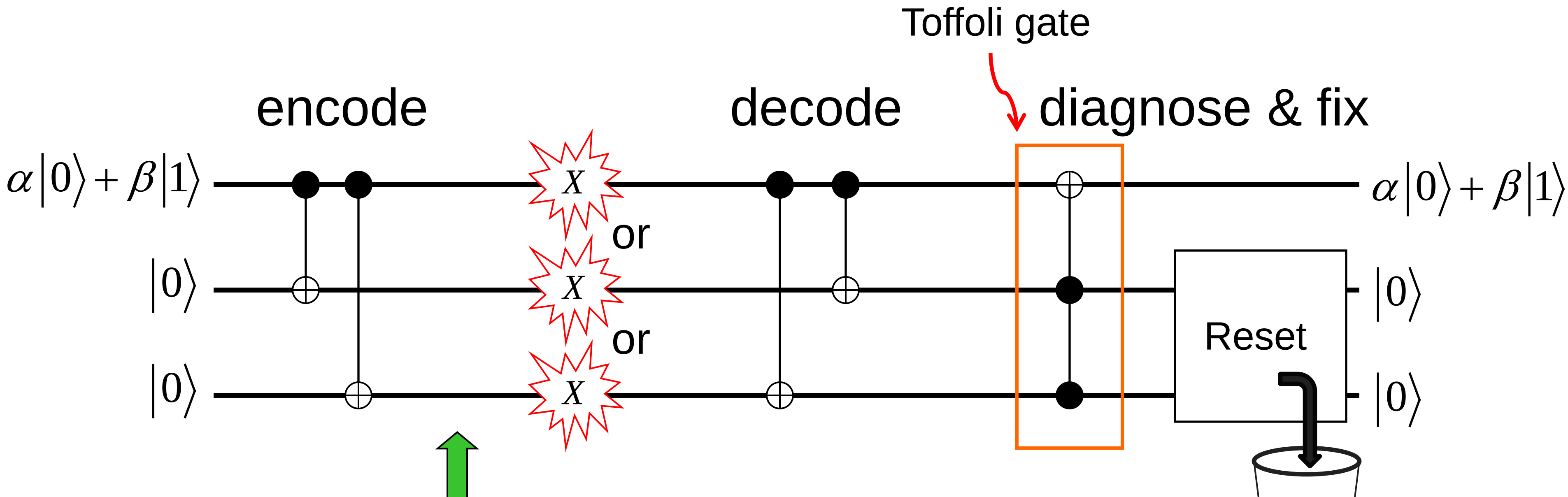
- Imposing $YYX = -1$
reduces the instruction sets to 8

| G | | H | | Z | |
|---|---|---|---|---|---|
| $X$ | $Y$ | $X$ | $Y$ | $X$ | $Y$ |
| +1 | +1 | +1 | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 | +1 | +1 |
| +1 | -1 | -1 | +1 | +1 | -1 |
| −1 | +1 | +1 | -1 | +1 | -1 |
| −1 | +1 | -1 | +1 | -1 | +1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | +1 | +1 | +1 | +1 |
| −1 | -1 | -1 | -1 | -1 | -1 |

$$\Rightarrow XXX = -1$$

None of the 8 remaining instruction sets
is consistent with $XXX = +1$ !!!!

# The bit-flip error correction code




$\alpha|000\rangle + \beta|111\rangle$

GHZ-like state

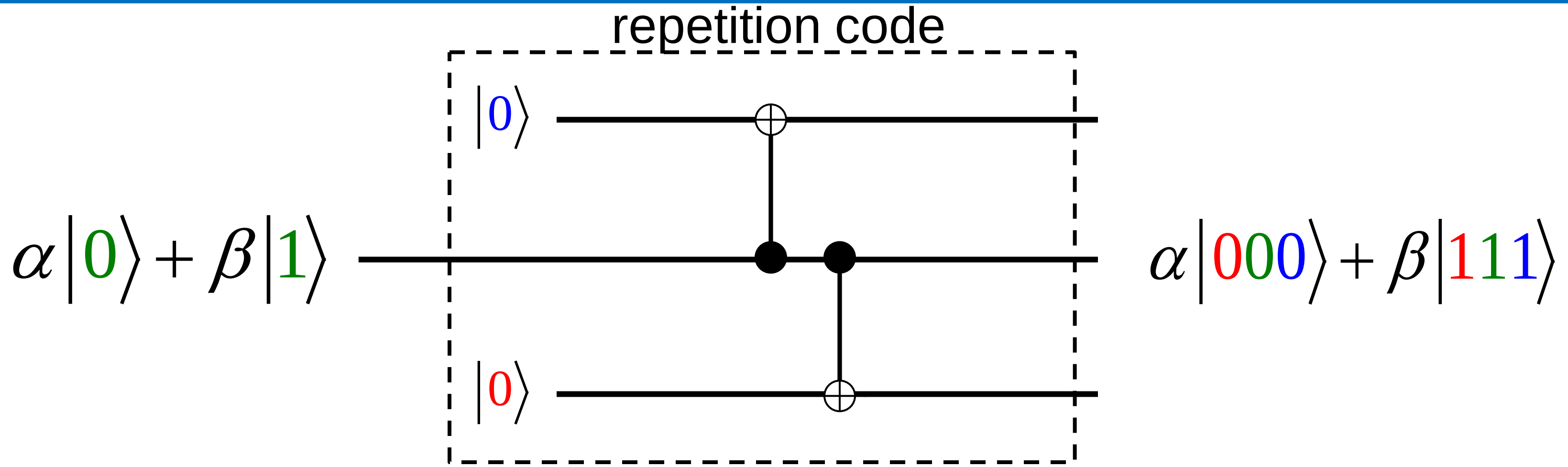
$\alpha|100\rangle + \beta|011\rangle$

$$\alpha|111\rangle + \beta|011\rangle$$
$$= (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

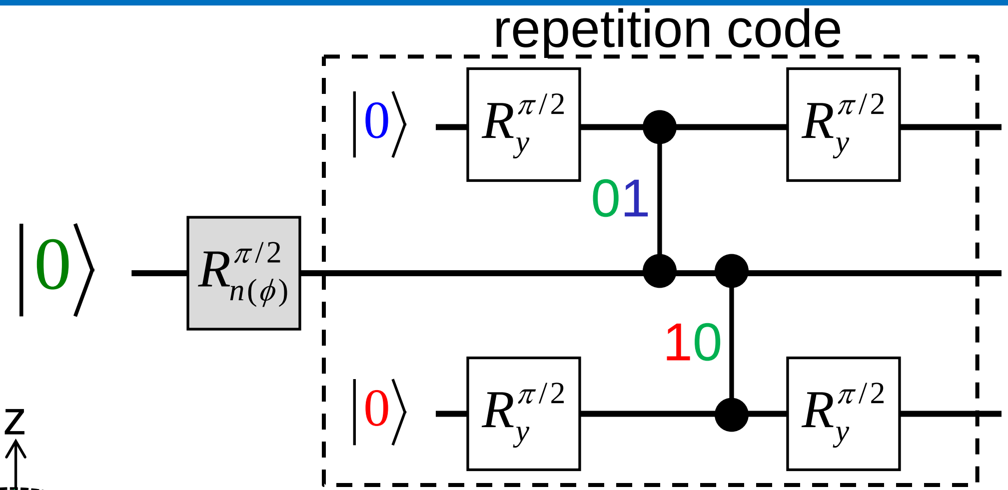
$(\alpha|0\rangle + \beta|1\rangle) \otimes |11\rangle$

Fast reset: Reed *et al*., APL (2010)

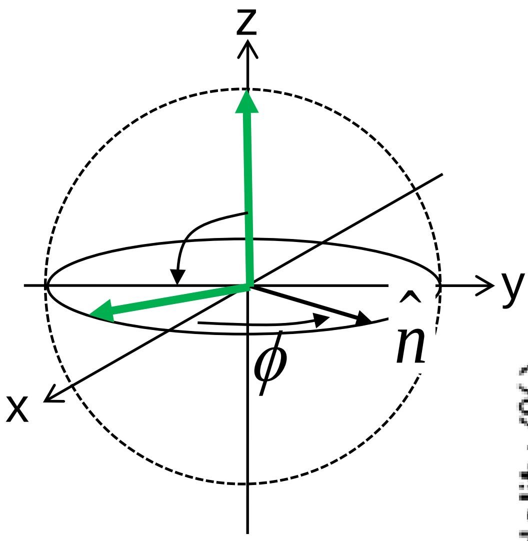
# Encoding a logical qubit for error correction

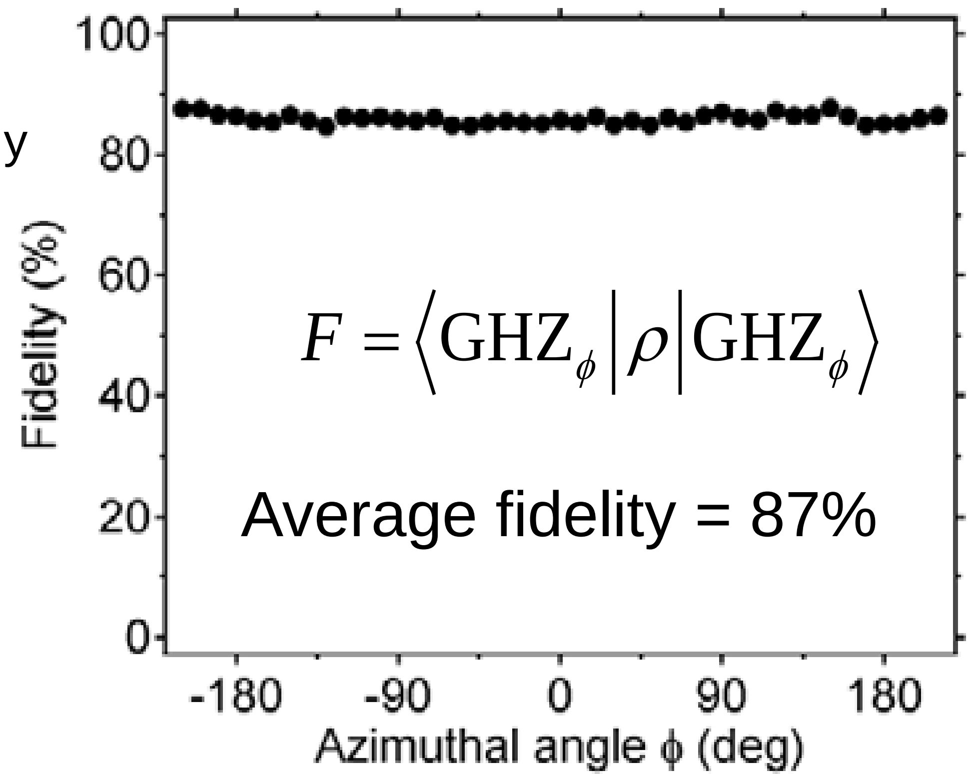


# Encoding a logical qubit for error correction

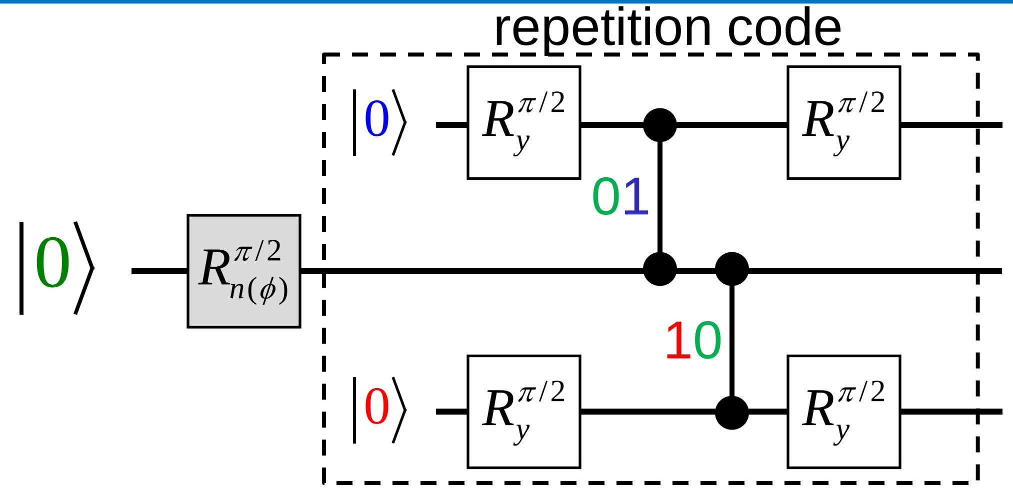



$$\left|\mathrm{GHZ}_\phi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|000\right\rangle - ie^{i\phi}\left|111\right\rangle\right)$$

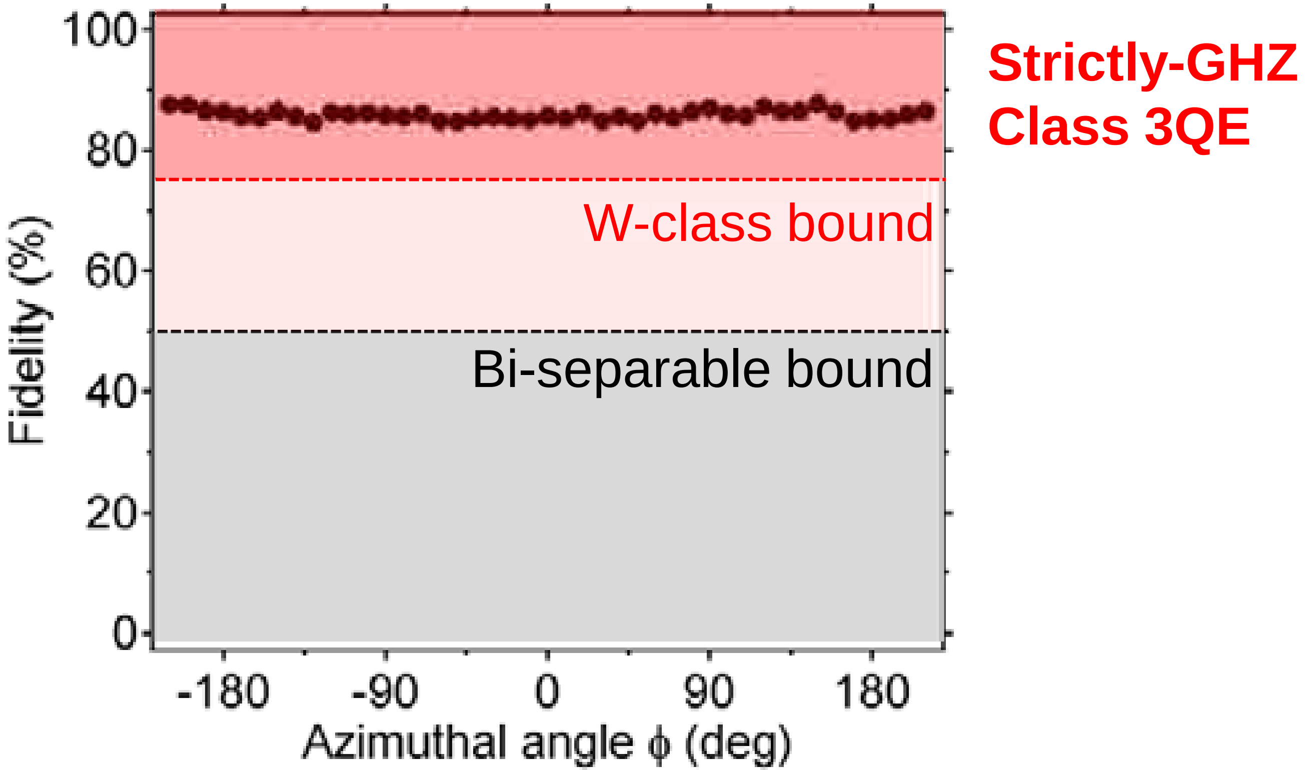


# Encoding a logical qubit for error correction



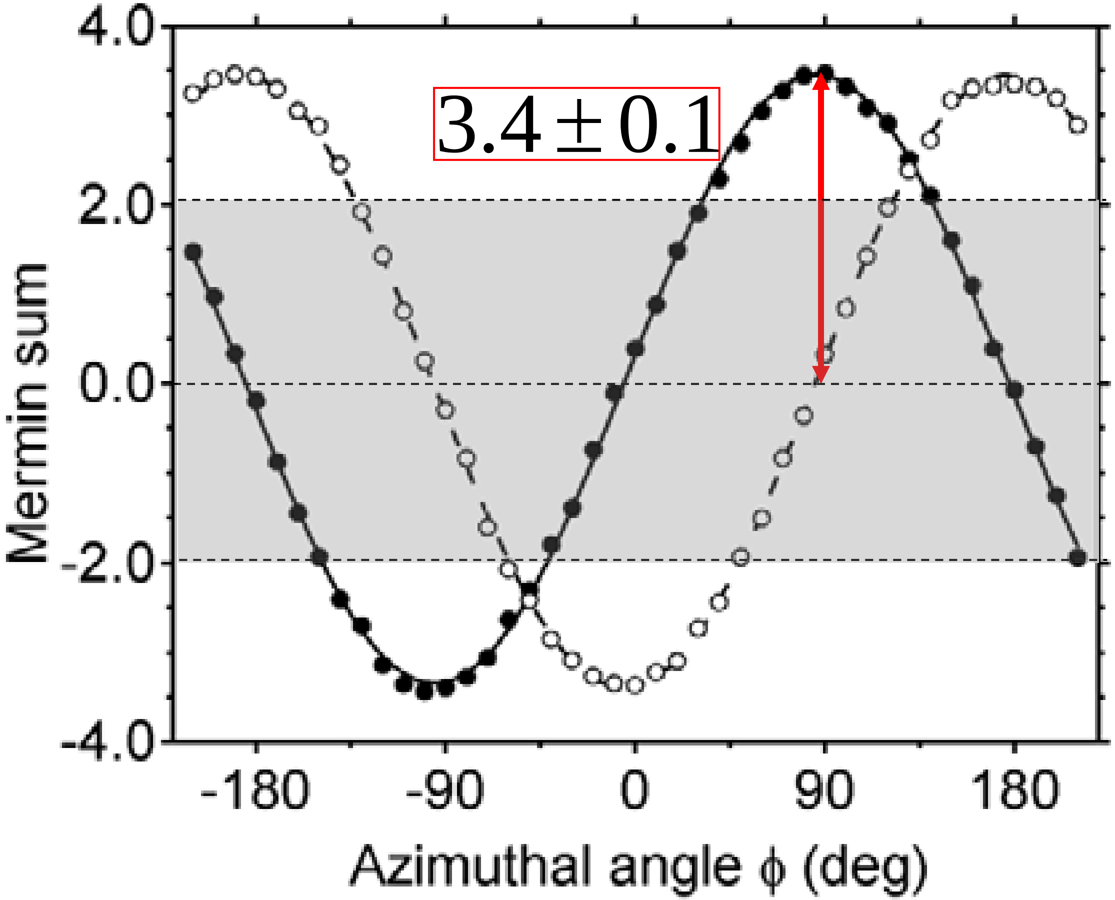

$$|\mathrm{GHZ}_\phi\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - ie^{i\phi}|111\rangle\right)$$



# Witnessing entanglement with Mermin-Bell inequalities

● $\langle M \rangle = \langle XXX \rangle - \langle XYY \rangle - \langle YXY \rangle - \langle YYX \rangle$

○ $\langle M \rangle = \langle YYY \rangle - \langle YXX \rangle - \langle XYX \rangle - \langle XXY \rangle$




$|\langle M \rangle| \leq 2$

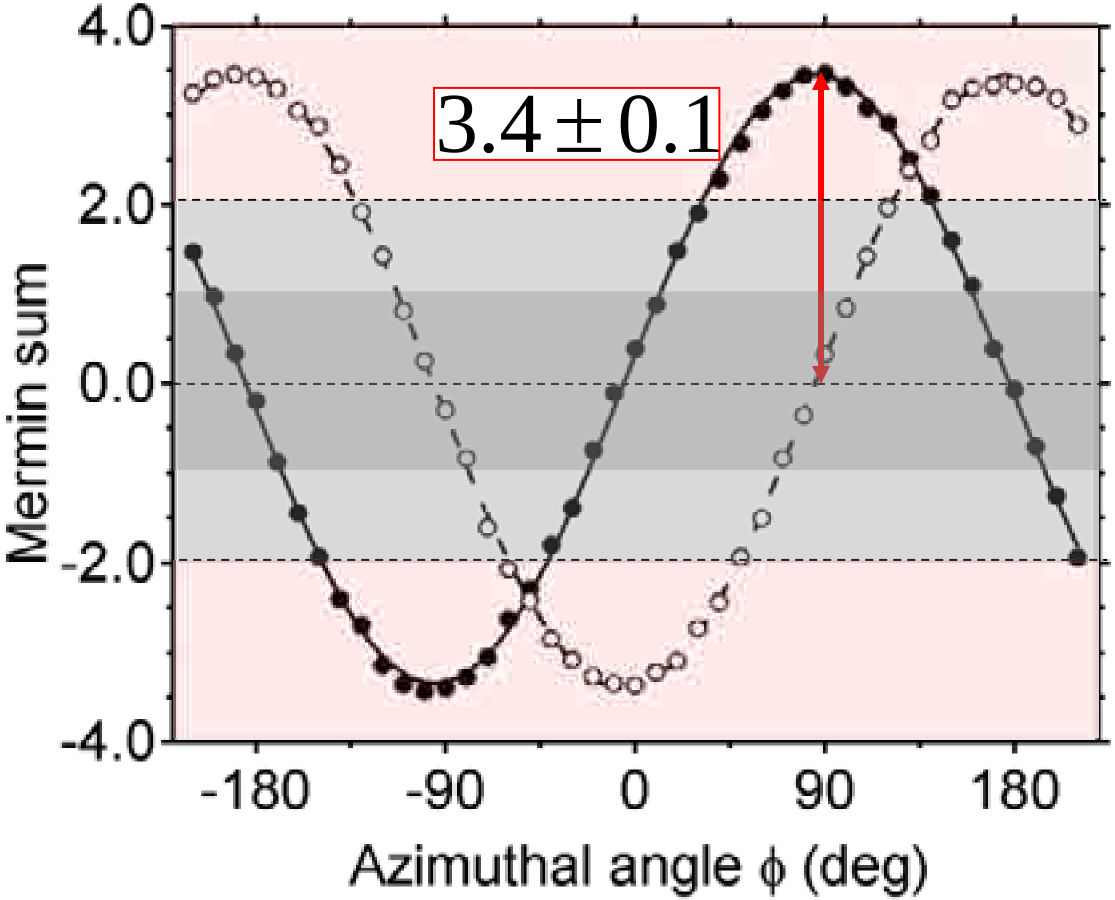
Local-Hidden-Variable bound

Mermin, PRL (1990)
Tóth & Gühne, PRA (2005)
Roy, PRL (2005)

# Witnessing entanglement with Mermin-Bell inequalities

● $\langle M \rangle = \langle XXX \rangle - \langle XYY \rangle - \langle YXY \rangle - \langle YYX \rangle$

○ $\langle M \rangle = \langle YYY \rangle - \langle YXX \rangle - \langle XYX \rangle - \langle XXY \rangle$




$|\langle M \rangle| \leq 2$
Bi-separable bound

$|\langle M \rangle| \leq 1$
Separable bound:

- Genuine 3-qubit entanglement

Mermin, PRL (1990)
Tóth & Gühne, PRA (2005)
Roy, PRL (2005)

# Outlook

- The quality of a quantum processor relies on its ability to generate near-perfect multi-qubit entanglement at intermediate steps of a computation.

- A quantum computer engineer needs to detect this entanglement as a way to benchmark or debug the processor.

- Full state tomography is OK for few-qubit registers, but witnesses are the scalable way of detecting entanglement.

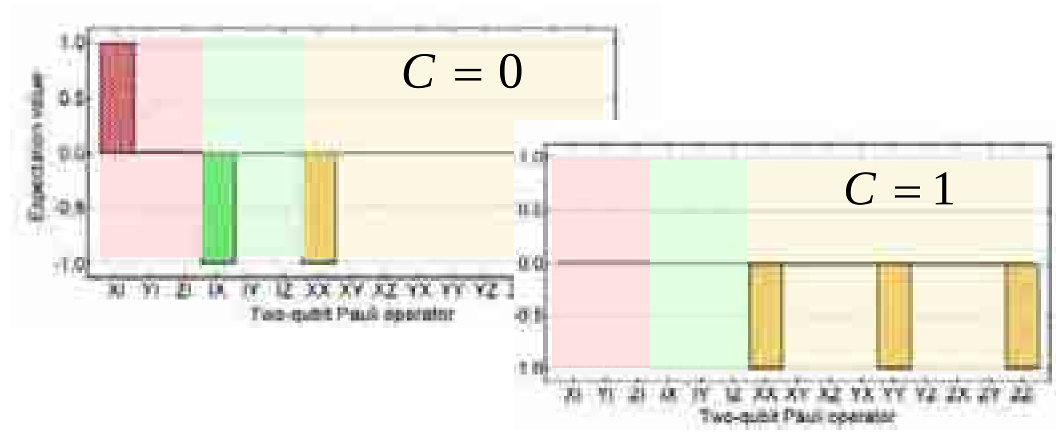
- Multi-qubit (3+) entanglement is required for quantum error correction

# Summary in pictures

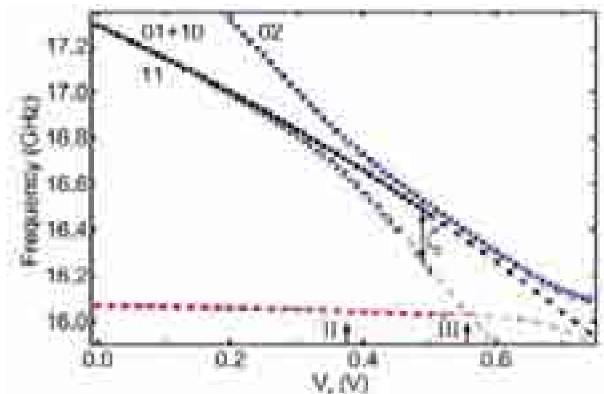
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

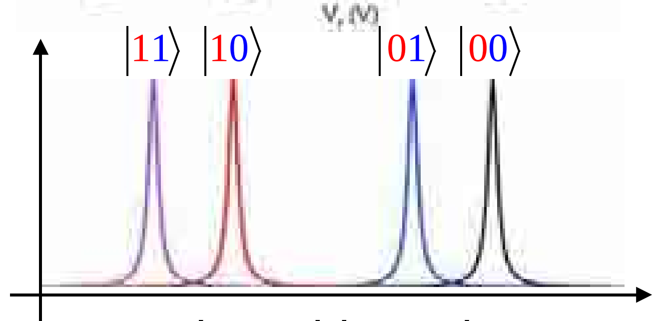
vs

$$|\psi\rangle = |\psi_1^a\rangle \otimes |\psi_2^a\rangle + |\psi_1^b\rangle \otimes |\psi_2^b\rangle$$

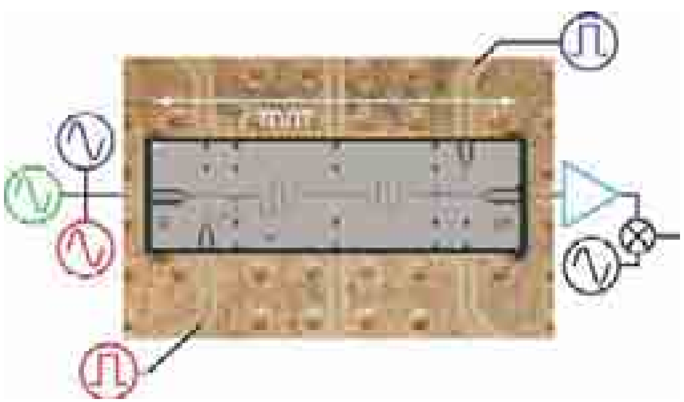



C-phase gate



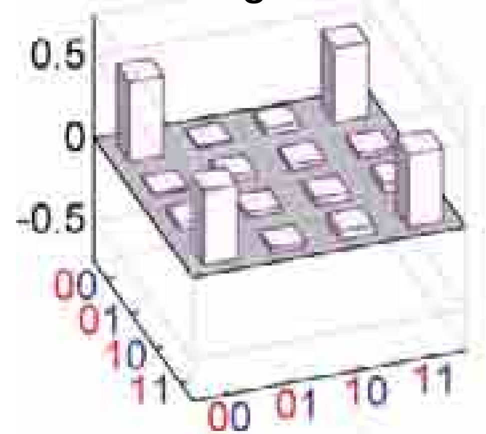




Joint qubit readout



Few-qubit processors based on circuit QED

Generation and detection of 2 & 3-qubit entanglement



# Yale circuit QED team members 2010

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