Metrology of Entangled States in Circuit QED

Applied Physics + Physics
Yale University

PI’s:
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Leo DiCarlo
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Theory
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IARPA
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Recent Reviews

‘Wiring up quantum systems’
R. J. Schoelkopf, S. M. Girvin

‘Superconducting quantum bits’
John Clarke, Frank K. Wilhelm

*Quantum Information Processing* **8** (2009)
ed. by A. Korotkov
Overview

• ‘Transmon’ qubit, insensitive to charge noise

• Circuit QED: using cavity bus to couple qubits

• Two qubit gates and generation of Bell’s states

• “Metrology of entanglement” – using joint cQED msmt.

• Demonstration of Grover and Deutsch-Josza algorithms
  DiCarlo et al., cond-mat/0903.2030
Quantum Computation and NMR of a Single ‘Spin’

Electrical circuit with two quantized energy levels is like a spin -1/2.

(After Konrad Lehnert)
‘Transmon’ Cooper Pair Box: Charge Qubit that Works!

Josephson junction (SQUID loop):

300 µm

Added metal = capacitor & antenna

\( E_J \gg E_C \)

\( |e\rangle \)

\( |g\rangle \)

plasma oscillation of 2 or 3 Cooper pairs: almost no static dipole

Transmon qubit insensitive to 1/f electric fields

* Theory: J. Koch et al., PRA (2007); Expt: J. Schreier et al., PRB (2008)

Flux qubit + capacitor: F. You et al., PRB (2006)
‘Transmon’ Cooper Pair Box: Charge Qubit that Works!

Josephson junction plasma oscillations are anharmonic:

\[ H = -E_J \text{[tunneling]} + 4E_C (\hat{n} - n_{\text{offset}})^2 \]

Transition frequency tunable via SQUID flux.

\[ \hbar \omega_{01} \approx \sqrt{8E_J E_C} \]
Outsmarting Noise: Sweet Spot

1\textsuperscript{st} coherence strategy: optimize design

But \( T_2 \) still < 500 ns due to second order noise!

Vion et al., Science 296, 886 (2002)
“Eliminating” Charge Noise with Better Design

\[
\hat{H} = 4E_C (\hat{n} - n_g)^2 - E_J \cos \hat{\varphi}
\]

Cooper-pair Box

"Transmon"

Koch et al., 2007; Houck et al., 2008
Single Qubit Rotations

Fidelity = 99%

J. Chow et al., PRL (2009):

\[ V = \Omega_{\text{Rabi}}^x (t) \cos(\omega_{01} t) \sigma^x + \Omega_{\text{Rabi}}^y (t) \sin(\omega_{01} t) \sigma^y \]

\[ T_1 = 1.5 \mu s \]
Ramsey Fringe and Qubit Coherence

Fidelity = 99%

J. Chow et al., PRL (2009):

\[ V = \Omega_{\text{Rabi}}(t) \cos(\omega_{01} t)\sigma^x \]

\[ T_2^* = 3.0 \mu s \]
Coherence in Transmon Qubit

\[ T_1 = 1.5 \mu s \]

\[ T_2^* = 2T_1 = 3.0 \mu s \]

\[ \frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\phi} \Rightarrow T_\phi > 35 \mu s \]

Random benchmarking of 1-qubit ops

Chow et al. *PRL* 2009:
Technique from Knill et al. for ions

Error per gate
\[ = 1.2 \% \]

Similar error rates in phase qubits (UCSB):
Lucero et al. *PRL* 100, 247001 (2007)
Cavity Quantum Electrodynamics (cQED)

\[ 2g = \text{vacuum Rabi freq.} \]
\[ \kappa = \text{cavity decay rate} \]
\[ \gamma = \text{“transverse” decay rate} \]

Strong Coupling = \( g > \kappa, \gamma \)

Jaynes-Cummings Hamiltonian

\[ \hat{H} = \hbar \omega_r (a^\dagger a + \frac{1}{2}) - \frac{\hbar \omega_a}{2} \hat{\sigma}_z - \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_\kappa + H_\gamma \]

Quantized Field
2-level system
Electric dipole Interaction
Dissipation
Coupling SC Qubits: Use a Circuit Element

- **Charge qubits:** NEC 2003
- **Phase qubits:** UCSB 2006

- **Flux qubits:** Delft 2007
- **Flux qubits:** Berkeley 2006, NEC 2007
  Or tunable bus, Chalmers

- **Concurrence:** \( \sim 55\% \)

- **Entangled states!**

**Diagram:**
- **Capacitor:**
- **Inductor:**
- **Tunable (SQUID) element:**
Qubits Coupled with a Quantum Bus

use microwave photons guided on wires!

“Circuit QED”

transmission line “cavity”

7 GHz in

Josephson-junction qubits

Expts: Majer et al., *Nature* 2007 (Charge qubits / Yale)
Sillanpaa et al., *Nature* 2007 (Phase qubits / NIST)
A Two-Qubit Processor

T = 10 mK

cavity: “entanglement bus,” driver, & detector

cavity: transmon qubits

flux bias lines control qubit frequency
1 ns resolution
DC - 2 GHz

1 ns resolution
How do we entangle two qubits?

$R_y(-\pi/2)$ rotation on each qubit yields superposition:

$$|\Psi\rangle = \frac{1}{2} (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$$= \frac{1}{2} (|00\rangle + |10\rangle + |01\rangle + |11\rangle)$$

‘Conditional Phase Gate’ entangler:

$$|\Psi\rangle = \frac{1}{2} (|00\rangle + |10\rangle + |01\rangle - |11\rangle)$$

No longer a product state!
How do we entangle two qubits?

\[
\begin{pmatrix}
+1 & 0 & 0 & 0 \\
0 & +1 & 0 & 0 \\
0 & 0 & +1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

\[
|\Psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle) = \frac{1}{\sqrt{2}}(|0 \rightarrow\rangle + |1 \leftarrow\rangle)
\]

\(R_y(+\pi/2)\) rotation on LEFT qubit yields:

\[
|\text{Bell}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)
\]

Other 3 Bell states similarly achieved.
Entanglement on Demand

\[ \frac{1}{\sqrt{2}} \left( \left| \psi \right> + \left| \phi \right> \right) \]

L'état quantique c'est Moi!
How do we realize the conditional phase gate?

\[
\begin{pmatrix}
+1 & 0 & 0 & 0 \\
0 & +1 & 0 & 0 \\
0 & 0 & +1 & 0 \\
0 & 0 & 0 & -1 \\
\end{pmatrix}
\]

\[|\Psi\rangle = \frac{1}{2} \left( |00\rangle + |01\rangle + |10\rangle - |11\rangle \right)\]

Use control lines to push qubits near a resonance:

A controlled z-z interaction also à la NMR
Key is to use 3\textsuperscript{rd} level of transmon (outside the logical subspace)

Coupling turned off.

Coupling turned on:
Near resonance with 3\textsuperscript{rd} level

\[ \omega_{01} \approx \omega_{12} \]

Energy is shifted if and only if both qubits are in excited state.
Adiabatic Conditional Phase Gate

- Avoided crossing (160 MHz)
  \[ |11\rangle \leftrightarrow |02\rangle \]

- A frequency shift
  \[ \frac{\zeta}{2\pi} = f_{01} + f_{10} - f_{11} \]
  \[ 1.2 \text{ MHz} \leq \frac{\zeta}{2\pi} \leq 150 \text{ MHz} \]

On/off ratio \(\approx 100:1\)

Use large on-off ratio of \(\zeta\) to implement 2-qubit phase gates.

\[ \int \zeta(t) \, dt = (2n + 1)\pi \]

Adjust timing so that amplitude for both qubits to be excited acquires a minus sign:

\[
\begin{pmatrix}
+1 & 0 & 0 & 0 \\
0 & +1 & 0 & 0 \\
0 & 0 & +1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
\Psi
\end{pmatrix} = \frac{1}{2}(|00\rangle + |10\rangle + |01\rangle - |11\rangle)
\]
Entanglement on Demand

\[ |0\rangle \quad R_y^{\pi/2} \quad cU_{ij} \quad R_y^{\pi/2} \quad |0\rangle \]

\[ \text{Re}(\rho) \]

<table>
<thead>
<tr>
<th>Bell state</th>
<th>Fidelity</th>
<th>Concurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>00\rangle +</td>
<td>11\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>00\rangle -</td>
<td>11\rangle)</td>
</tr>
<tr>
<td>(</td>
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<td>10\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>01\rangle -</td>
<td>10\rangle)</td>
</tr>
</tbody>
</table>

ETH: Leek et al., PRL (2009)
How do we read out the qubit state and measure the entanglement?
Two Qubit Joint Readout via Cavity

Cavity frequency $f_c$ pulled by qubits

“Strong dispersive cQED”: Schuster et al., 2007
Two Qubit Joint Readout via Cavity

Initial polarization of qubit? > 99.7% (Bishop et al., 2009) -> reset fidelity is high!
Cavity Pull is **linear** in spin polarizations

![Diagram showing cavity transmission](image)

\[
\Delta \omega = a\sigma_L^z + b\sigma_R^z
\]

Complex transmitted amplitude is **non-linear** in cavity pull:

\[
t = \frac{\kappa / 2}{\omega_{\text{drive}} - \omega_{\text{cavity}} - \Delta \omega + i\kappa / 2}
\]

Most general non-linear function of two Ising spin variables:

\[
t = \beta_0 + \beta_1 \sigma_L^z + \beta_2 \sigma_R^z + \beta_{12} \sigma_L^z \otimes \sigma_R^z
\]
Joint Readout

\[ V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle \]

\[ \beta_1 \sim 1; \quad \beta_2 \sim 0.8; \quad \beta_{12} \sim 0.5 \]
Joint Readout

\[ V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle \]

\[ \langle \sigma_z^L \rangle = e^{-t/T_L} \cos(\Omega_L t) \]

\[ \langle \sigma_z^R \rangle = e^{-t/T_R} \cos(\Omega_R t) \]
State Tomography

\[ V_H \sim \langle M \rangle = \beta_1 \langle \sigma^L_z \rangle + \beta_2 \langle \sigma^R_z \rangle + \beta_{12} \langle \sigma^L_z \otimes \sigma^R_z \rangle \]

Combine joint readout with one-qubit “analysis” rotations

\[ \langle \sigma^L_z \rangle \sim V_H (\text{Ident.}) + V_H (Y^R_\pi) \quad \text{π-pulse on right} \]

\[ \langle \sigma^R_z \rangle \sim V_H (\text{Ident.}) + V_H (Y^L_\pi) \quad \text{π-pulse on left} \]

\[ \langle \sigma^L_z \sigma^R_z \rangle \sim V_H (\text{Ident.}) + V_H (Y^R_\pi, Y^L_\pi) \quad \text{π on both} \]

Possible to acquire correlation information even with single, ensemble averaged msmt.!

Rotate qubits to map other correlations onto z-z.

See similar from Zurich group: Fillip et al., PRL 102, 200402 (2009).
Measuring the Two-Qubit State

Total of 16 msmts.: \[ I, Y^L_\pi, X^L_{\pi/2}, Y^L_{\pi/2} \] and combinations \[ I, Y^R_\pi, X^R_{\pi/2}, Y^R_{\pi/2} \]

(Almost) raw data

Ground state: \[ |\psi\rangle = |00\rangle = |\uparrow\uparrow\rangle \]
\[ \langle \sigma^z_L \rangle = \langle \sigma^z_R \rangle = \langle \sigma^z_L \sigma^z_R \rangle = 1 \]
Measuring the Two-Qubit State

Apply $\pi$-pulse to invert state of right qubit

One qubit excited: $|\psi\rangle = |01\rangle = |\uparrow\downarrow\rangle$

$$\langle \sigma_L^z \rangle = +1$$
$$\langle \sigma_R^z \rangle = \langle \sigma_L^z \sigma_R^z \rangle = -1$$
Measuring the Two-Qubit State

Now apply a two-qubit gate to *entangle* the qubits

Entangled state: \( |\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \)

\[
\begin{align*}
\langle \sigma^z_L \rangle &= \langle \sigma^z_R \rangle = 0 \\
\langle \sigma^z_L \sigma^z_R \rangle &= +1 \\
\langle \sigma^y_L \sigma^y_R \rangle &= +1 \\
\langle \sigma^x_L \sigma^x_R \rangle &= -1
\end{align*}
\]
Witnessing Entanglement

CHSH operator = entanglement witness

\[ CHSH = XX' - XZ' + ZZ' + ZX' \]

If variables take on the values ±1 and exist even independent of measurement then

\[ CHSH = X(X' - Z') + Z(X' + Z') \]

Either: \[ CHSH = 0 \] \[ = \pm 2 \]
Or: \[ = \pm 2 \] \[ = 0 \]

Classically:

\[ |CHSH| \leq 2 \]
Witnessing Entanglement

CHSH operator = entanglement witness

\[ \langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle \]

- Green: \( XX' - XZ' + ZX' + ZZ' \)
- Blue: \( XX' + XZ' - ZX' + ZZ' \)

Clauser, Horne, Shimony & Holt (1969)

Separable bound:

\[ |CHSH| \leq 2 \]

not? Bell’s violation!

(loopholes abound)

but state is clearly highly entangled!

(and no likelihood req.)

Bell state

\[ 2.44 \pm 0.05 \]
Control: Analyzing Product States

CHSH operator = entanglement witness

\[ \langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle \]

- Green line: \( XX' - XZ' + ZX' + ZZ' \)
- Blue line: \( XX' + XZ' - ZX' + ZZ' \)

Clauser, Horne, Shimony & Holt (1969)

no entanglement!
Using entanglement on demand to run first quantum algorithm on a solid state quantum processor
General Features of a Quantum Algorithm

1) Start in superposition: all values at once!
2) Build complex transformation out of one-qubit and two-qubit “gates”
3) Somehow* make the answer we want result in a definite state at end!

*use interference: the magic of the properly designed algorithm
The Search Problem

\[ f(x) = \begin{cases} 
-1, & x \neq x_0 \\
1, & x = x_0 
\end{cases} \]

“Find \( x_0 \)!”

“Find the queen!”
The Search Problem

\[ f(x) = \begin{cases} 
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The Search Problem

\[ f(x) = \begin{cases} 
-1, & x \neq x_0 \\
1, & x = x_0 \end{cases} \]

“Find \( x_0 \)!”

“Find the queen!”

Position: 

\[
\begin{array}{c}
\text{Position:} & 0 & \text{I} & \text{II} & \text{III} \\
\end{array}
\]
The Search Problem

Classically, takes on average 2.25 guesses to succeed…

Use QM to “peek” under all the cards, find queen on first try!

“Find the queen!”
Grover’s Algorithm

“unknown” unitary operation: \[ O \ket{\psi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \ket{\psi} \]

Challenge: Find the location of the -1 (!!!) (= queen)

Previously implemented in NMR: Chuang et al., 1998
Ion traps: Brickman et al., 2003

10 pulses w/ nanosecond resolution, total 104 ns duration
\[ |\psi_{\text{ideal}}\rangle = |00\rangle \]

Begin in ground state:
Create a maximal superposition: look everywhere at once!

\[ |\psi_{\text{ideal}}\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \]
\[ |\psi_{\text{ideal}}\rangle = \frac{1}{2} (|00\rangle + |01\rangle - |10\rangle + |11\rangle) \]

Apply the “unknown” function, and mark the solution

\[
cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]
Some more 1-qubit rotations…

Now we arrive in one of the four Bell states

\[ |\psi_{\text{ideal}}\rangle = \frac{1}{\sqrt{2}} \left( |00\rangle + |11\rangle \right) \]
Another (but known) 2-qubit operation now undoes the entanglement and makes an interference pattern that holds the answer!

\[ |\psi_{\text{ideal}}\rangle = \frac{1}{2} (|00\rangle - |01\rangle + |10\rangle - |11\rangle) \]

Grover Step-by-Step
Final 1-qubit rotations reveal the answer:

The binary representation of “2”!

The correct answer is found >80% of the time!

\[ |\psi_{\text{ideal}} \rangle = |10\rangle \]
Grover with Other Oracles

Oracle

$$\hat{O} = c U_{00}$$

\[ F = \langle \psi_{\text{ideal}} | \rho | \psi_{\text{ideal}} \rangle \] to ideal output

(average over 10 repetitions)
Circuit QED Team Members

Joe Schreier
Blake Johnson
Jared Schwede
Jens Koch
Jay Gambetta
Steve Girvin
Leo DiCarlo
Lev Bishop
Emily Chan
David Schuster
Hannes Majer
Jerry Chow
Andrew Houck
Michel Devoret

Funding:
Summary

- Grover algorithm with Fidelity $F > 80\%$
- CHSH as entanglement witness
- Rudimentary two-qubit processor
- Entanglement on demand
  \[ F = 87-94\% \]
  \[ C = 81-94\% \]

Additional Slides Follow
Multiplexed Qubit Control and Read-Out

![Graph showing polarization as a function of Rabi pulse length with oscillations at frequencies \( \omega_1 = 6.617 \text{ GHz} \) and \( \omega_2 = 6.529 \text{ GHz} \).]

Single Oscillation

![Graph showing homodyne voltage as a function of time with different states represented by different colors.]

\( \pi_1 \) and \( \pi_2 \) pulses are used to control the qubit states.
Witnessing Entanglement

Bell state

Product state
Measuring the Two-Qubit State

Now apply a two-qubit gate to entangle the qubits

Concurrence *directly*:  
(for pure states)  
\[ C = \sqrt{\frac{Q - 1}{2}} \]

\[ Q = \langle XX \rangle^2 + \langle XY \rangle^2 + \langle XZ \rangle^2 + \langle YX \rangle^2 + \langle YY \rangle^2 + \ldots + \langle ZZ \rangle^2 \]
Measurement with \( \sim 5 \) photons in cavity; 
SNR \( \sim 4 \) in one qubit lifetime \( (T_1) \)

\( T_1 \sim 300 \) ns, low Q cavity on sapphire
• Measurement after pi/2 pulse bimodal, halfway between
Deutsch-Jozsa Algorithm

\[ f_0(x) = 0 \]
\[ f_0(x) = x \]
\[ f_1(x) = 0 \]
\[ f_1(x) = 1 - x \]

|0\rangle \rightarrow R_y^{\pi/2} \rightarrow l \rightarrow U_i \rightarrow l \rightarrow R_y^{\pi/2} \rightarrow R_{x,y}^{0,\pi/2,\pi} \rightarrow \text{Joint Dispersive Readout} \]

Constant functions

Balanced functions

Answer is encoded in the state of left qubit

The correct answer is found >84% of the time.
The cost of entanglement

1. Cryogenic HEMT amp
2. Room Temp Amps
1. Two-channel digitizer
1. Two-channel AWG
1. Four-channel AWG
2. Scalar signal generators
2. Vector signal generators
1. Low-frequency generator
1. Rubidium frequency standard
2. Yokogawa DC sources
1. DC power supply
1. Amp biasing servo
1. Computer
$10^3$ Coffee pods
One-Qubit Gates

Preparation
1-qubit rotations
Measurement

Apply microwave pulse resonant with qubit
Spectroscopy of Qubits Interacting with Cavity

Qubit-qubit swap interaction
Majer et al., Nature (2007)

Cavity-qubit interaction
Vacuum Rabi splitting

Flux bias line controls qubit transition frequency
Spectroscopy of Qubits Interacting with Cavity

Qubits mostly separated and non-interacting due to frequency difference

\[ T_{1,r} = 0.79 \mu s \]
\[ T_{2,r}^* = 1.15 \mu s \]
\[ T_{1,1} = 1.3 \mu s \]
\[ T_{2,1}^* = 1.8 \mu s \]
One-Qubit Gates

J. Chow et al., PRL (2009): Fidelity = 99%
Two-Qubit Gate: Turn On Interactions

Use control lines to push qubits near a resonance:

A controlled z-z interaction also ala NMR
“Qubits” and cavity both have multiple levels…
On/Off Ratio for Two-Qubit Coupling

\[ \zeta = -2g_L^2g_R^2 \left( \frac{1}{(\omega_{01}^L - \omega_C)(\omega_{01}^R - \omega_C)^2} + \frac{1}{(\omega_{01}^R - \omega_C)(\omega_{01}^L - \omega_C)^2} + \frac{1}{(\omega_{01}^R - \omega_{12}^L)(\omega_{01}^L - \omega_C)^2} + \frac{1}{(\omega_{01}^L - \omega_{12}^R)(\omega_{01}^R - \omega_C)^2} \right) \]

Diverges at Point II

4\textsuperscript{th}-order in qubit-cavity coupling!

> 100 MHz

1 MHz

3-level transmons

spectroscopic measurement
○ time-domain measurement
 dashed - exact diagonalization
 purple - perturbation theory (3-level)
 red - perturbation theory (2-level)
State Tomography

\[ V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle \]

Combine joint readout with one-qubit “analysis” rotations

\[ \langle \sigma_z^L \rangle \sim V_H (\text{Ident.}) + V_H (Y_{\pi}^R) \quad \rightarrow \quad \pi\text{-pulse on right} \]

\[ \langle \sigma_z^R \rangle \sim V_H (\text{Ident.}) + V_H (Y_{\pi}^L) \quad \rightarrow \quad \pi\text{-pulse on left} \]

\[ \langle \sigma_z^L \sigma_z^R \rangle \sim V_H (\text{Ident.}) + V_H (Y_{\pi}^R, Y_{\pi}^L) \quad \rightarrow \quad \pi\text{ on both} \]

Possible to acquire correlation info., even with single, ensemble averaged msmt.!

See similar from Zurich group: Fillip et al., PRL 102, 200402 (2009).
Measuring the Two-Qubit State

Total of 16 msmts.: \( I, Y^L_\pi, X^L_{\pi/2}, Y^L_{\pi/2} \) and combinations \( I, Y^R_\pi, X^R_{\pi/2}, Y^R_{\pi/2} \)

(Almost) raw data

Ground state: \( |\psi\rangle = |00\rangle \)

Density matrix

Max. likelihood (nonlinear!)
Apply $\pi$-pulse to invert state of right qubit

Measuring the Two-Qubit State

One qubit excited:  $|\psi\rangle = |01\rangle$
Measuring the Two-Qubit State

Now apply a two-qubit gate to \textit{entangle} the qubits

Entangled state: \[ |\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \]

What's the entanglement metric?

"Concurrence": \[ C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \]

\( \lambda \) are e-values of \( \sqrt{\rho \tilde{\rho}} \)

What's the entanglement metric?

"Concurrence": \[ C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \]

\( \lambda \) are e-values of \( \sqrt{\rho \tilde{\rho}} \)
Microwave pulses at \( \omega_{01} \) can be used for single qubit rotations.

Anharmonicity:

\[
\omega_{01} - \omega_{12} = 455\text{MHz} \sim E_C
\]

Sufficient to control the artificial atom as a two level system: Qubit

*Slide courtesy of J. Schreier and R. Schoelkopf*