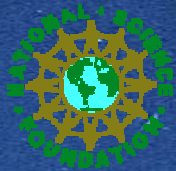


Metrology of Entangled States in Circuit QED

Applied Physics + Physics
Yale University

PI's:

Rob Schoelkopf
Michel Devoret
Steven Girvin



IARPA

Expt.

Leo DiCarlo

Andrew Houck
David Schuster
Hannes Majer

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Joe Schreier
Blake Johnson
Luigi Frunzio

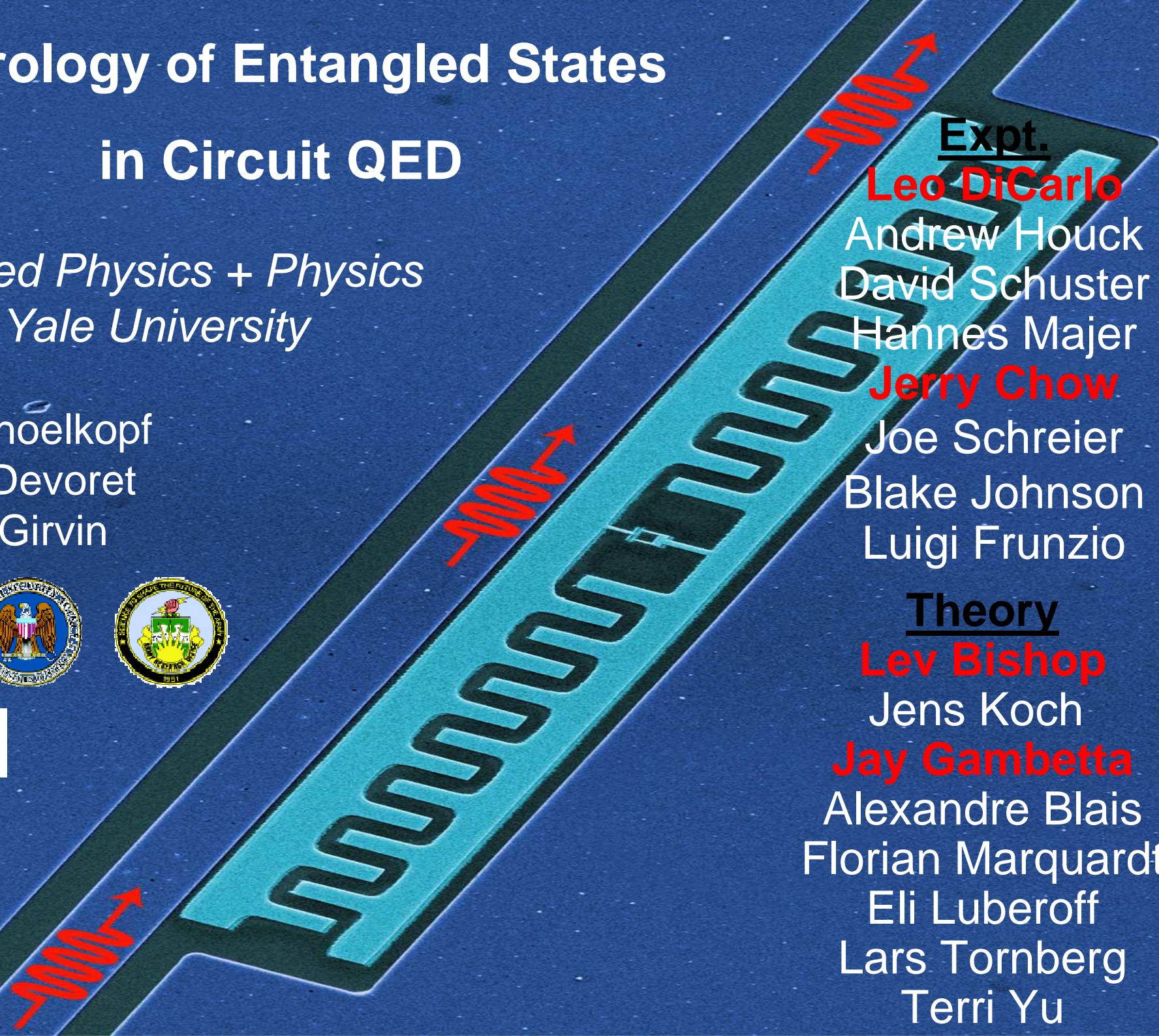
Theory

Lev Bishop

Jens Koch

Jay Gambetta

Alexandre Blais
Florian Marquardt
Eli Luberoﬀ
Lars Tornberg
Terri Yu



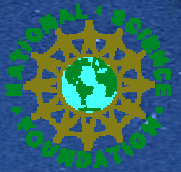
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Vienna

IQC/Waterloo
U. Sherbrooke

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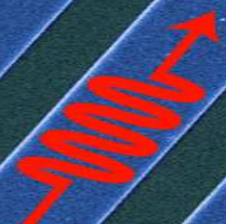
Alexandre Blais

Florian Marquardt

Eli Luberoff

Lars Tornberg

Terri Yu



Recent Reviews

‘Wiring up quantum systems’

R. J. Schoelkopf, S. M. Girvin

Nature **451**, 664 (2008)

‘Superconducting quantum bits’

John Clarke, Frank K. Wilhelm

Nature **453**, 1031 (2008)

Quantum Information Processing **8** (2009)

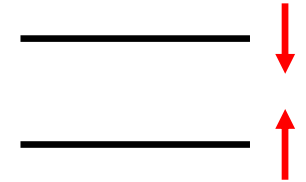
ed. by A. Korotkov

Overview

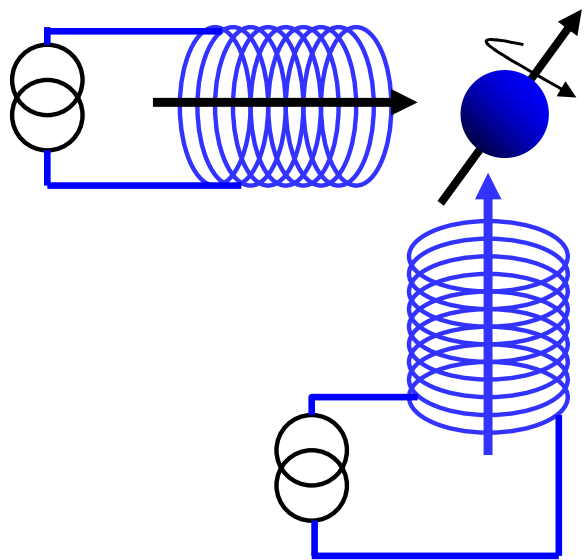
- ‘Transmon’ qubit, insensitive to charge noise
- Circuit QED: using cavity bus to couple qubits
- Two qubit gates and generation of Bell’s states
- “Metrology of entanglement” – using joint cQED msmt.
- Demonstration of Grover and Deutsch-Josza algorithms
[DiCarlo et al., cond-mat/0903.2030](#)
Nature, (in press, June 2009)

Quantum Computation and NMR of a Single 'Spin'

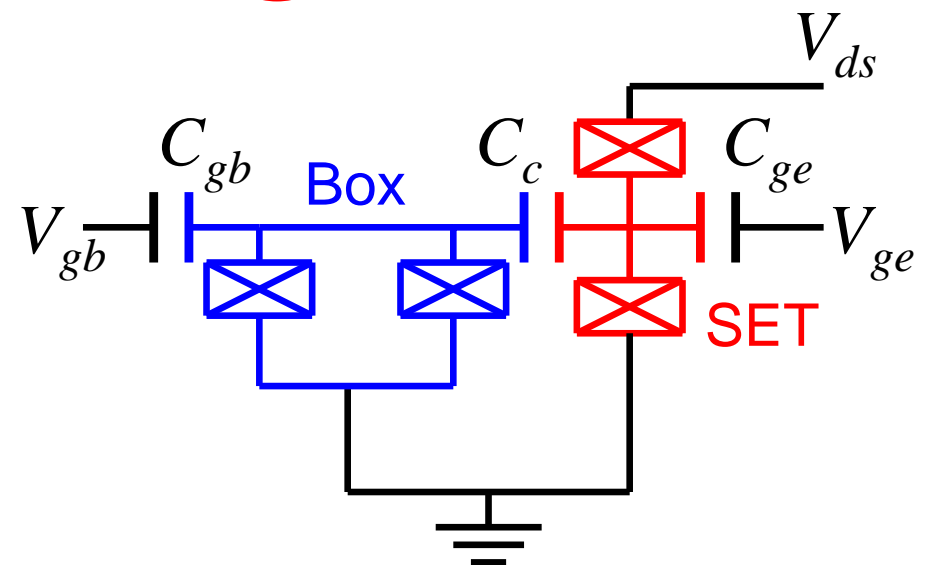
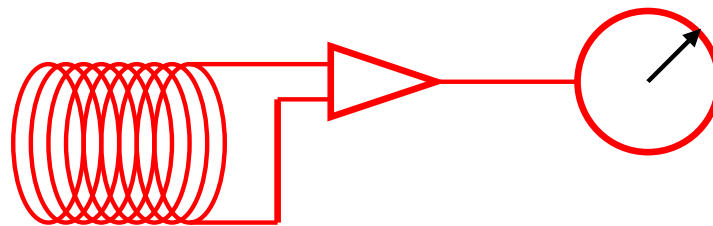
Electrical circuit with two quantized energy levels is like a spin $-1/2$.



Single Spin $1/2$

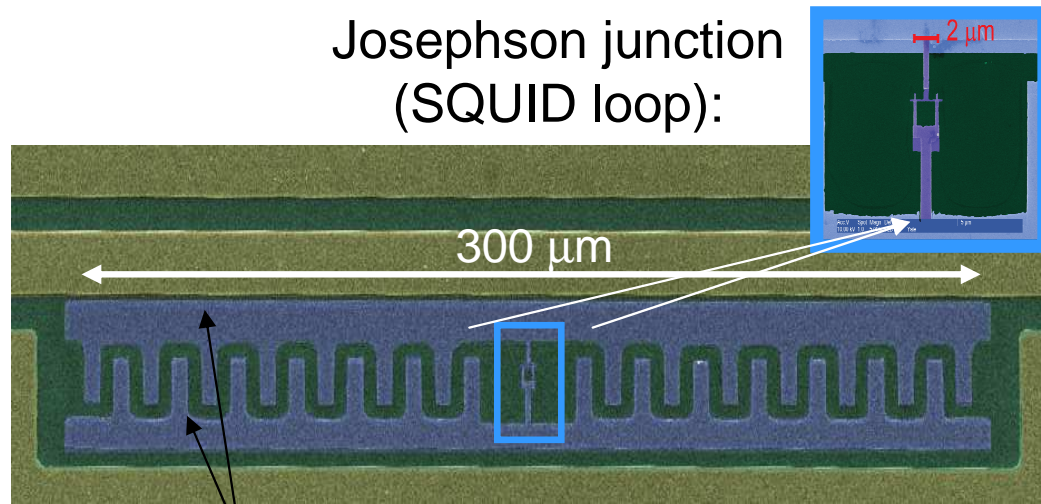


Quantum Measurement



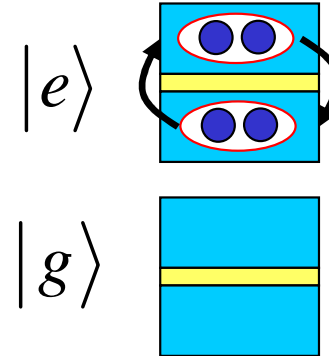
(After Konrad Lehnert)

'Transmon' Cooper Pair Box: Charge Qubit that Works!



Added metal
= capacitor & antenna

$$E_J \gg E_C$$



plasma oscillation of
2 or 3 Cooper pairs:
almost no static dipole

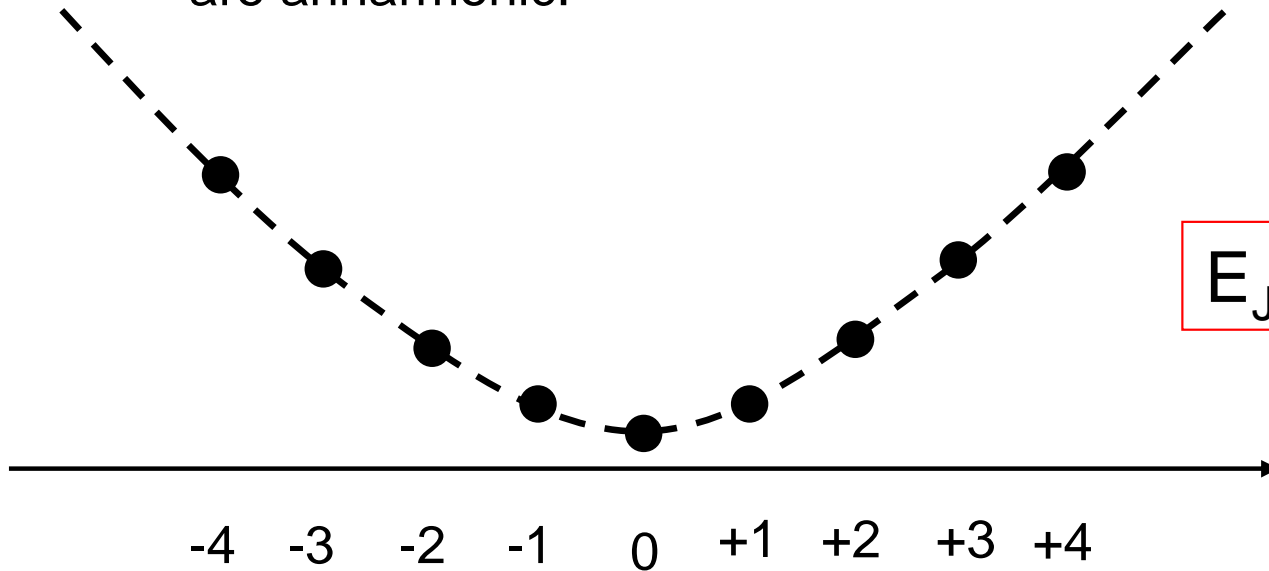
Transmon qubit insensitive to $1/f$ electric fields

* Theory: J. Koch et al., PRA (2007); Expt: J. Schreier et al., PRB (2008)

Flux qubit + capacitor: F. You et al., PRB (2006)

'Transmon' Cooper Pair Box: Charge Qubit that Works!

Josephson junction plasma oscillations are anharmonic:



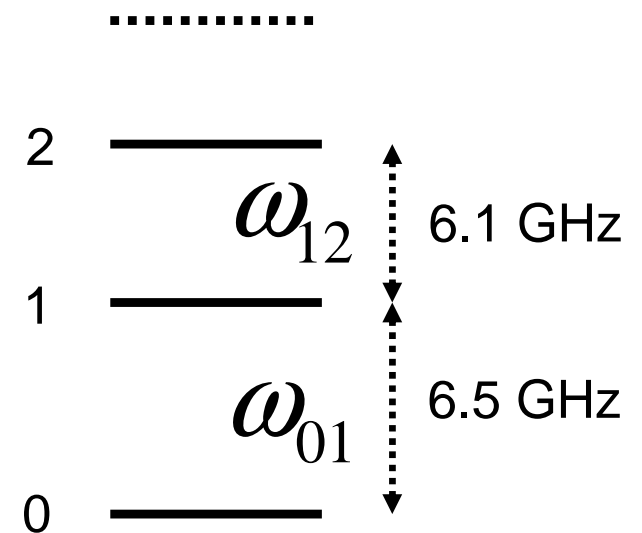
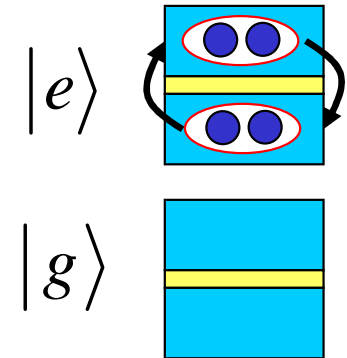
$$E_J \gg E_C$$

n = Number of pairs that have tunneled

$$H = -E_J [\text{tunneling}] + 4E_C (\hat{n} - n_{\text{offset}})^2$$

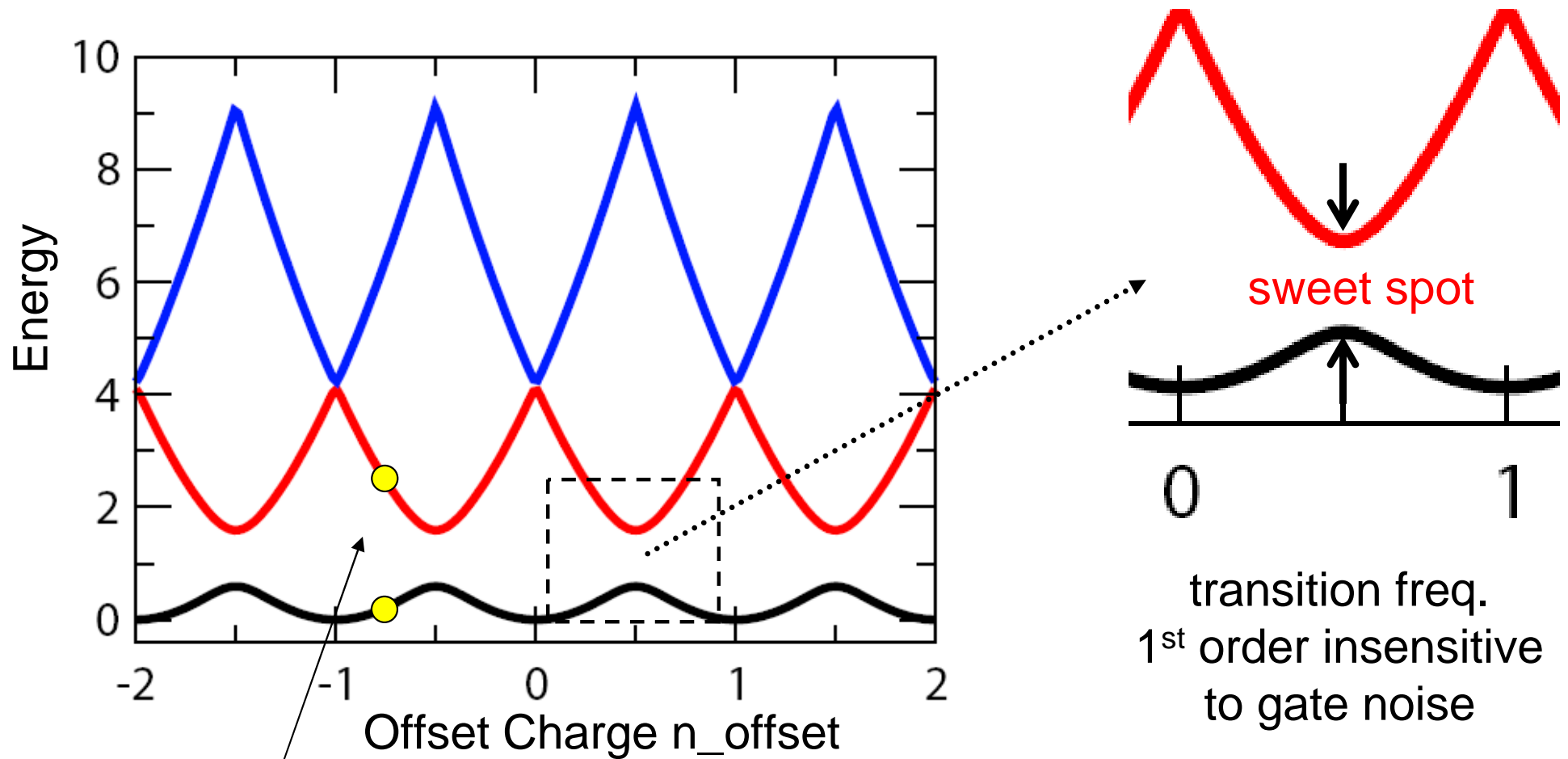
Transition frequency tunable via SQUID flux.

$$\hbar\omega_{01} \approx \sqrt{8E_J E_C}$$



Outsmarting Noise: Sweet Spot

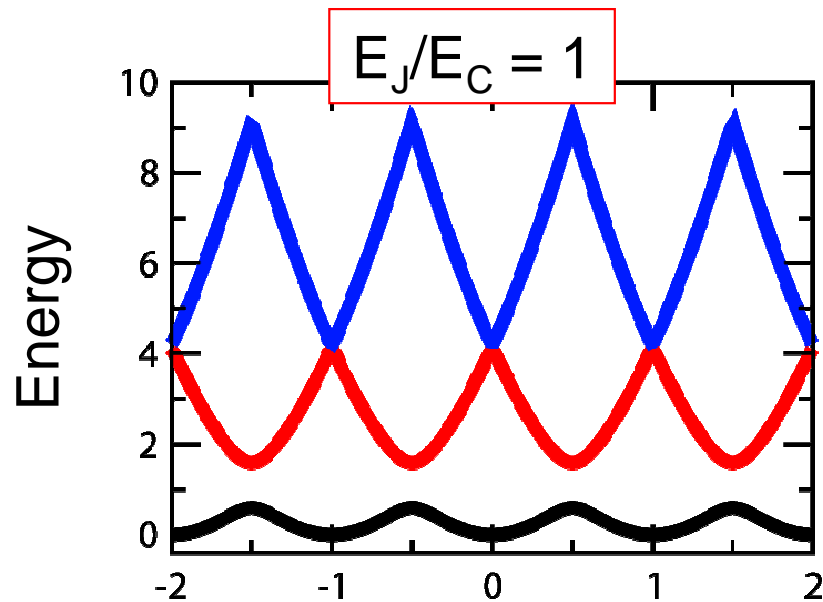
1st coherence strategy: optimize design



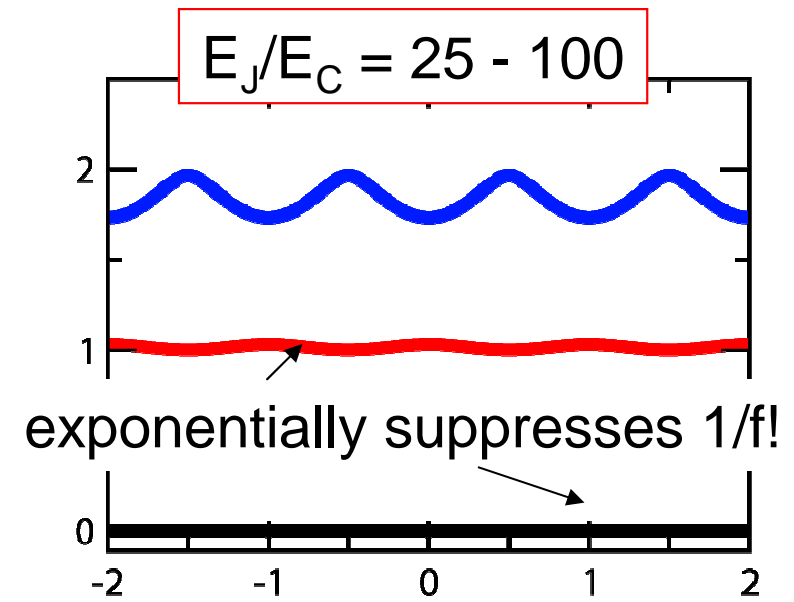
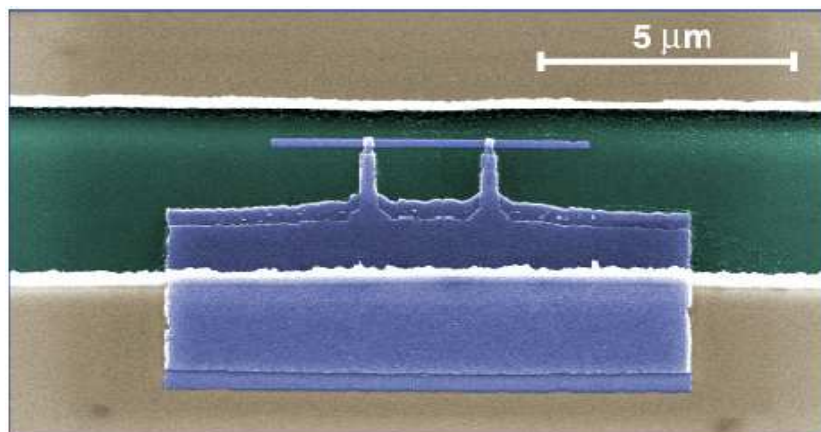
Strong sensitivity of frequency to charge noise!
But $T_2^{\text{stim}} < 500$ ns due to second-order noise!

“Eliminating” Charge Noise with Better Design

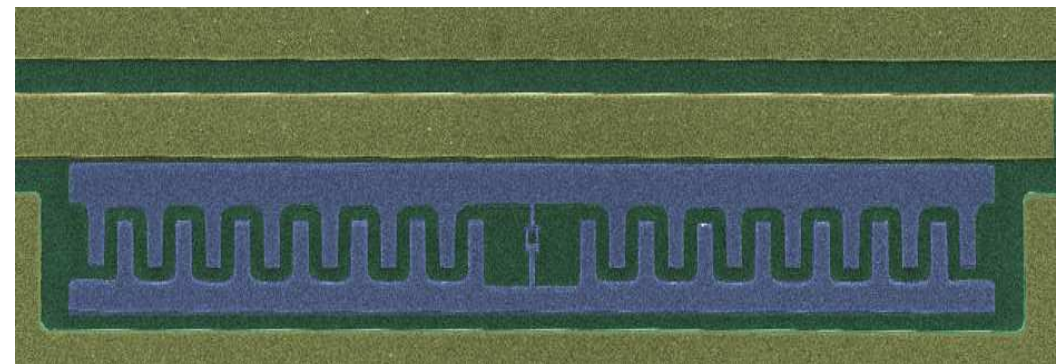
$$\hat{H} = 4E_C (\hat{n} - n_g)^2 - E_J \cos \hat{\varphi}$$



Cooper-pair Box



“Transmon”

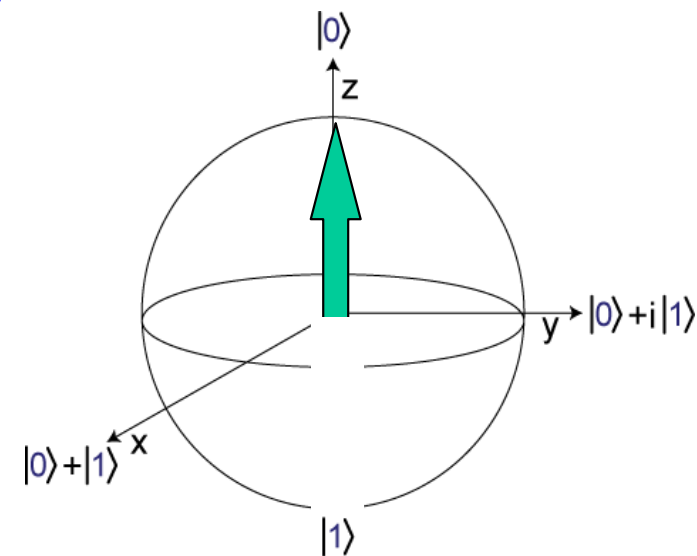
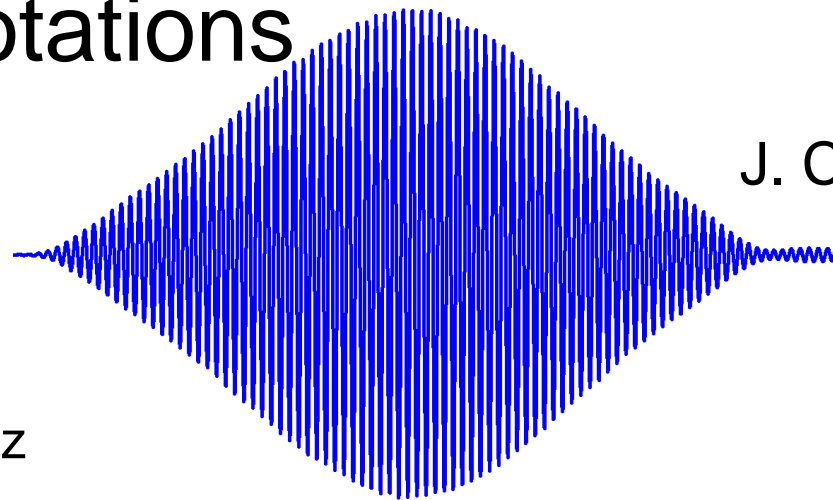
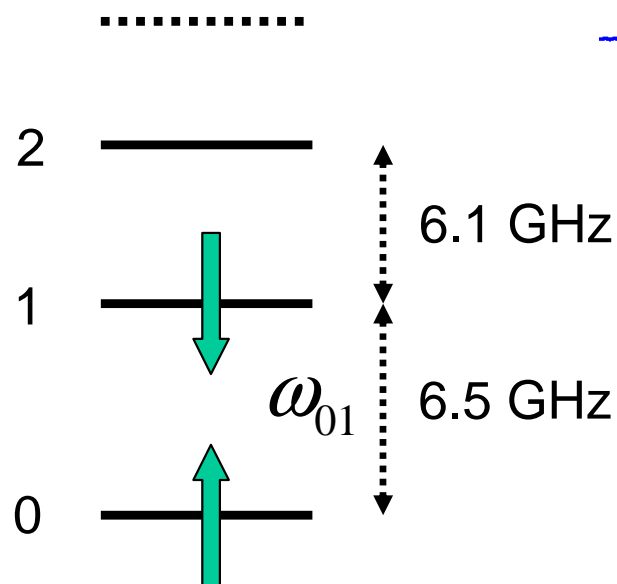


Koch et al., 2007; Houck et al., 2008

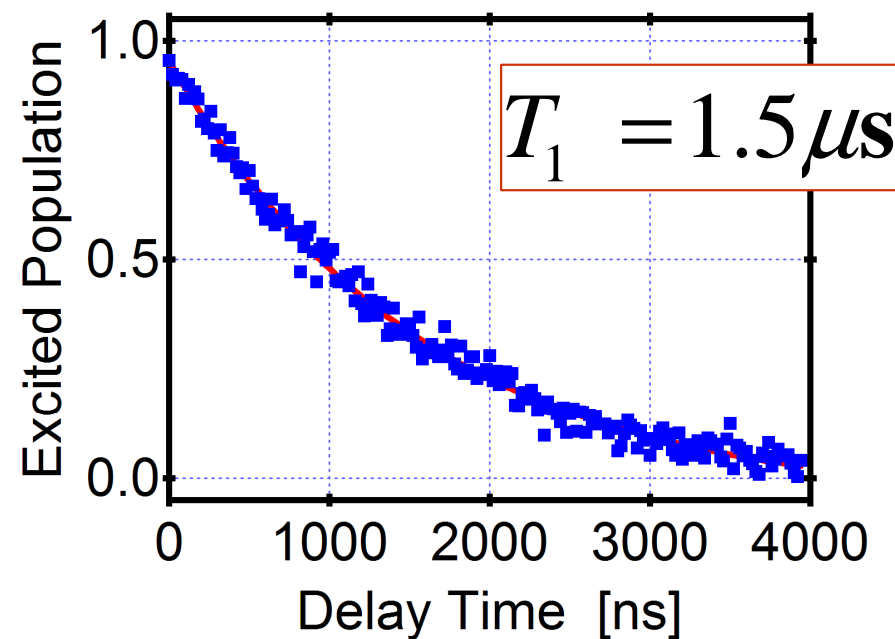
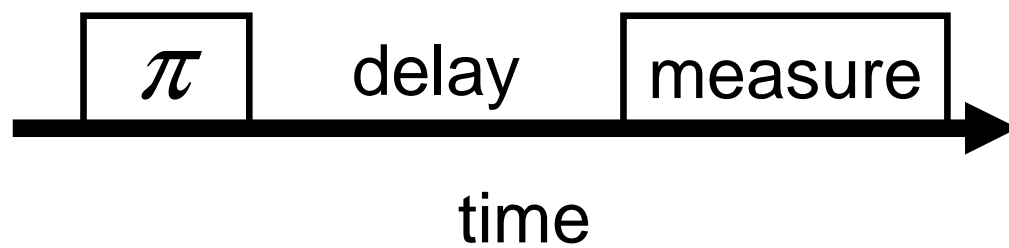
Single Qubit Rotations

Fidelity = 99%

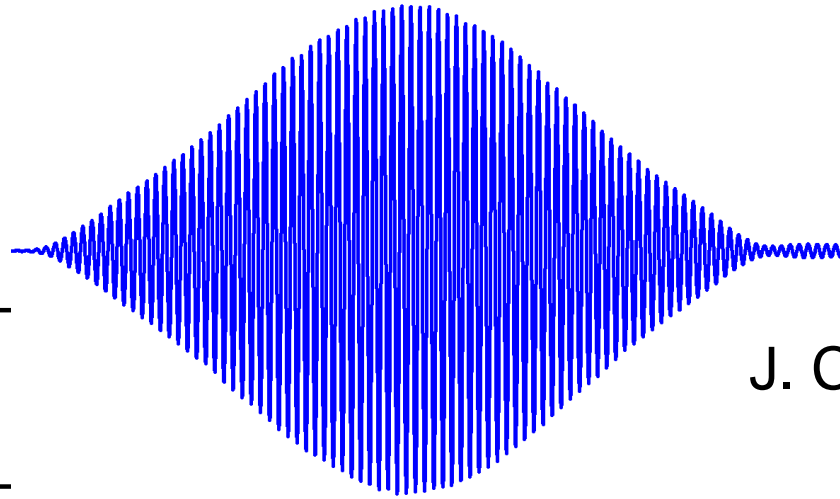
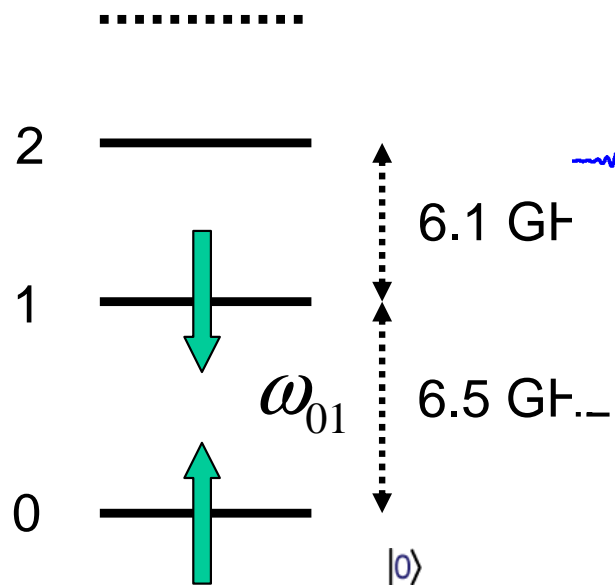
J. Chow *et al.*, PRL (2009):



$$V = \Omega_{\text{Rabi}}^x(t) \cos(\omega_{01}t) \sigma^x + \Omega_{\text{Rabi}}^y(t) \sin(\omega_{01}t) \sigma^y$$



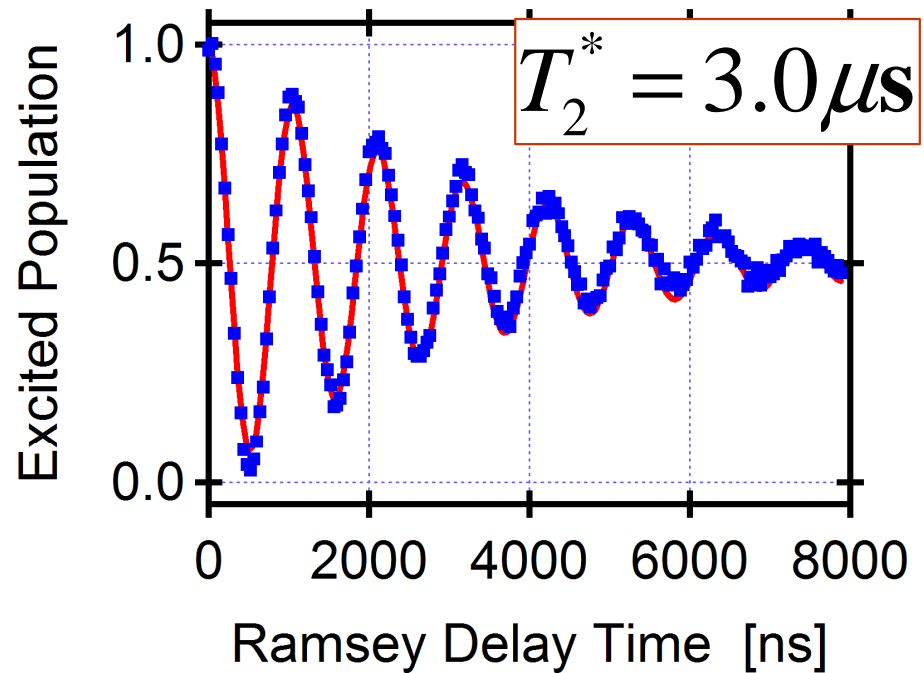
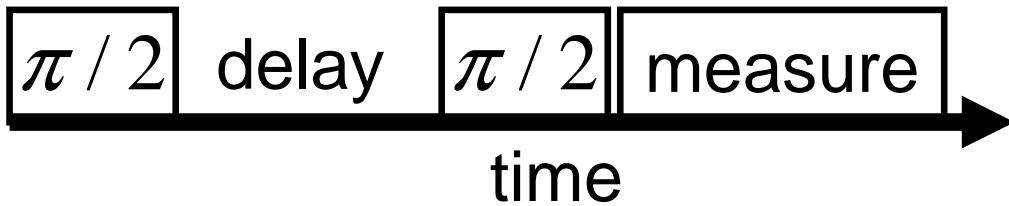
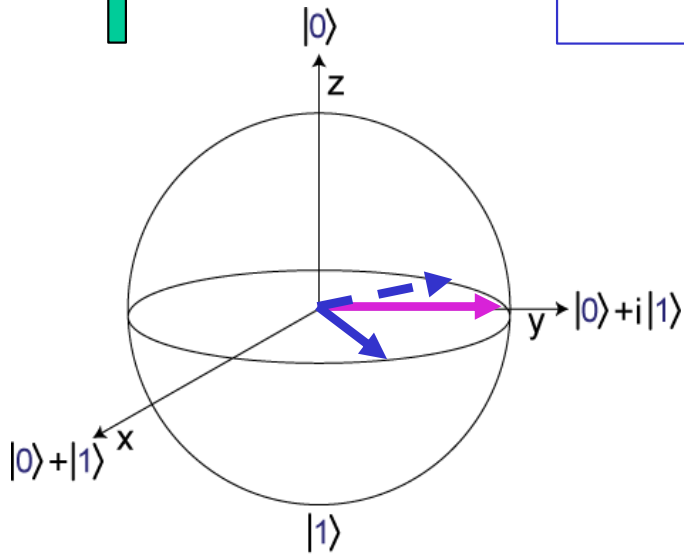
Ramsey Fringe and Qubit Coherence



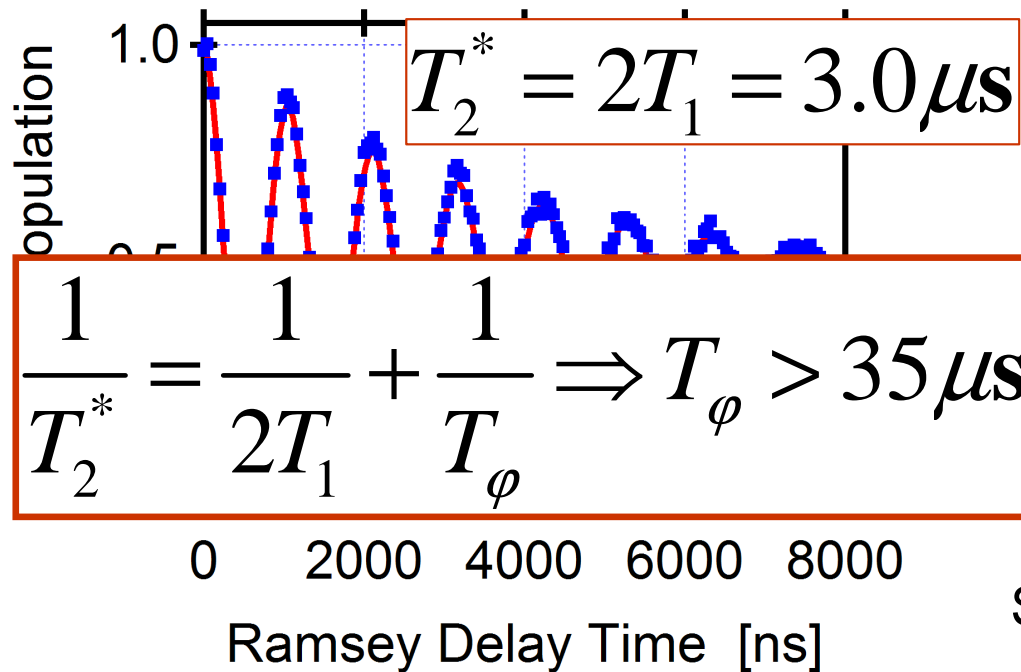
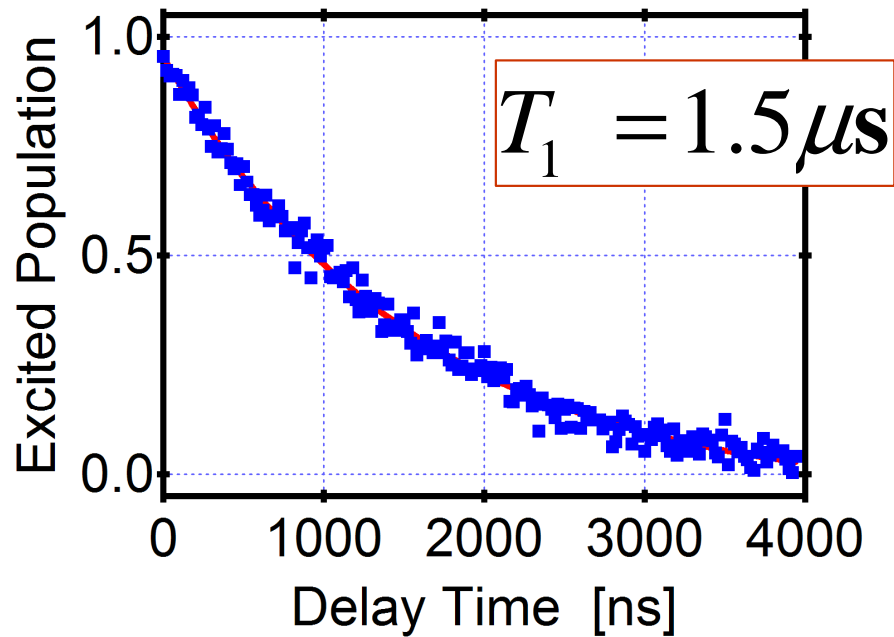
Fidelity = 99%

J. Chow *et al.*, *PRL* (2009):

$$V = \Omega_{\text{Rabi}}(t) \cos(\omega_{01}t) \sigma^x$$



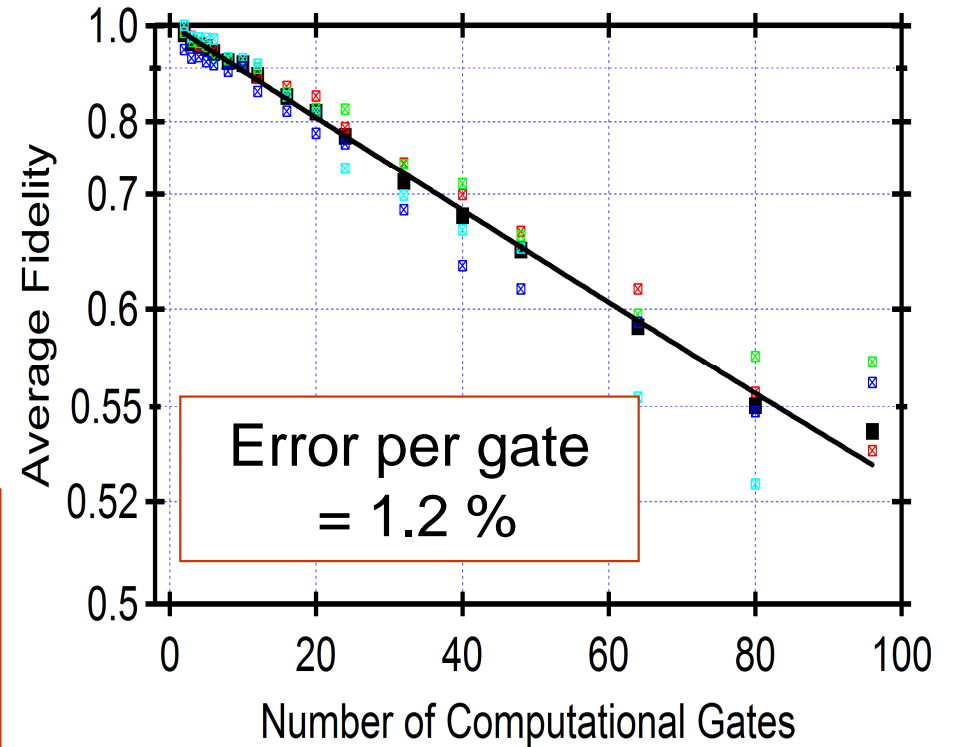
Coherence in Transmon Qubit



$$\frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\phi} \Rightarrow T_\phi > 35 \mu\text{s}$$

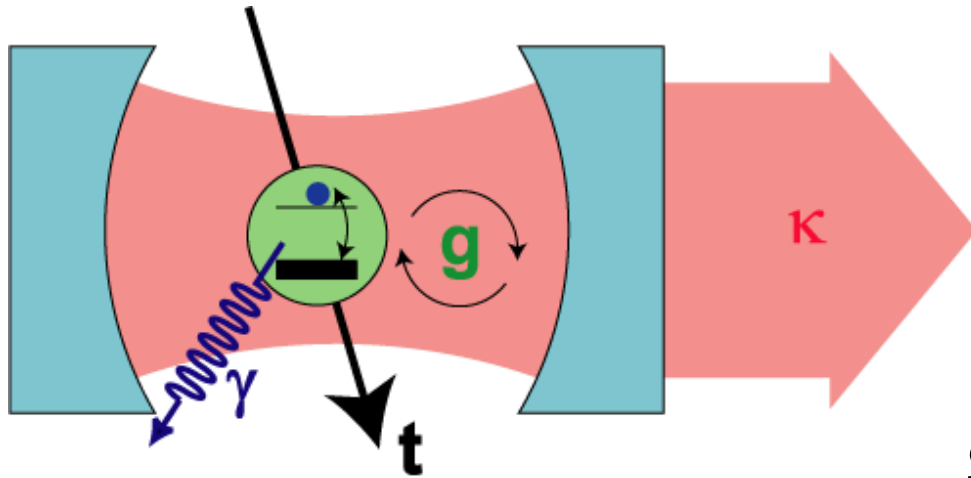
Random benchmarking of 1-qubit ops

Chow et al. *PRL* 2009:
Technique from Knill et al. for ions



Similar error rates in phase qubits (UCSB):
Lucero et al. *PRL* 100, 247001 (2007)

Cavity Quantum Electrodynamics (cQED)



$2g$ = vacuum Rabi freq.

κ = cavity decay rate

γ = “transverse” decay rate

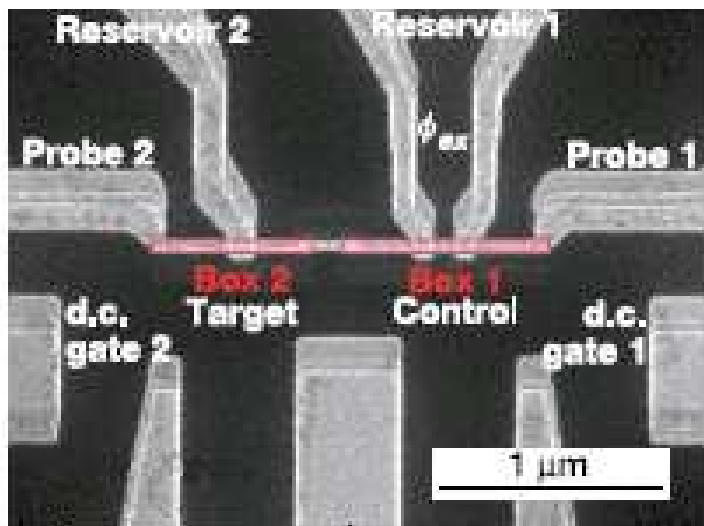
Strong Coupling = $g > \kappa, \gamma$

Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_r (a^\dagger a + \frac{1}{2}) - \frac{\hbar\omega_a}{2} \hat{\sigma}_z - \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_\kappa + H_\gamma$$

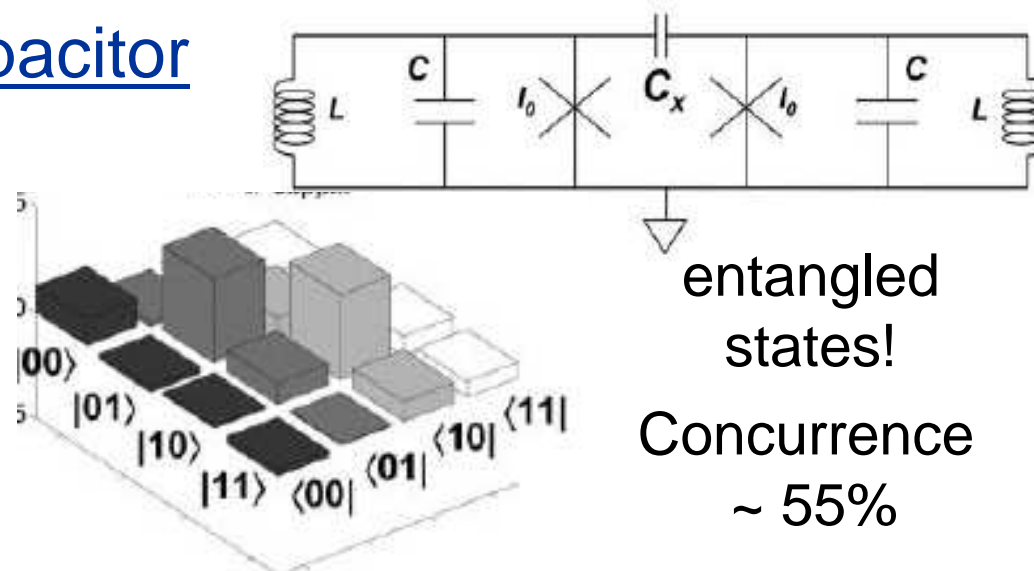
Quantized Field 2-level system Electric dipole Interaction Dissipation

Coupling SC Qubits: Use a Circuit Element



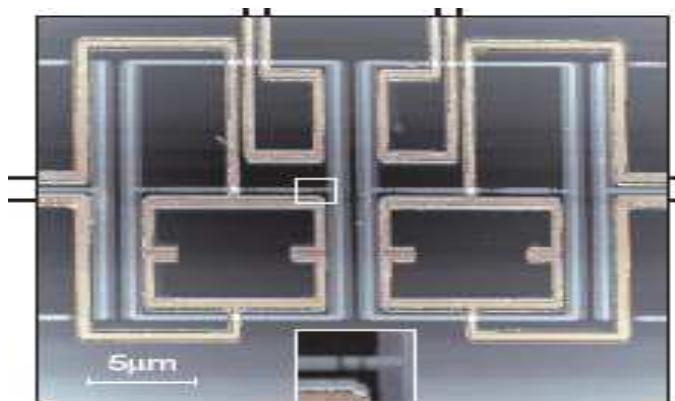
Charge qubits: NEC 2003

a capacitor



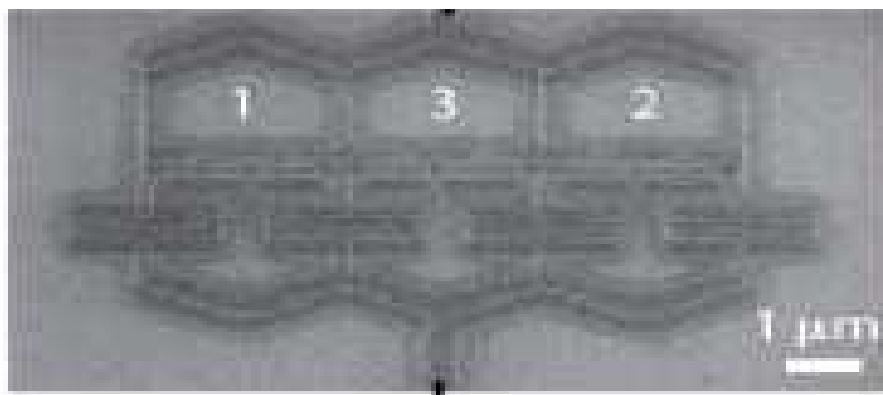
Phase qubits: UCSB 2006

an inductor



Flux qubits: Delft 2007

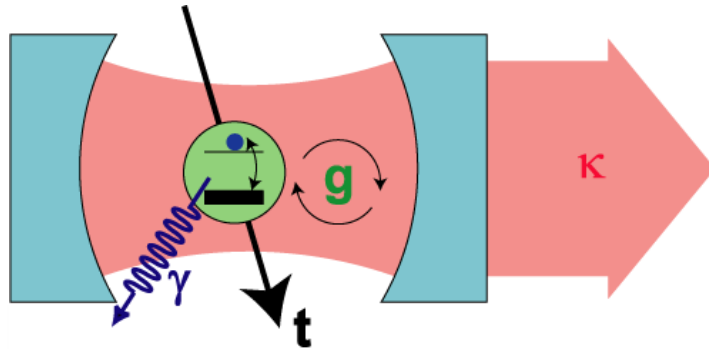
tunable (SQUID) element



Flux qubits: Berkeley 2006, NEC 2007
Or tunable bus, Chalmers

Qubits Coupled with a Quantum Bus

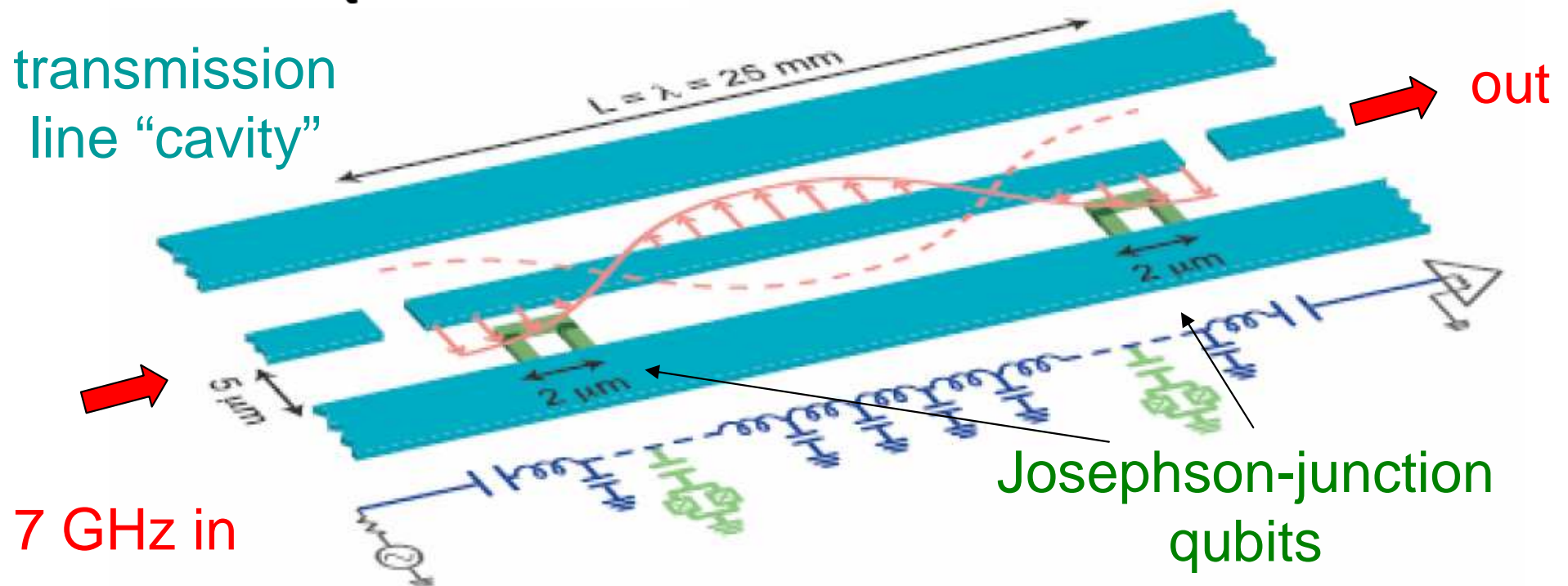
use microwave photons guided on wires!



“Circuit QED”

Blais *et al.*, *Phys. Rev. A* (2004)

transmission
line “cavity”

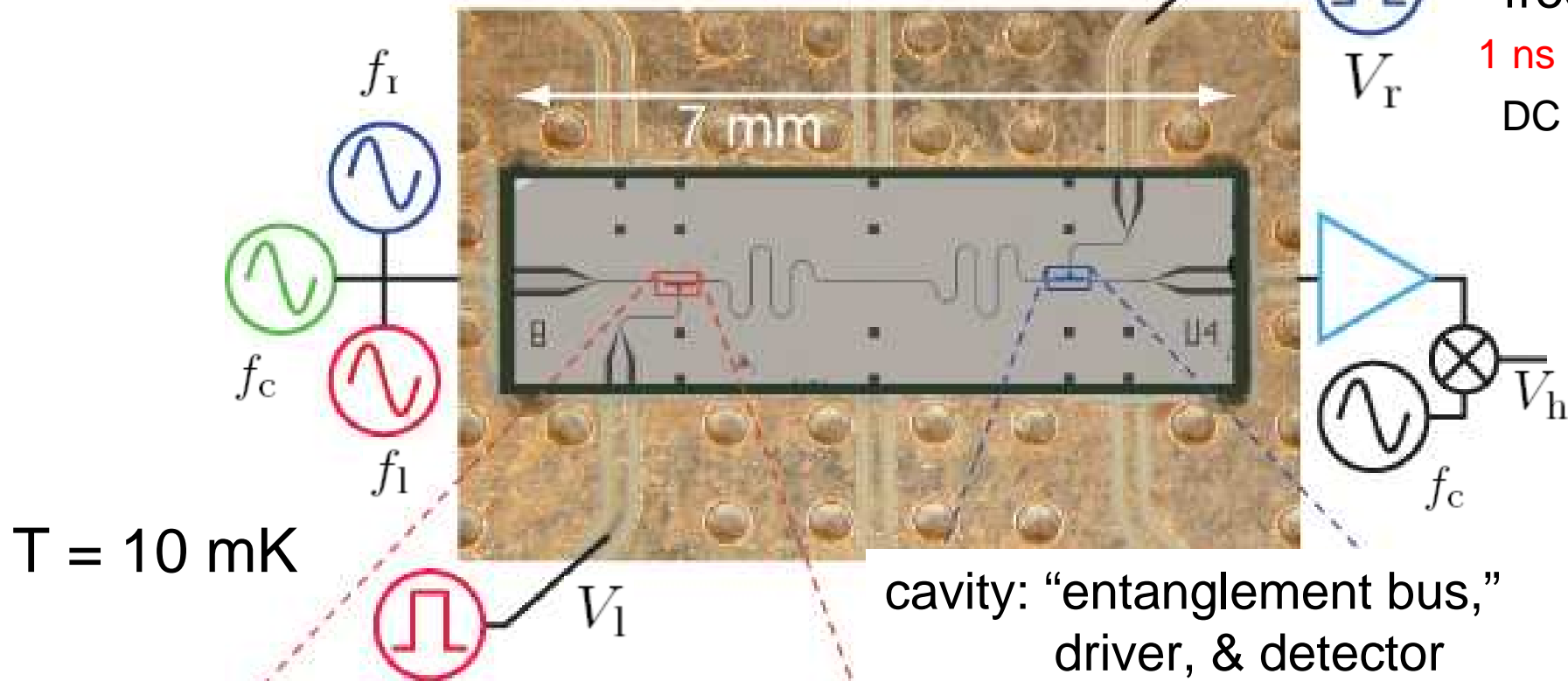


Expts: Majer *et al.*, *Nature* 2007 (Charge qubits / Yale)

Sillanpaa *et al.*, *Nature* 2007 (Phase qubits / NIST)

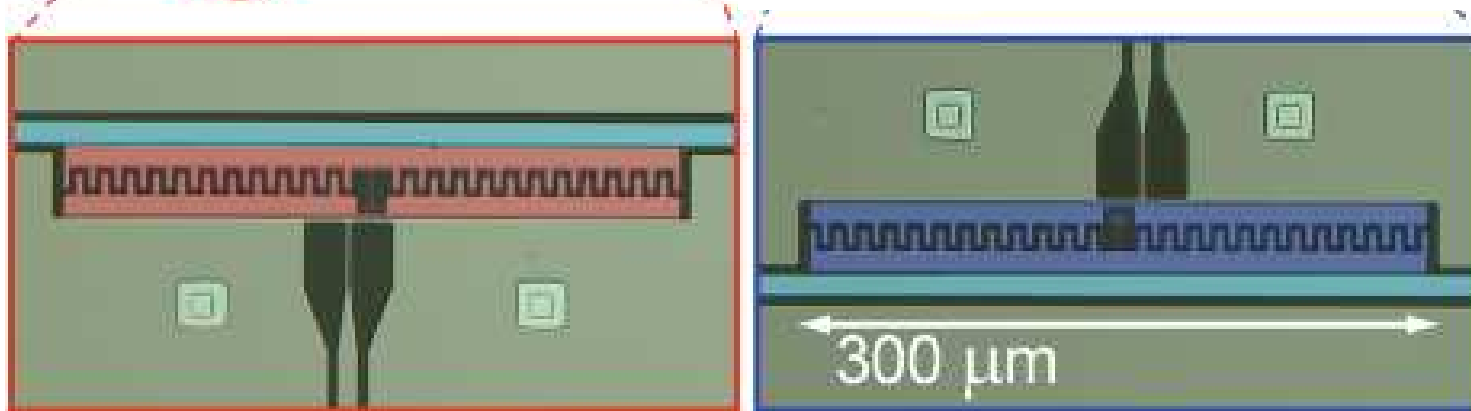
A Two-Qubit Processor

flux bias lines
control qubit
frequency
1 ns resolution
DC - 2 GHz



T = 10 mK

cavity: "entanglement bus,"
driver, & detector




transmon qubits

How do we entangle two qubits?

$R_Y(-\pi/2)$ rotation on each qubit yields superposition:


$$\begin{aligned} |\Psi\rangle &= \frac{1}{2}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \\ &= \frac{1}{2}(|00\rangle + |10\rangle + |01\rangle + |11\rangle) \end{aligned}$$

'Conditional Phase Gate' entangler:

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}(|00\rangle + |10\rangle + |01\rangle - |11\rangle)$$


No longer a product state!

How do we entangle two qubits?

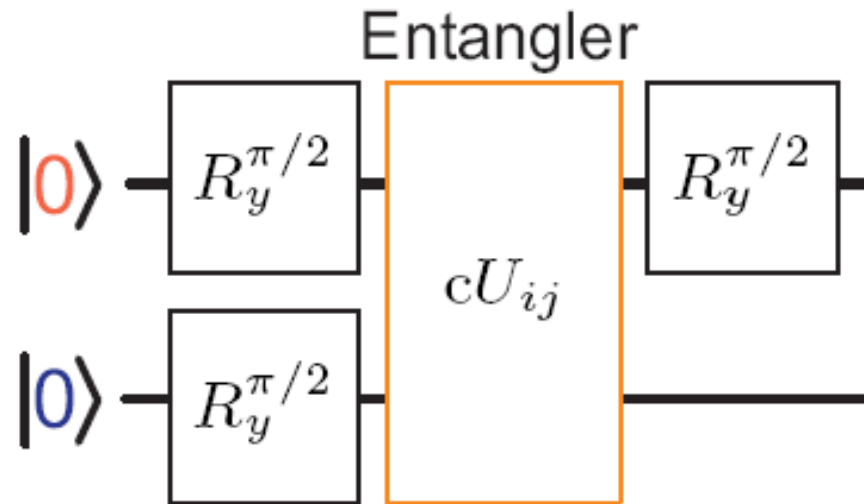
$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle - |11\rangle) = \frac{1}{\sqrt{2}} (|0 \rightarrow\rangle + |1 \leftarrow\rangle)$$


$R_Y(+\pi/2)$ rotation on **LEFT** qubit yields:

$$|\text{Bell}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Other 3 Bell states similarly achieved.


Entanglement on Demand




$$\frac{1}{\sqrt{2}} \left(\left| \begin{array}{|c|c|} \hline \text{[Blue Box]} & \text{[Red Box]} \\ \hline \end{array} \right\rangle + \left| \begin{array}{|c|c|} \hline \text{[Blue Box]} & \text{[Red Box]} \\ \hline \end{array} \right\rangle \right)$$

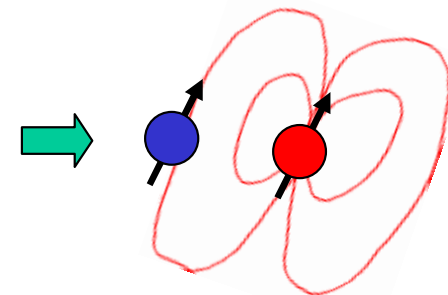
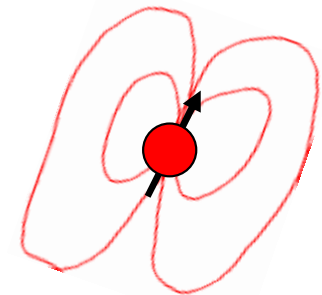
L'état quantique c'est Moi!

How do we realize the conditional phase gate?

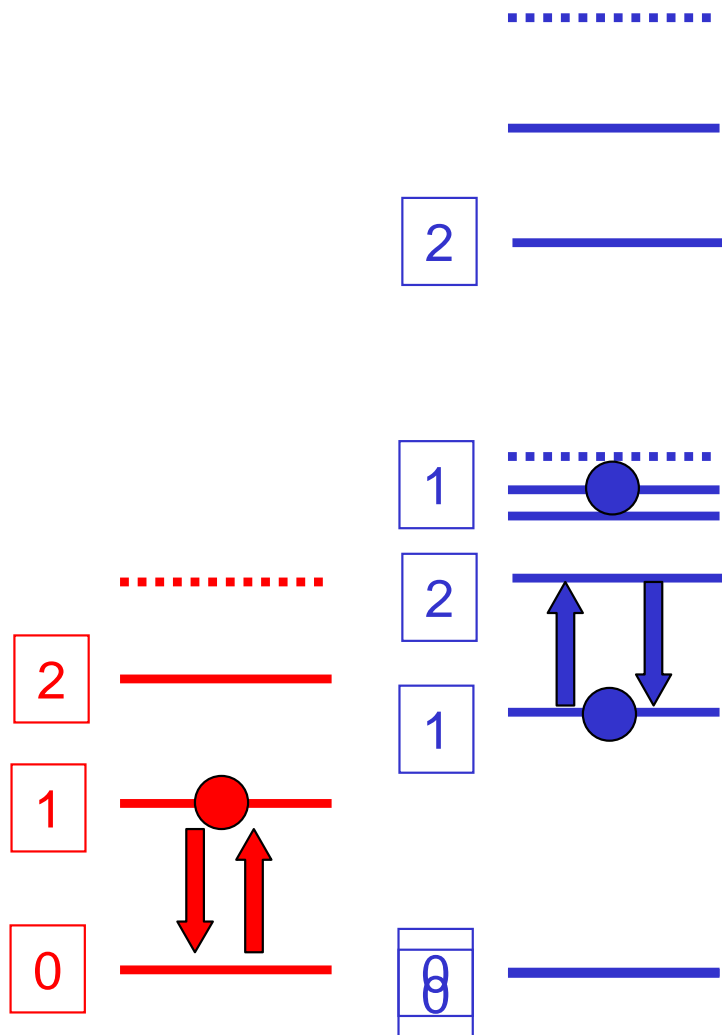
$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$


Use control lines to push qubits near a resonance:


 A controlled z-z interaction also à la NMR



Key is to use 3rd level of transmon (outside the logical subspace)



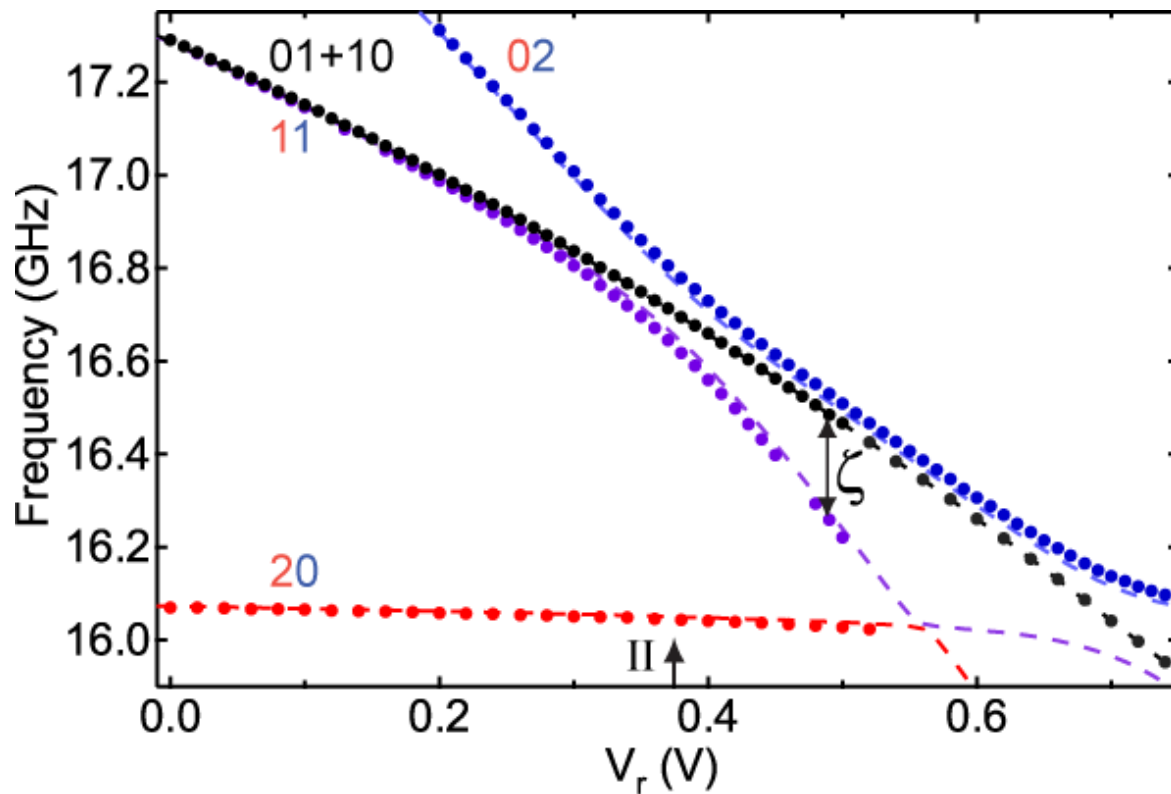
Coupling turned off.

Coupling turned on:
Near resonance with 3rd level

$$\omega_{01} \approx \omega_{12}$$

Energy is shifted if and only if
both qubits are in excited state.

Adiabatic Conditional Phase Gate



- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |02\rangle$$

- A frequency shift

$$\zeta/2\pi = f_{01} + f_{10} - f_{11}$$

$$1.2 \text{ MHz} \leq \zeta/2\pi \lesssim 150 \text{ MHz}$$


On/off ratio $\approx 100:1$

Use large on-off ratio of ζ to implement 2-qubit phase gates.

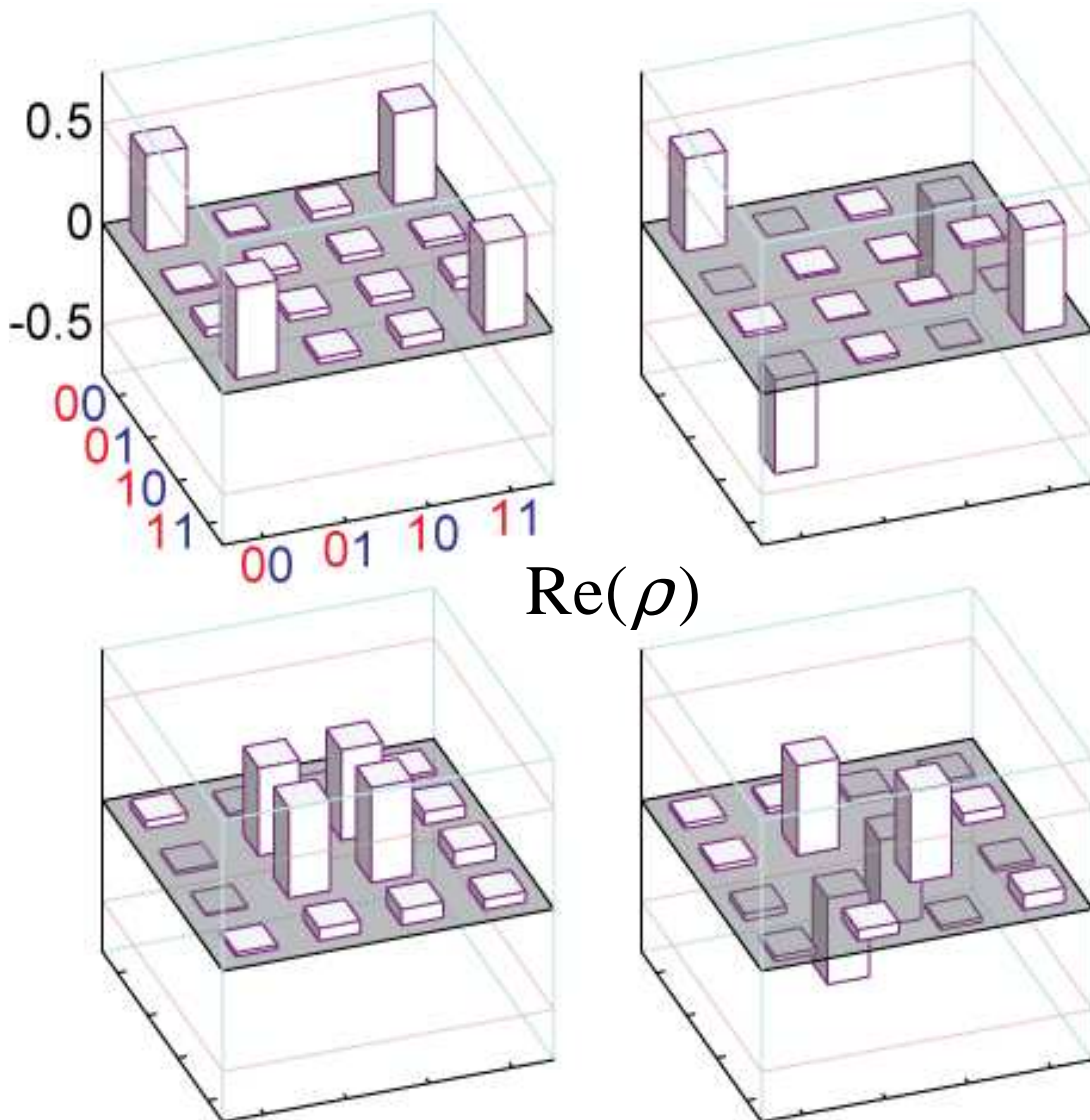
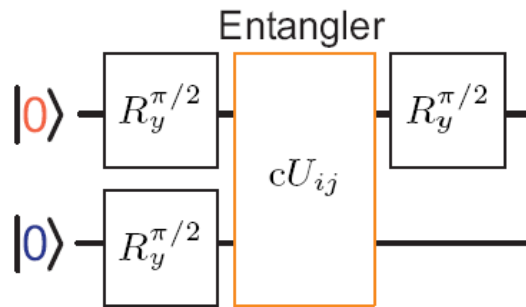
$$\int \zeta(t) dt = (2n + 1)\pi$$

Strauch et al. *PRL* (2003): proposed use of excited states in phase qubits

Adjust timing so that amplitude for both qubits to be excited acquires a minus sign:

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2} (|00\rangle + |10\rangle + |01\rangle - |11\rangle)$$


Entanglement on Demand

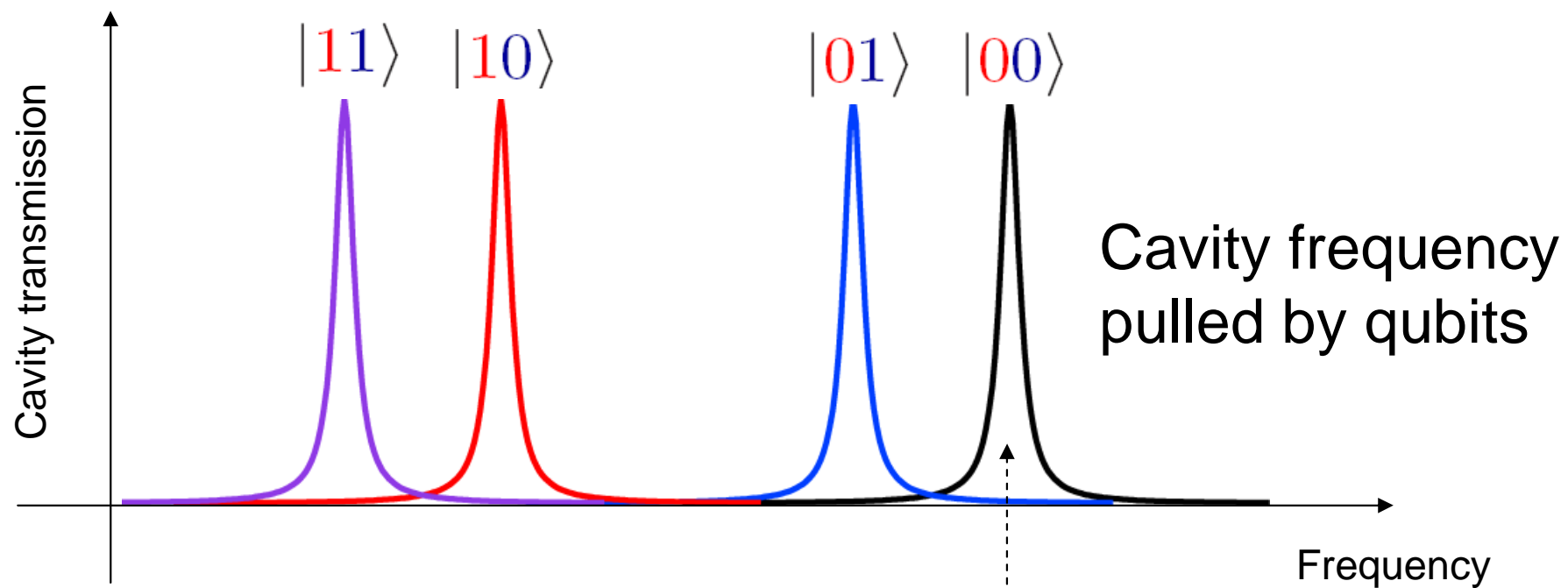
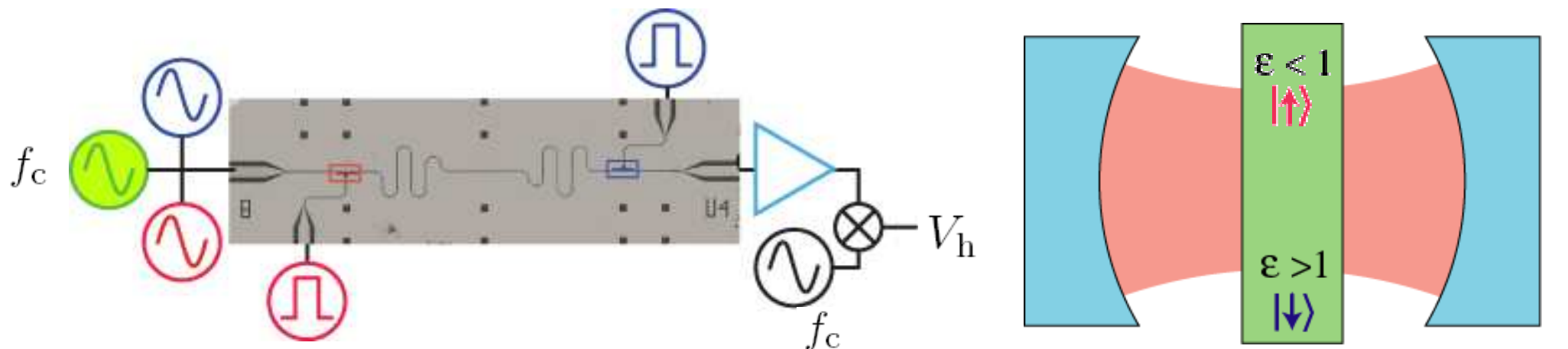


Bell state	Fidelity	Concurrence
$ 00\rangle + 11\rangle$	91%	88%
$ 00\rangle - 11\rangle$	94%	94%
$ 01\rangle + 10\rangle$	90%	86%
$ 01\rangle - 10\rangle$	87%	81%

UCSB: Steffen *et al.*, Science (2006)
 ETH: Leek *et al.*, PRL (2009)

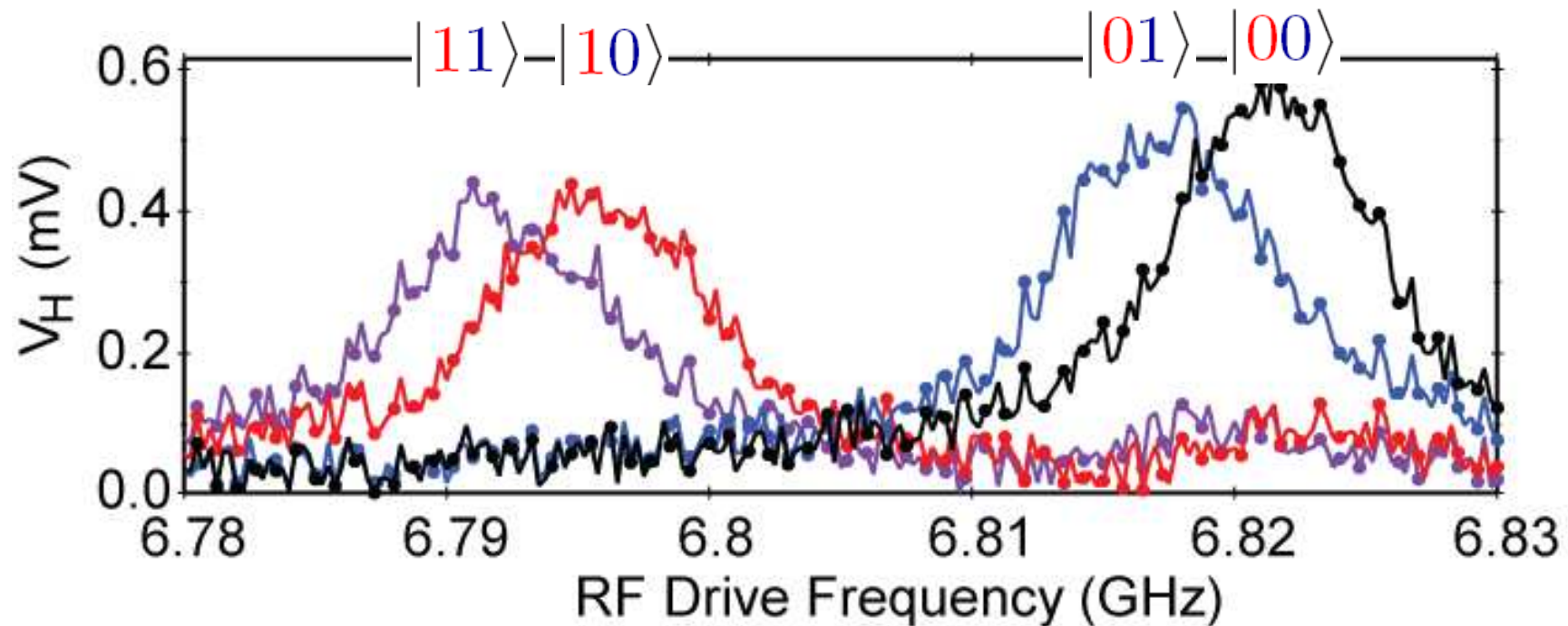
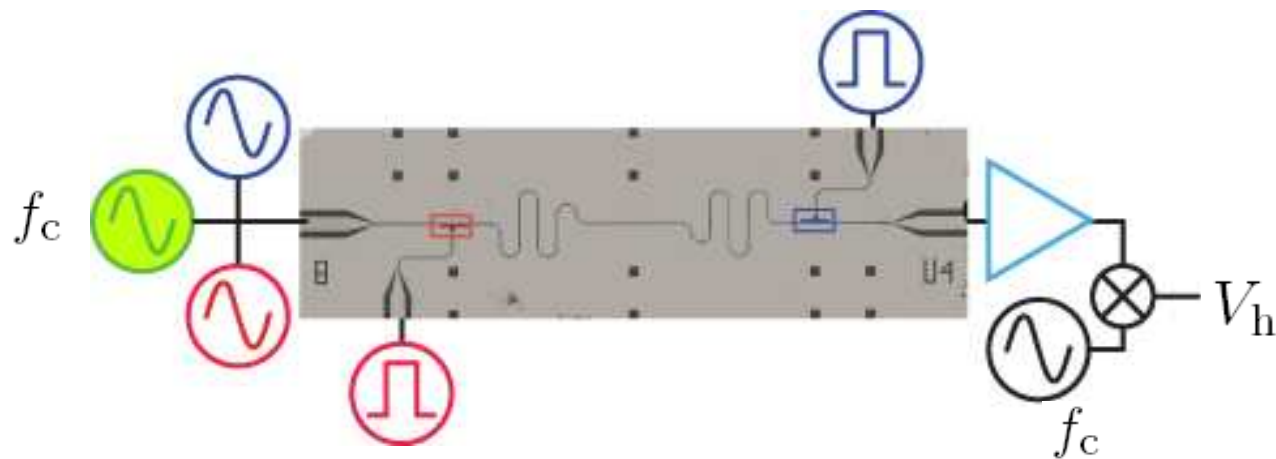
How do we read out the qubit state and
measure the entanglement?

Two Qubit Joint Readout via Cavity



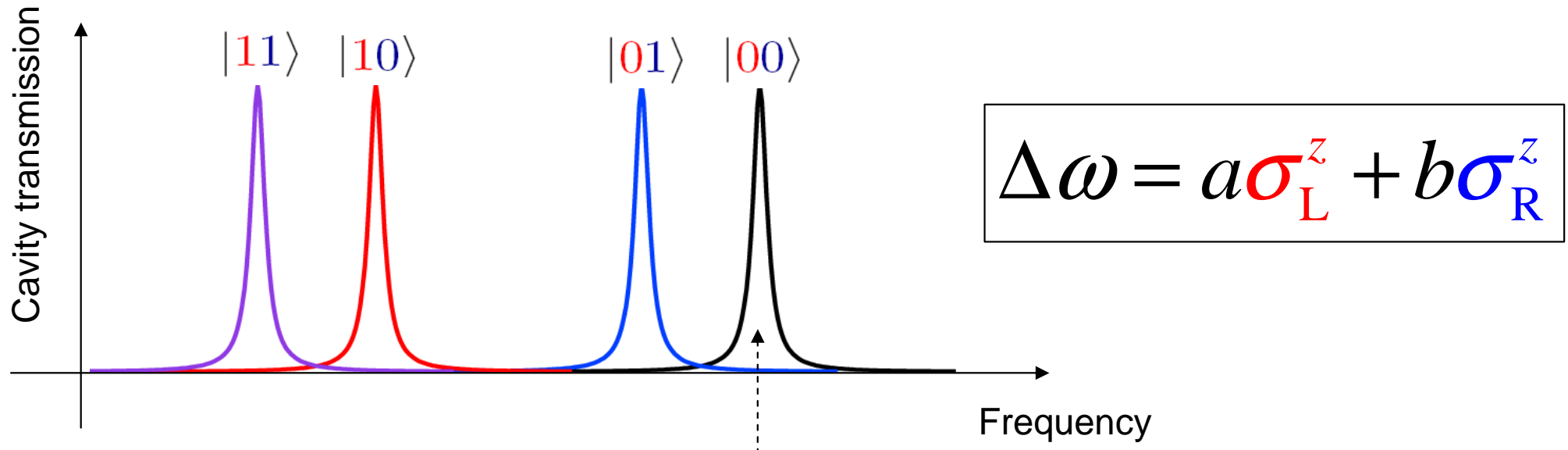
“Strong dispersive cQED”: Schuster et al., 2007

Two Qubit Joint Readout via Cavity



Initial polarization of qubit? $> 99.7\%$ (Bishop et al., 2009) \rightarrow reset fidelity is high!

Cavity Pull is linear in spin polarizations



Complex transmitted amplitude is non-linear in cavity pull:

$$t = \frac{\kappa/2}{\omega_{\text{drive}} - \omega_{\text{cavity}} - \Delta\omega + i\kappa/2}$$

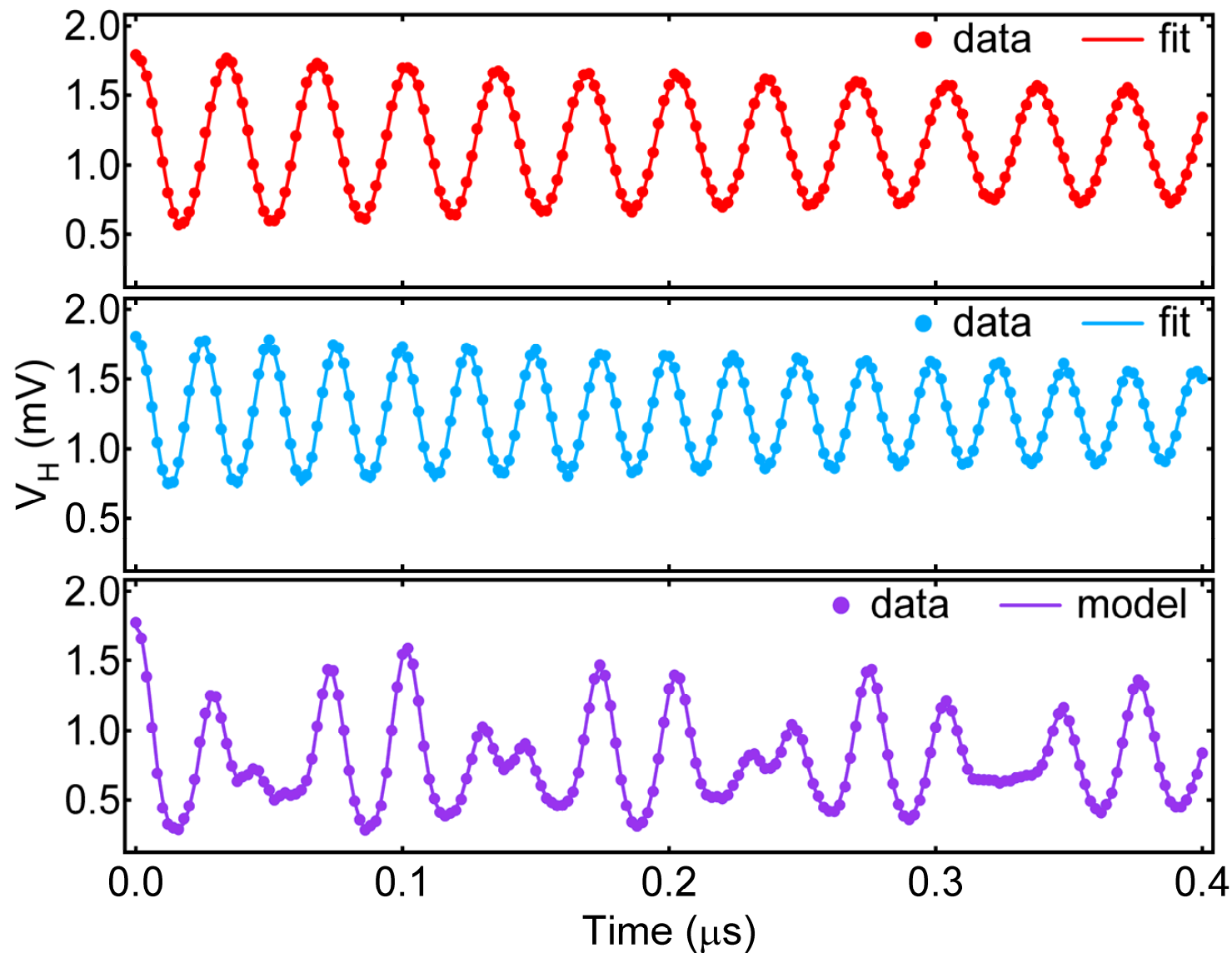
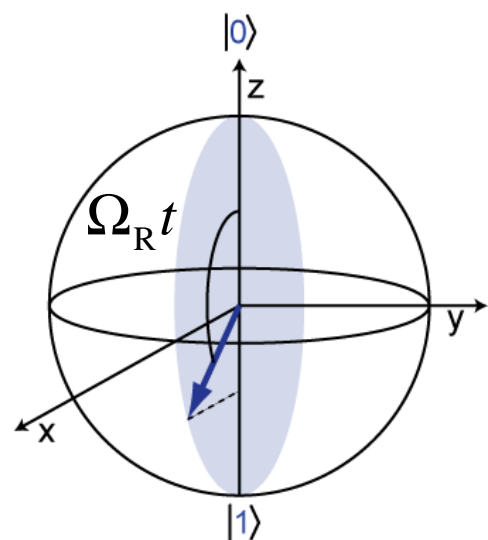
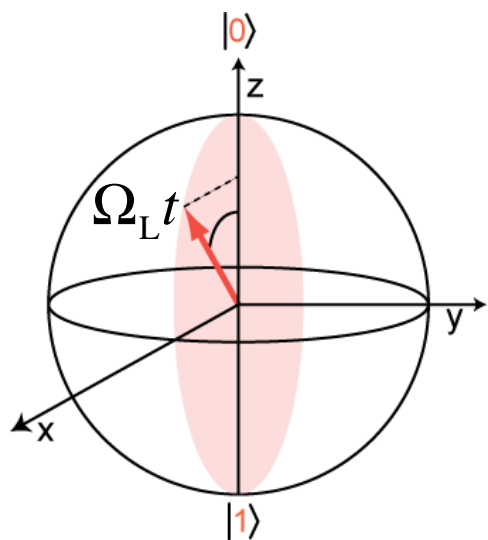
Most general non-linear function of two Ising spin variables:

$$t = \cancel{\beta_0} + \beta_1 \sigma_L^z + \beta_2 \sigma_R^z + \beta_{12} \sigma_L^z \otimes \sigma_R^z$$

Joint Readout

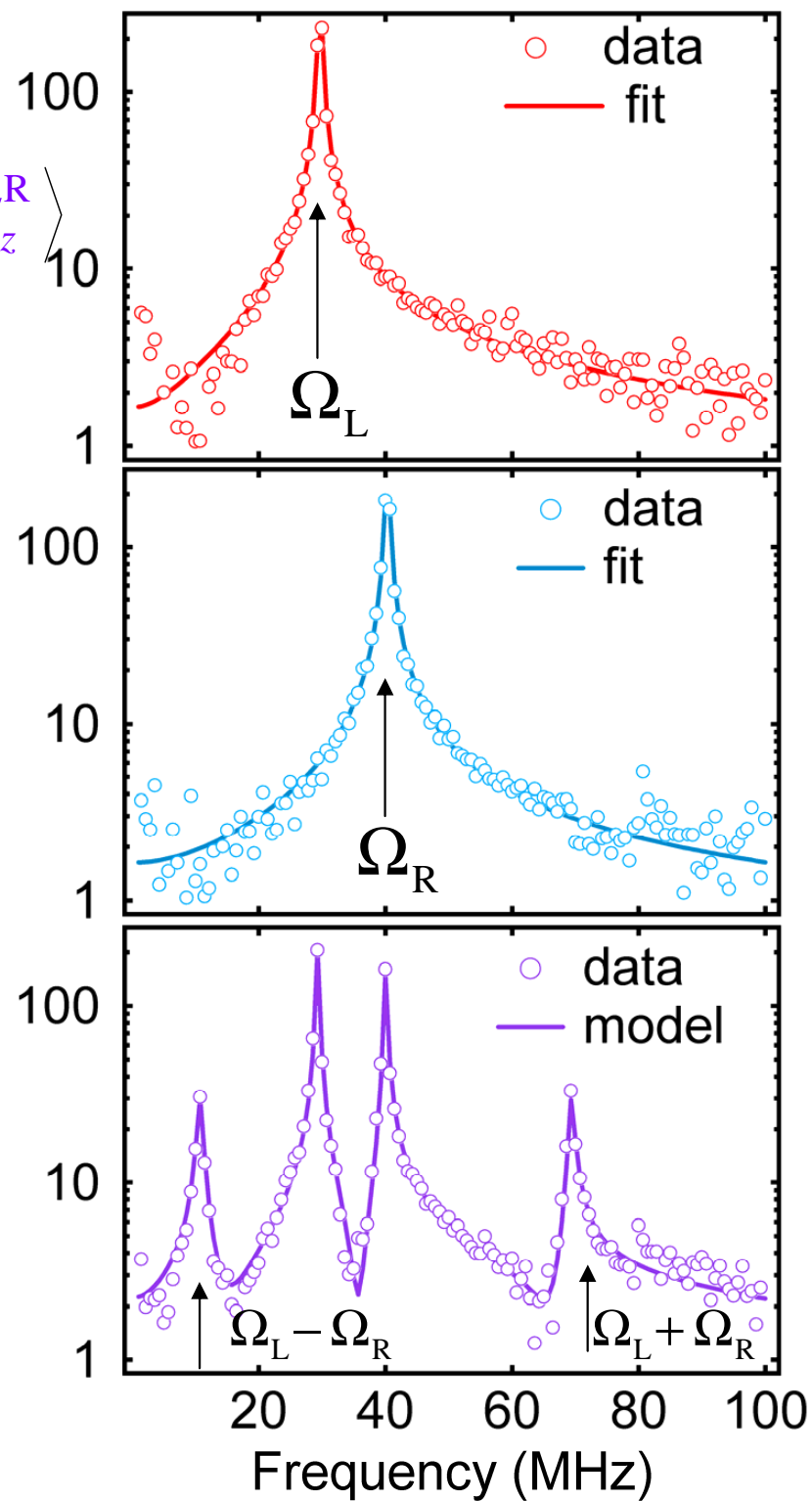
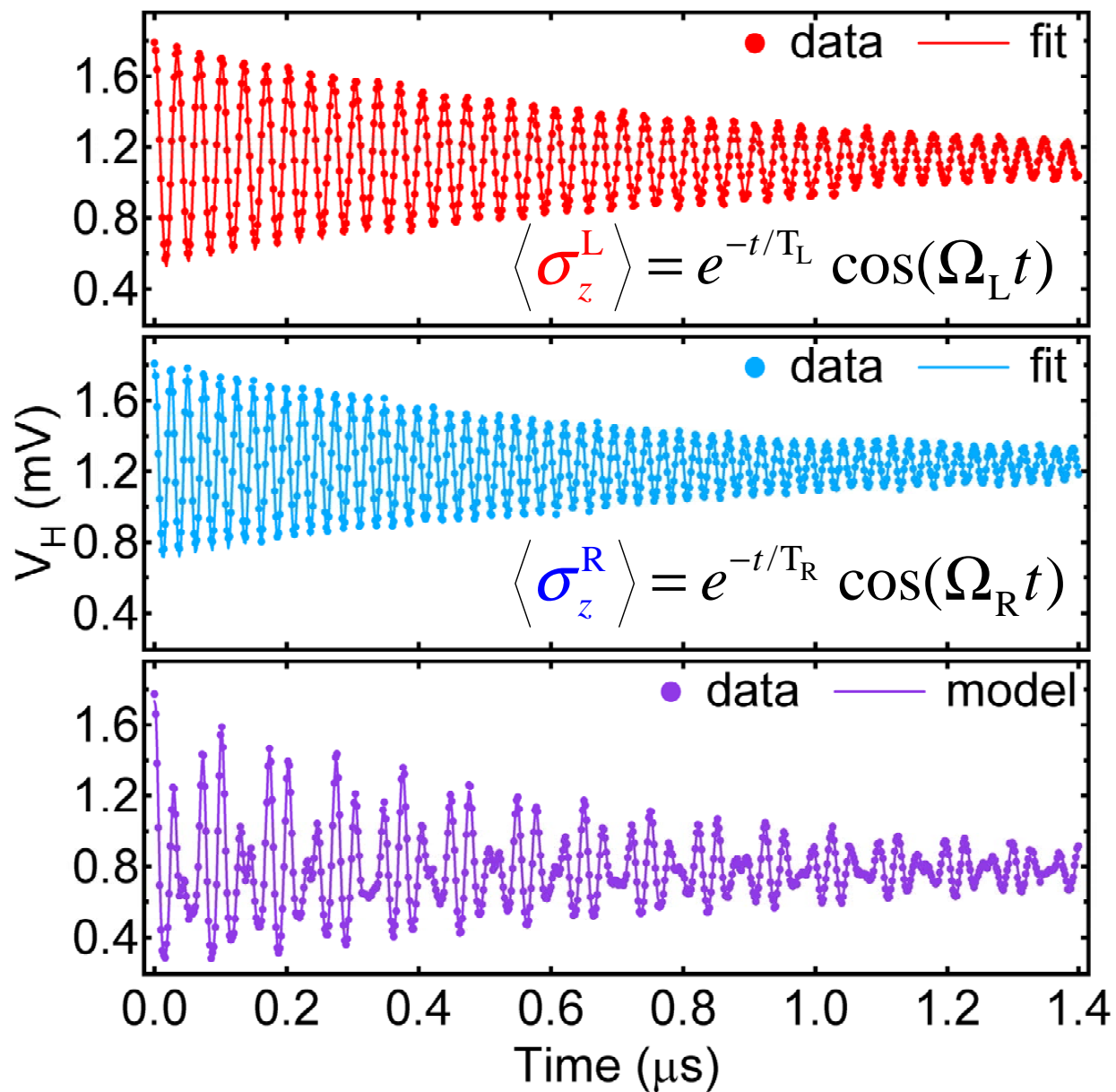
$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$

$$\beta_1 \sim 1; \quad \beta_2 \sim 0.8; \quad \beta_{12} \sim 0.5$$

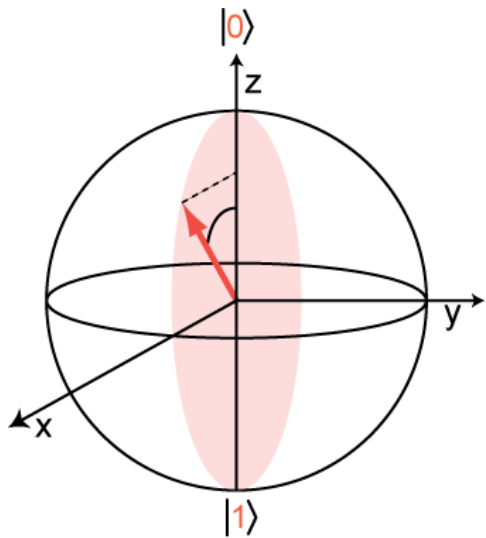


Joint Readout

$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$



State Tomography



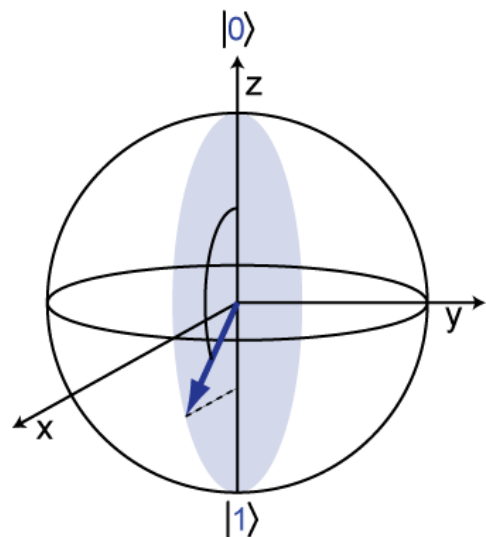
$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$

Combine joint readout with one-qubit “analysis” rotations

$$\langle \sigma_z^L \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^R) \leftarrow \pi\text{-pulse on right}$$

$$\langle \sigma_z^R \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^L) \leftarrow \pi\text{-pulse on left}$$

$$\langle \sigma_z^L \sigma_z^R \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^R, Y_\pi^L) \leftarrow \pi \text{ on both}$$



Possible to acquire correlation information even with single, ensemble averaged msmt.!

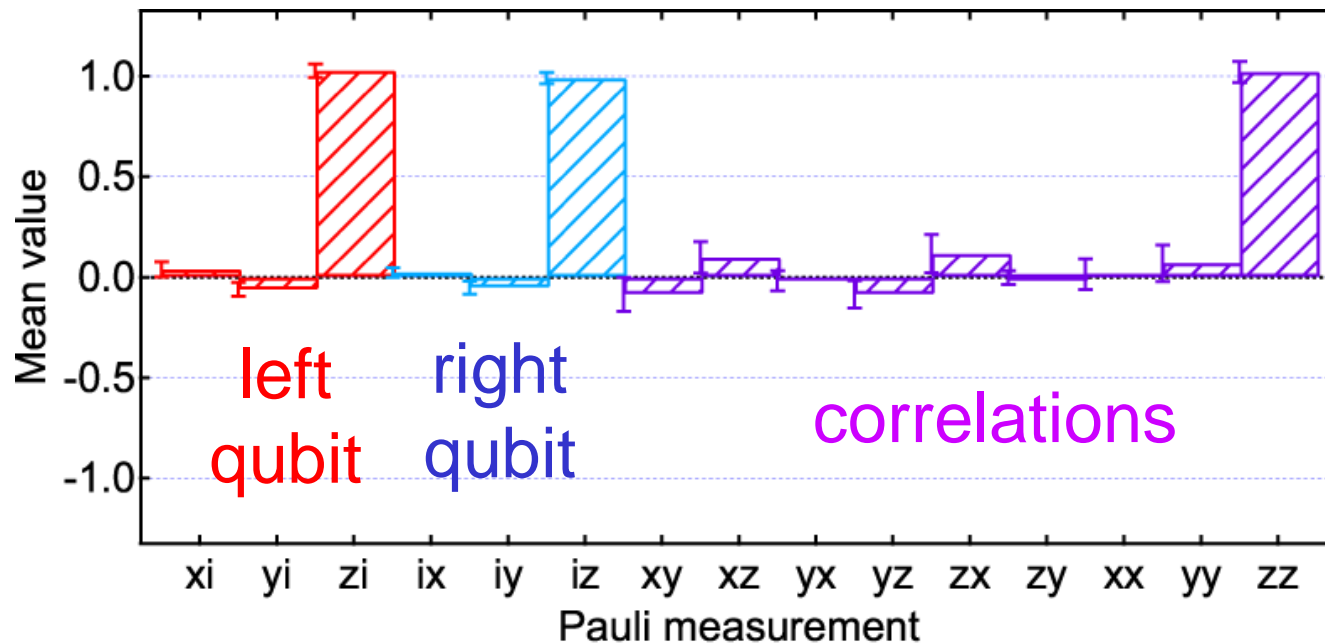
Rotate qubits to map other correlations onto z-z.

See similar from Zurich group: Phillip et al., PRL **102**, 200402 (2009).

Measuring the Two-Qubit State

Total of 16 msmts.: $I, Y_{\pi}^L, X_{\pi/2}^L, Y_{\pi/2}^L$ and combinations
 $I, Y_{\pi}^R, X_{\pi/2}^R, Y_{\pi/2}^R$

(almost) raw data

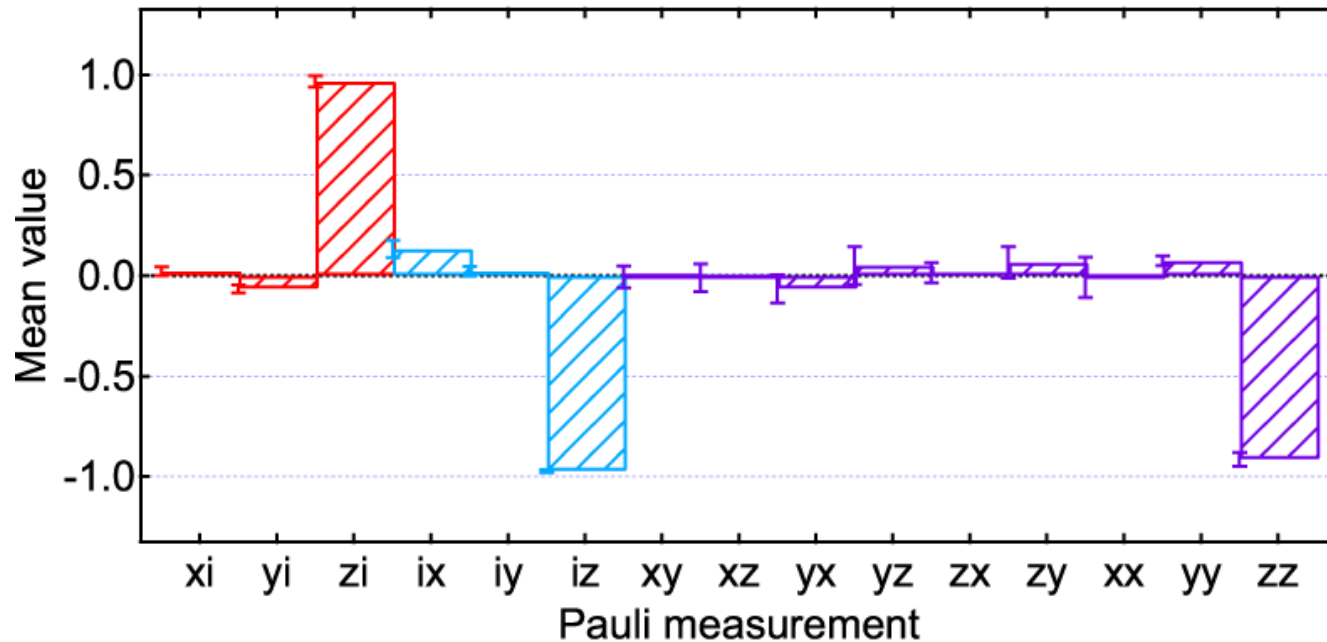


Ground state: $|\psi\rangle = |00\rangle = |\uparrow\uparrow\rangle$

$$\langle \sigma_L^z \rangle = \langle \sigma_R^z \rangle = \langle \sigma_L^z \sigma_R^z \rangle = 1$$

Measuring the Two-Qubit State

Apply π -pulse to invert state of **right** qubit



One qubit excited: $|\psi\rangle = |01\rangle = |\uparrow\downarrow\rangle$

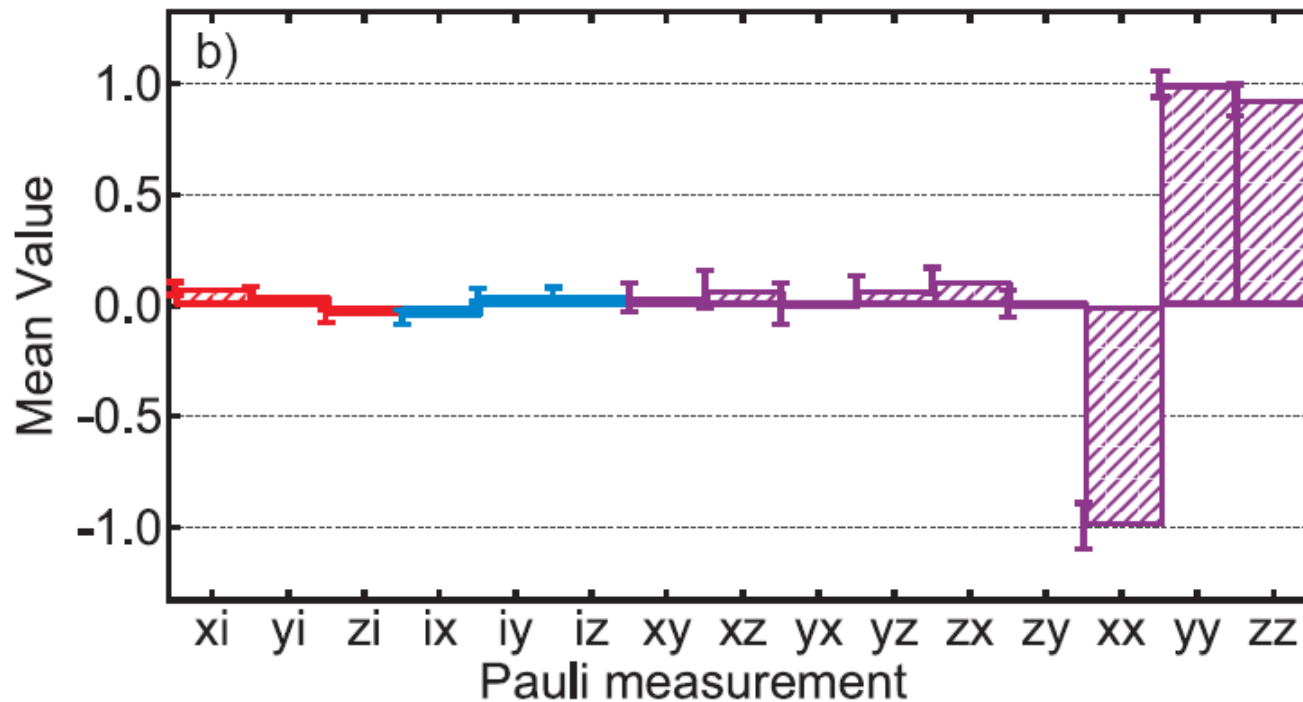
$$\langle \sigma_L^z \rangle = +1$$

$$\langle \sigma_R^z \rangle = \langle \sigma_L^z \sigma_R^z \rangle = -1$$

Measuring the Two-Qubit State

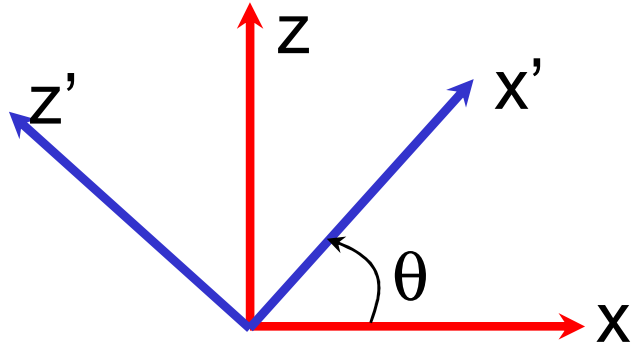
Now apply a two-qubit gate to *entangle* the qubits

Entangled state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$



$$\langle \sigma_L^z \rangle = \langle \sigma_R^z \rangle = 0$$
$$\langle \sigma_L^z \sigma_R^z \rangle = +1$$
$$\langle \sigma_L^y \sigma_R^y \rangle = +1$$
$$\langle \sigma_L^x \sigma_R^x \rangle = -1$$

Witnessing Entanglement



Clauser, Horne,
Shimony & Holt (1969)

CHSH operator = entanglement witness

$$CHSH = XX' - XZ' + ZX' + ZZ'$$

If variables take on the values ± 1
and exist even independent of
measurement then

$$CHSH = X(X' - Z') + Z(X' + Z')$$

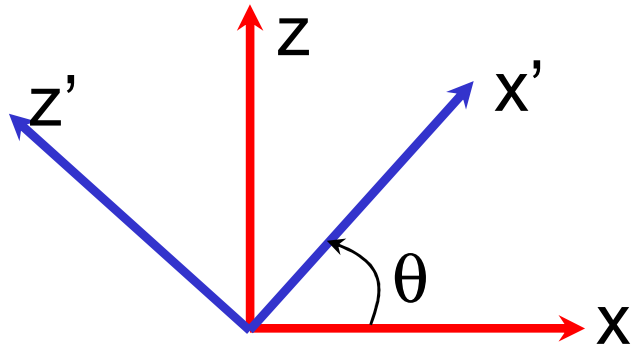
Either: $= 0$ $= \pm 2$

Or: $= \pm 2$ $= 0$

Classically:

$$|CHSH| \leq 2$$

Witnessing Entanglement



CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

— $XX' - XZ' + ZX' + ZZ'$

— $XX' + XZ' - ZX' + ZZ'$

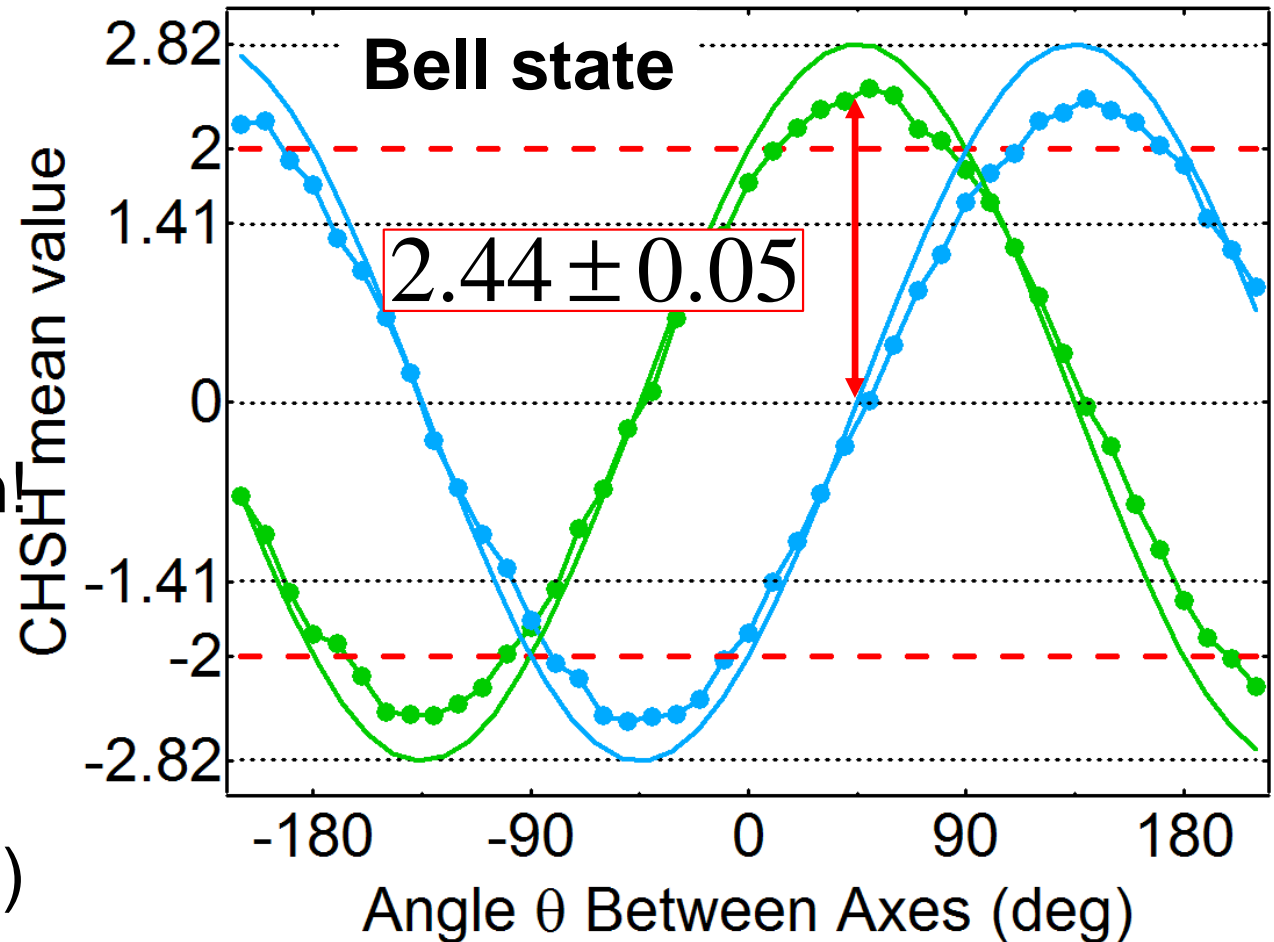
Clauser, Horne,
Shimony & Holt (1969)

Separable bound:

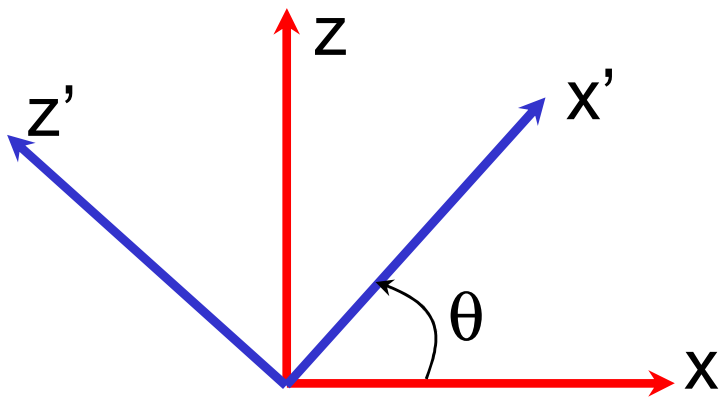
$$|CHSH| \leq 2$$

not? Bell's violation!
(loopholes abound)

but state is clearly
highly entangled!
(and no likelihood req.)



Control: Analyzing Product States



Clauser, Horne,
Shimony & Holt (1969)

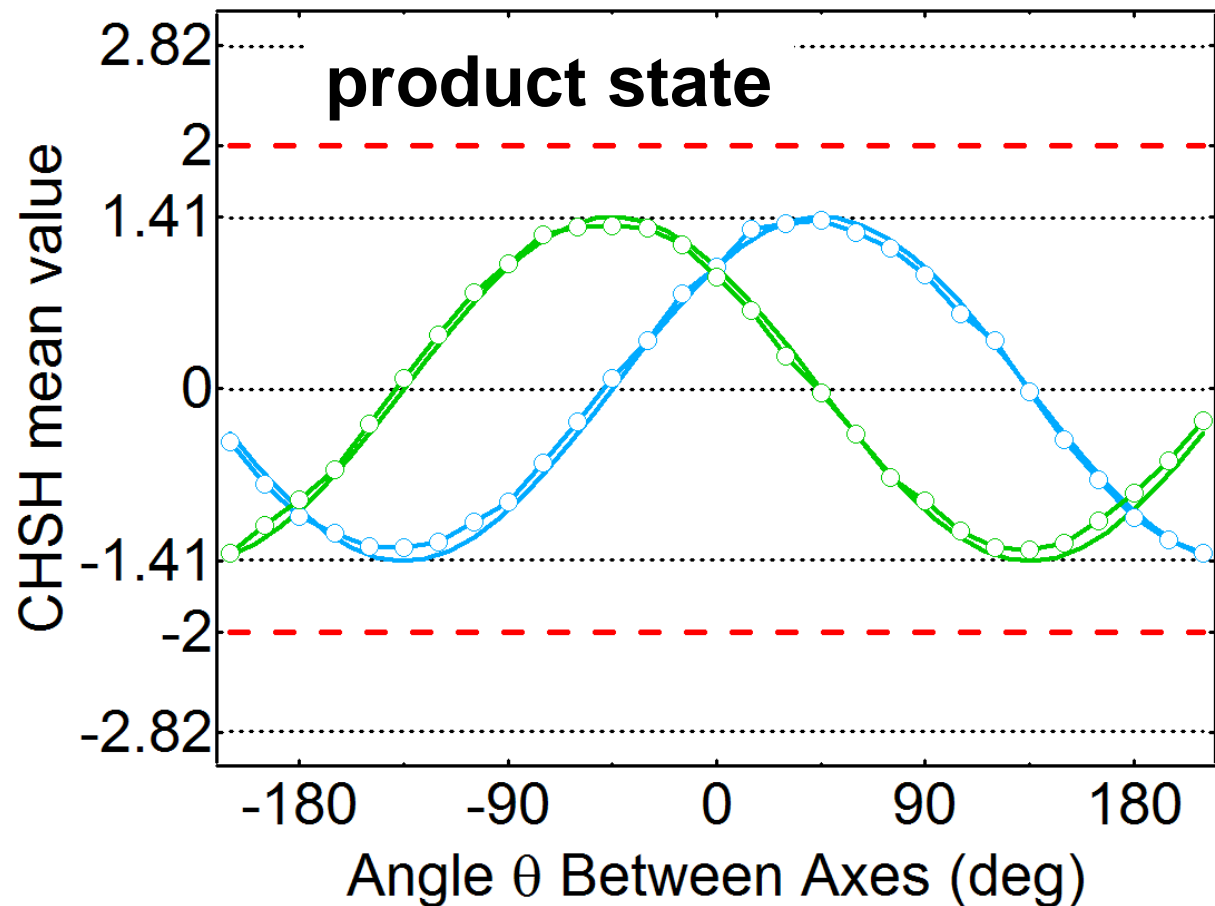
no entanglement!

CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

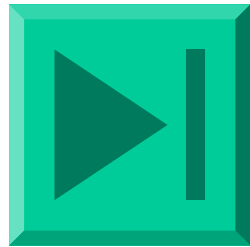
— $XX' - XZ' + ZX' + ZZ'$

— $XX' + XZ' - ZX' + ZZ'$

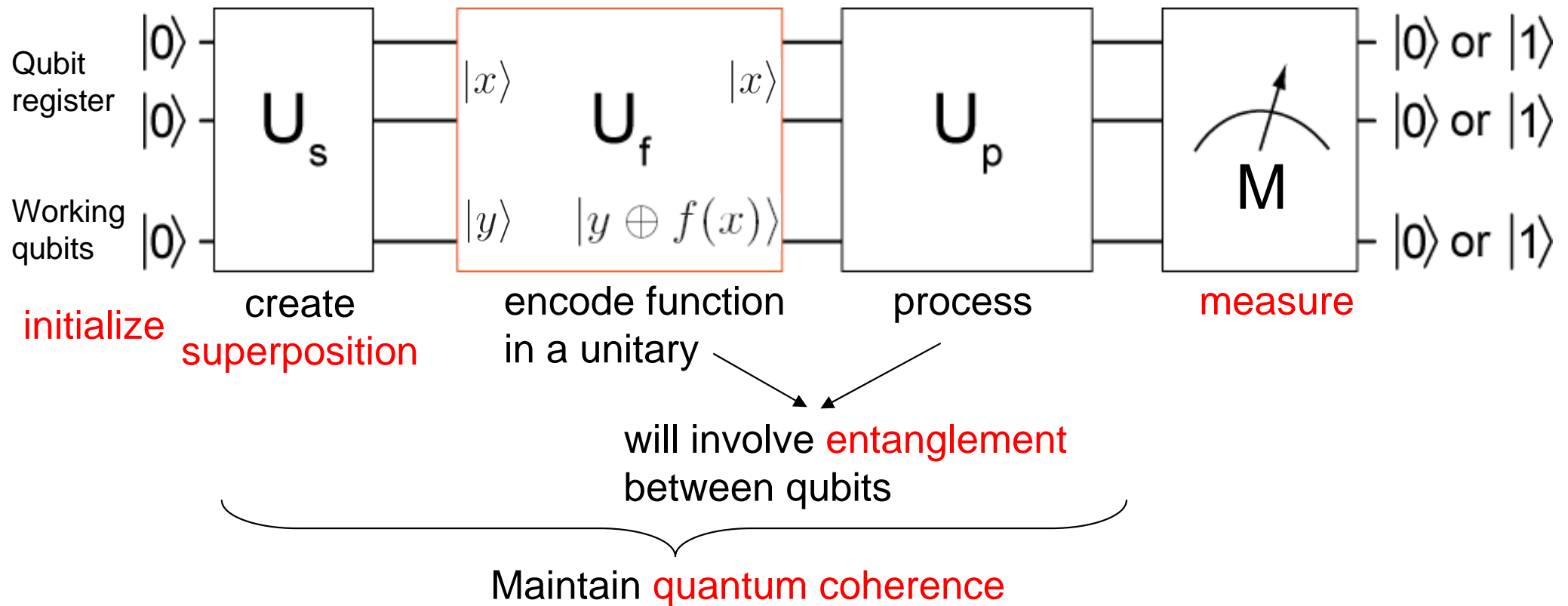


Using entanglement on demand to
run first quantum algorithm on a
solid state quantum processor

Skip to Summary



General Features of a Quantum Algorithm



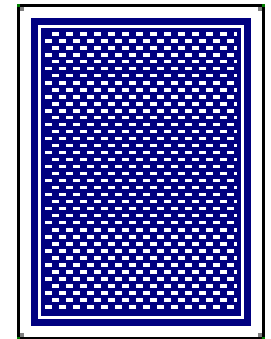
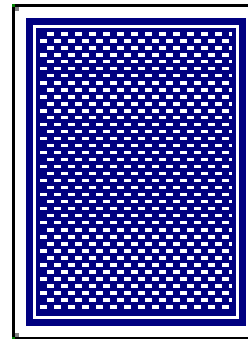
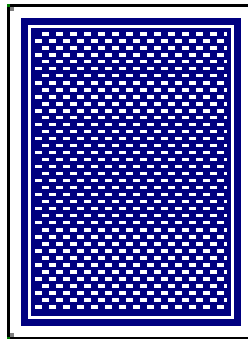
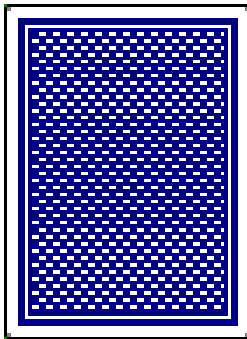
- 1) Start in superposition: all values at once!
- 2) Build complex transformation out of one-qubit and two-qubit “gates”
- 3) Somehow* make the answer we want result in a definite state at end!

*use interference: the magic of the properly designed algorithm

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

“Find x_0 !”



Position: 0

I

II

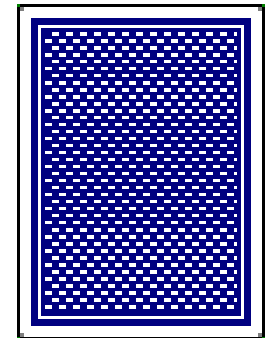
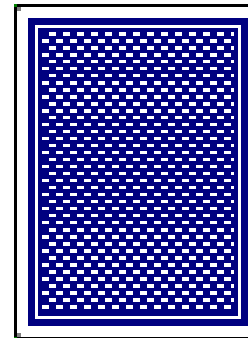
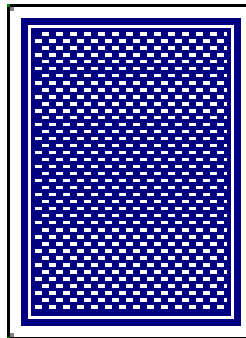
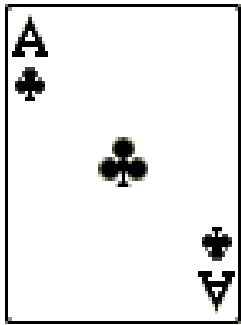
III

“Find the queen!”

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

“Find x_0 !”



Position: 0

I

II

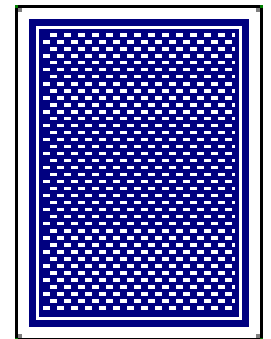
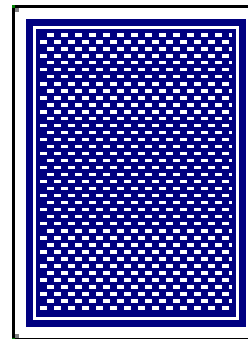
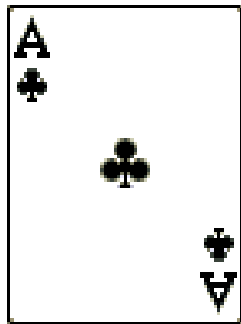
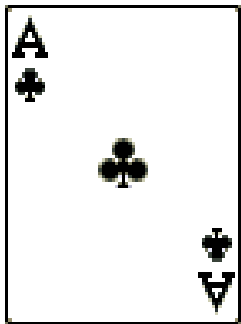
III

“Find the queen!”

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

“Find x_0 !”



Position: 0

I

II

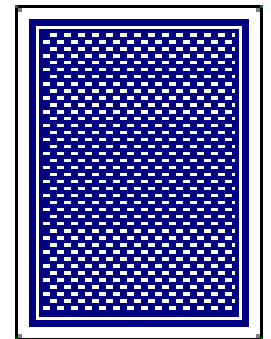
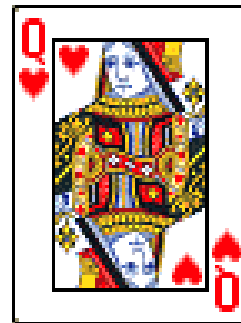
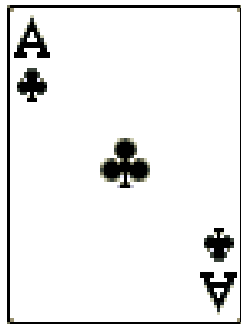
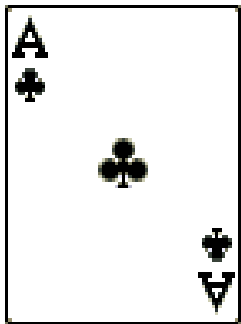
III

“Find the queen!”

The Search Problem

$$f(x) = \begin{cases} -1, & x \neq x_0 \\ 1, & x = x_0 \end{cases}$$

“Find x_0 !”



Position: 0

I

II

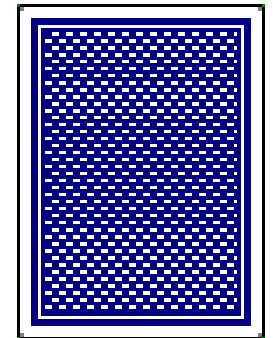
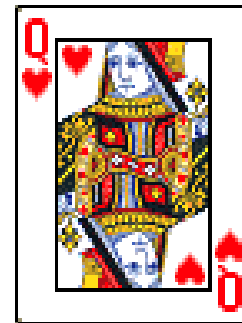
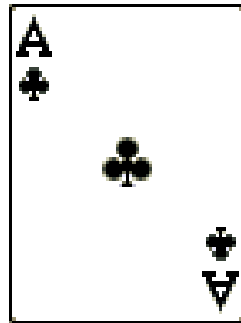
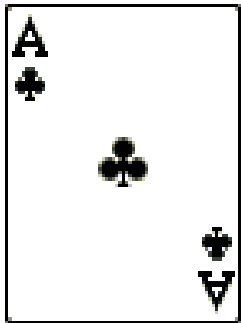
III

“Find the queen!”

The Search Problem

Classically, takes on average 2.25 guesses to succeed...

Use QM to “peek” under all the cards, find queen on first try!



Position: 0

I

II

III

“Find the queen!”

Grover's Algorithm

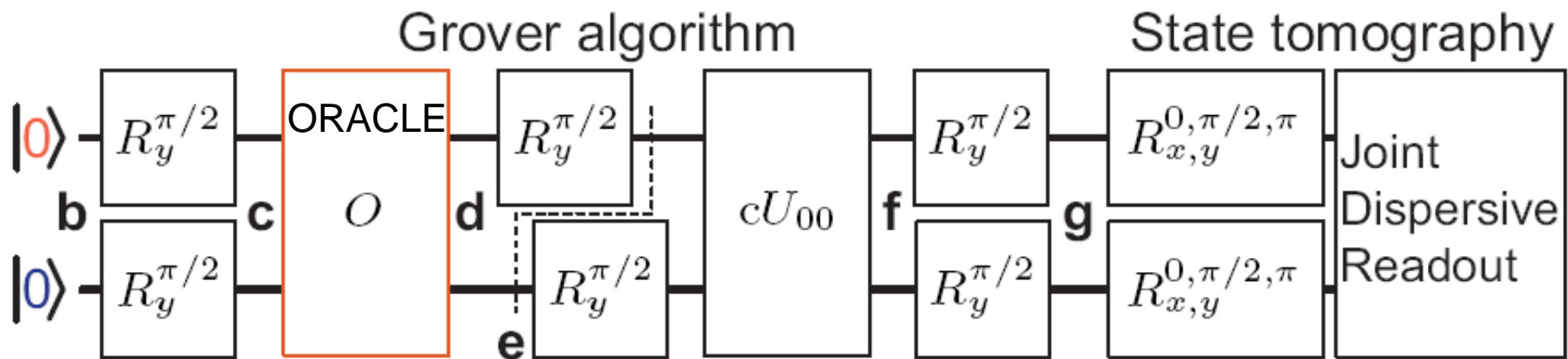
“unknown”
unitary
operation: →

$$O|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$

Challenge:
Find the location
of the -1 !!!
(= queen)

Previously implemented in NMR: Chuang et al., 1998

Ion traps: Brickman et al., 2003

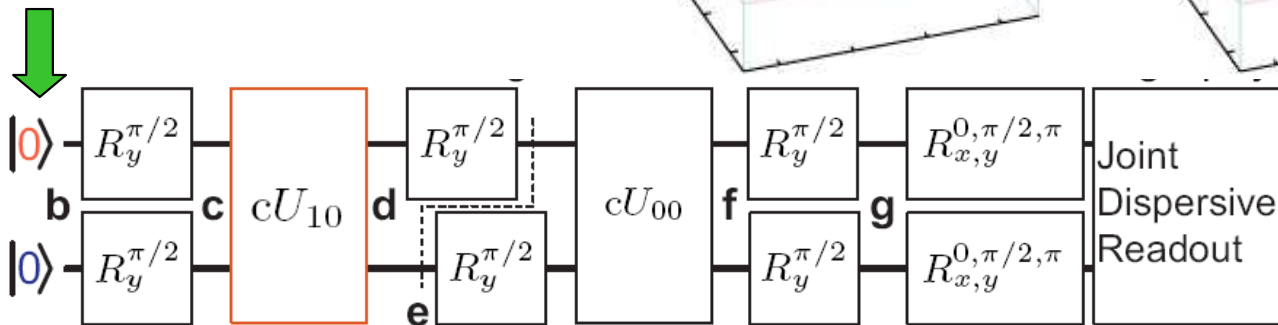
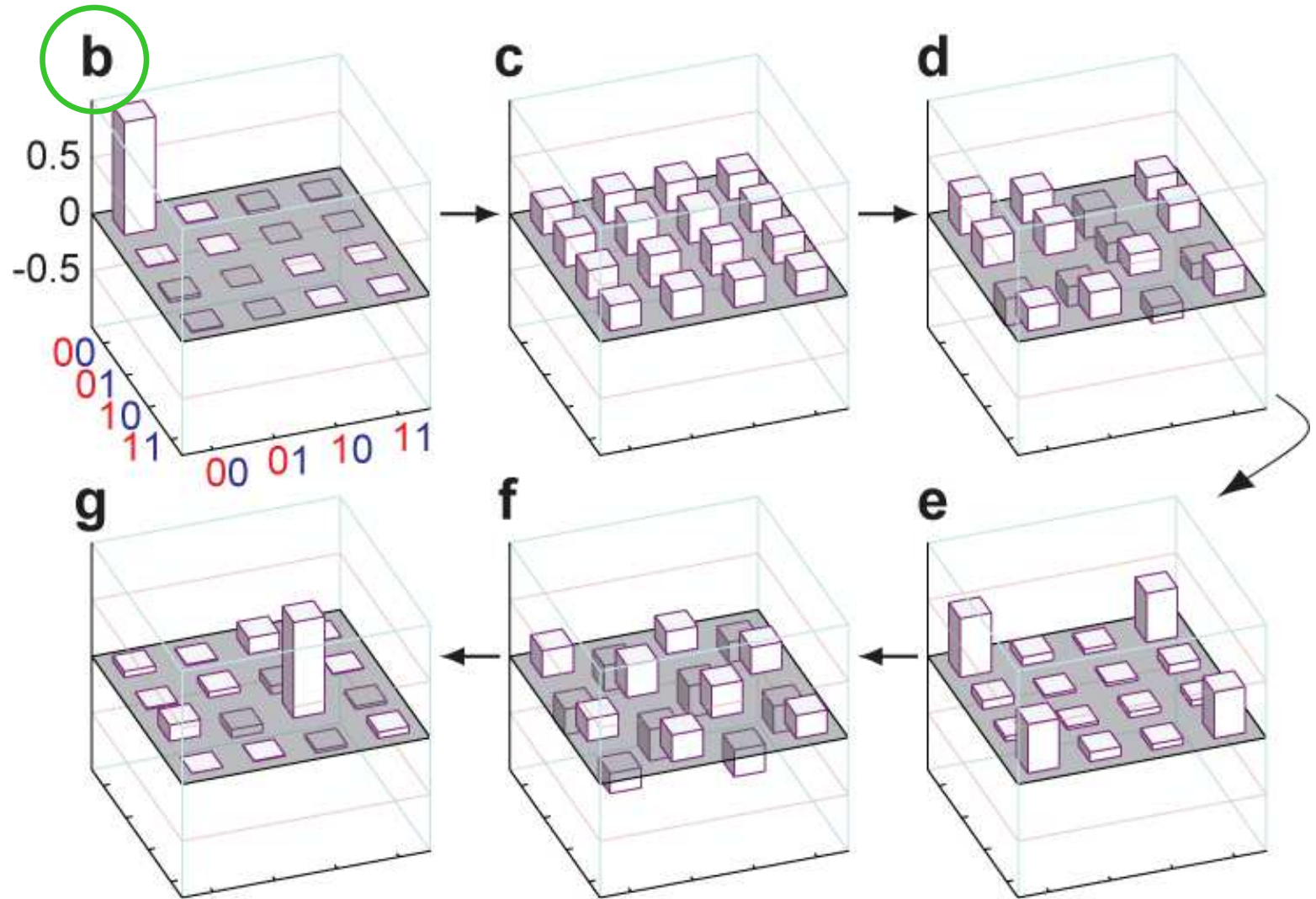


10 pulses w/ nanosecond resolution, total 104 ns duration

Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = |00\rangle$$

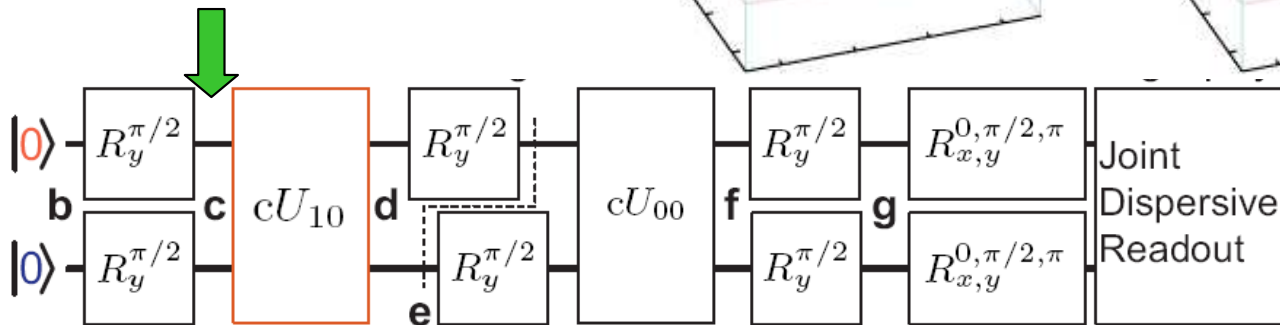
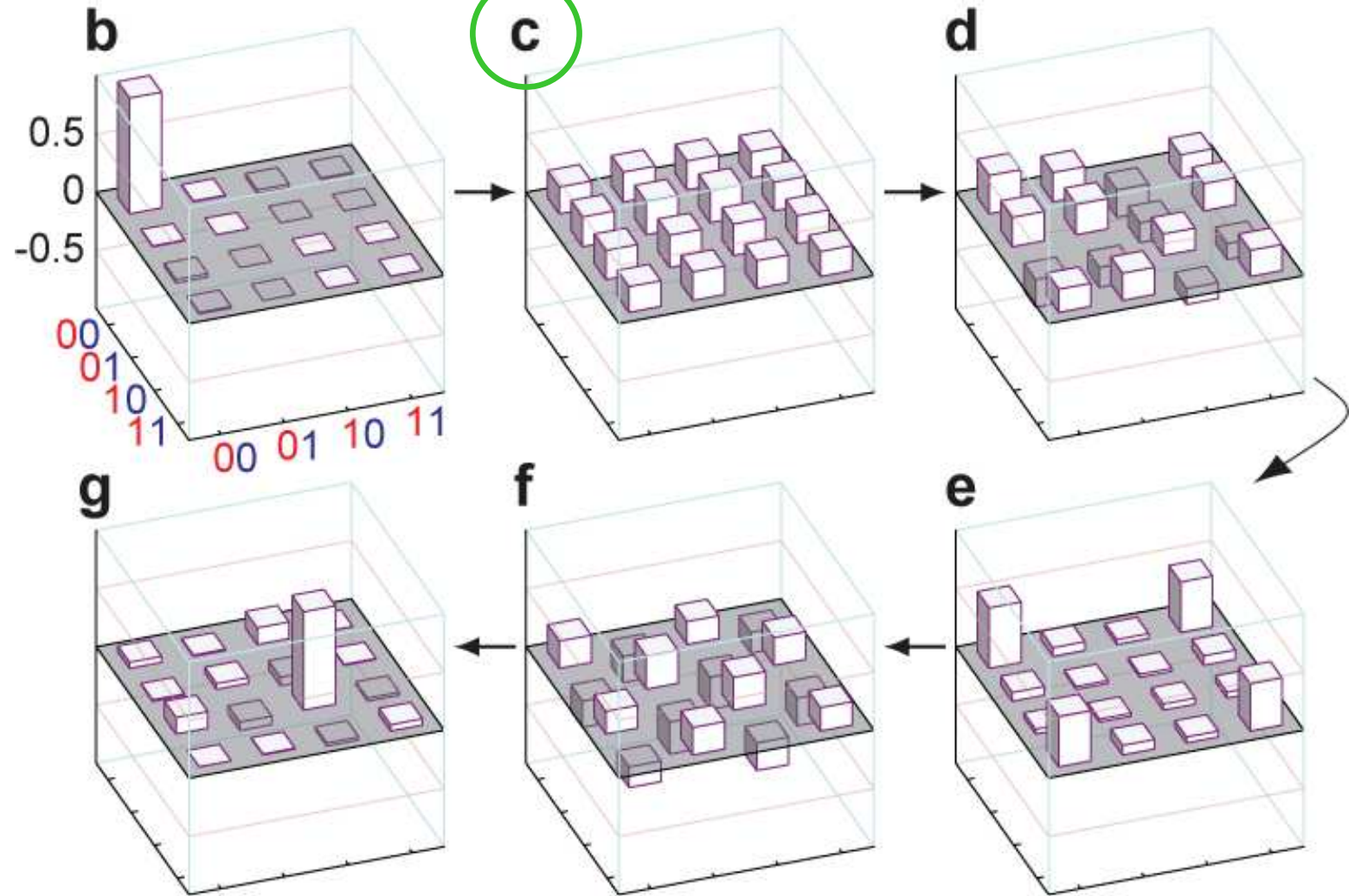
Begin in ground state:



Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Create a maximal superposition:
look everywhere
at once!

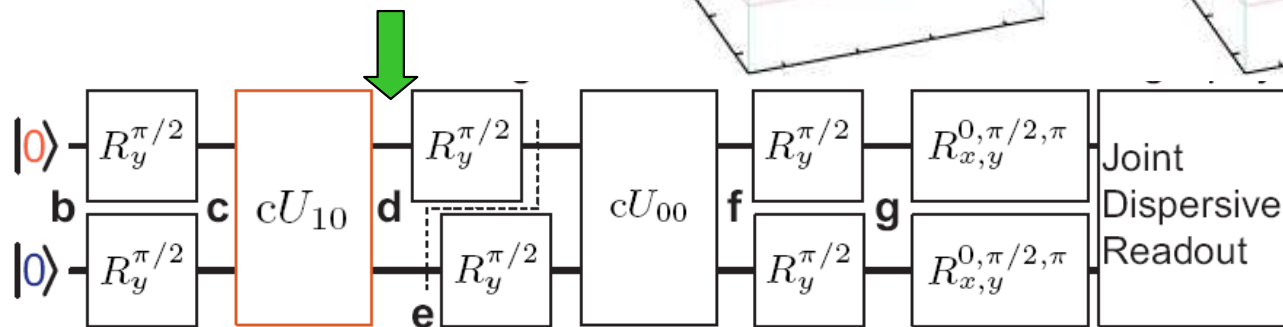
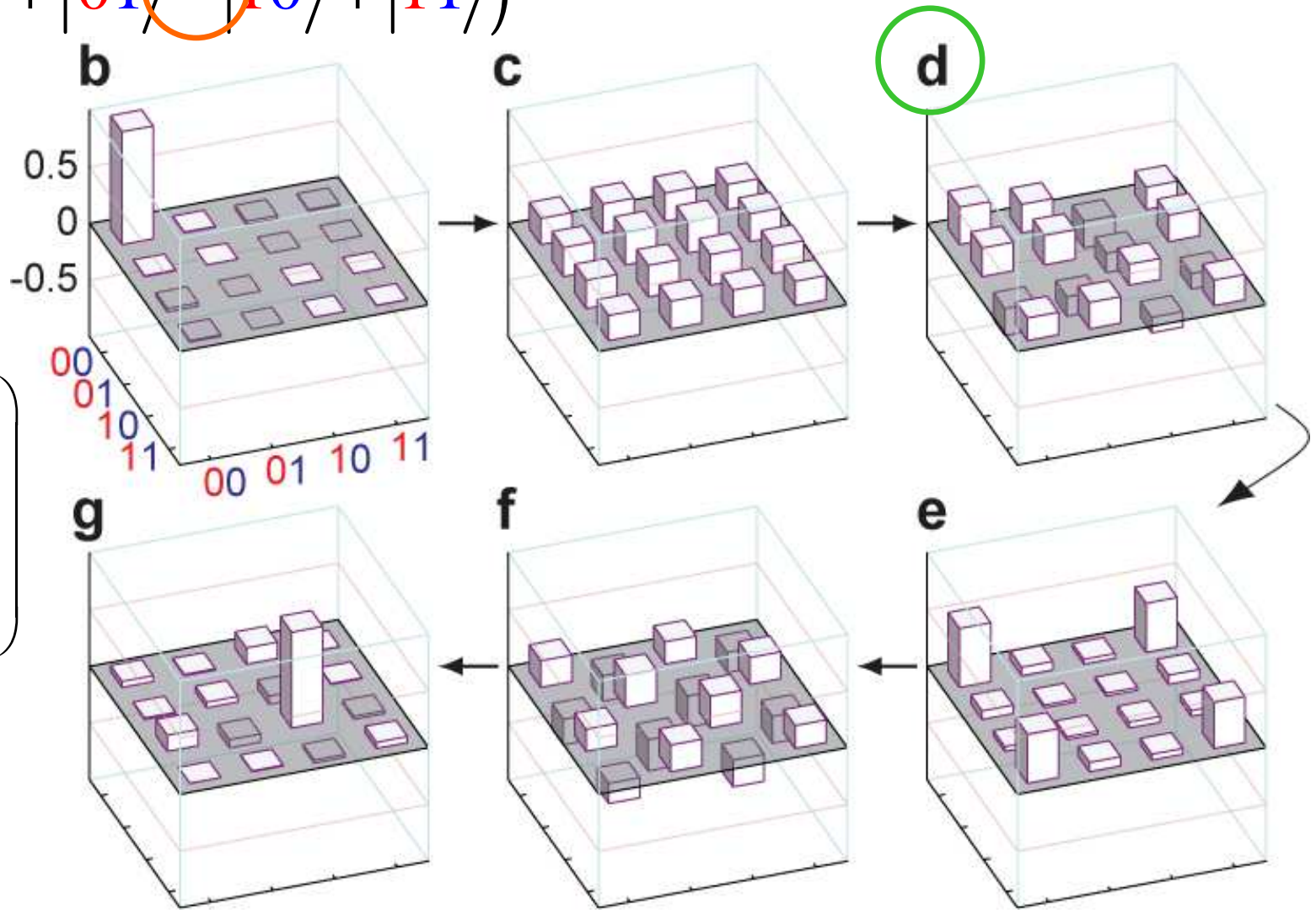


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2} (|00\rangle + |01\rangle - |10\rangle + |11\rangle)$$

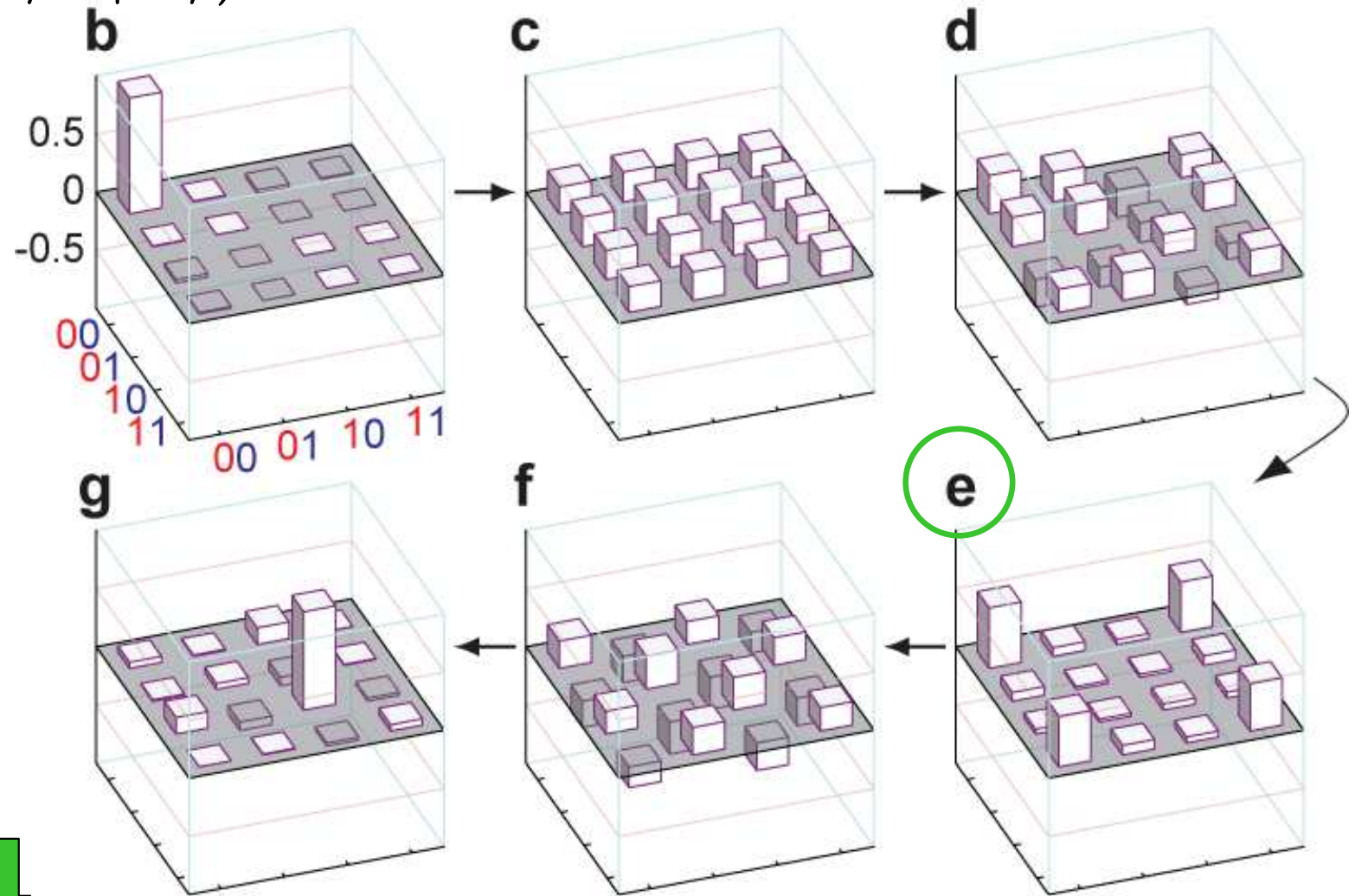
Apply the “unknown” function, and mark the solution

$$cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



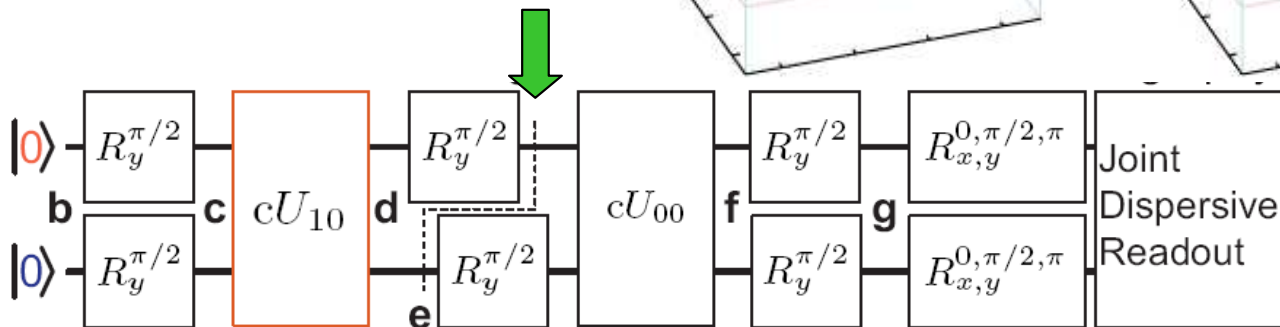
Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$



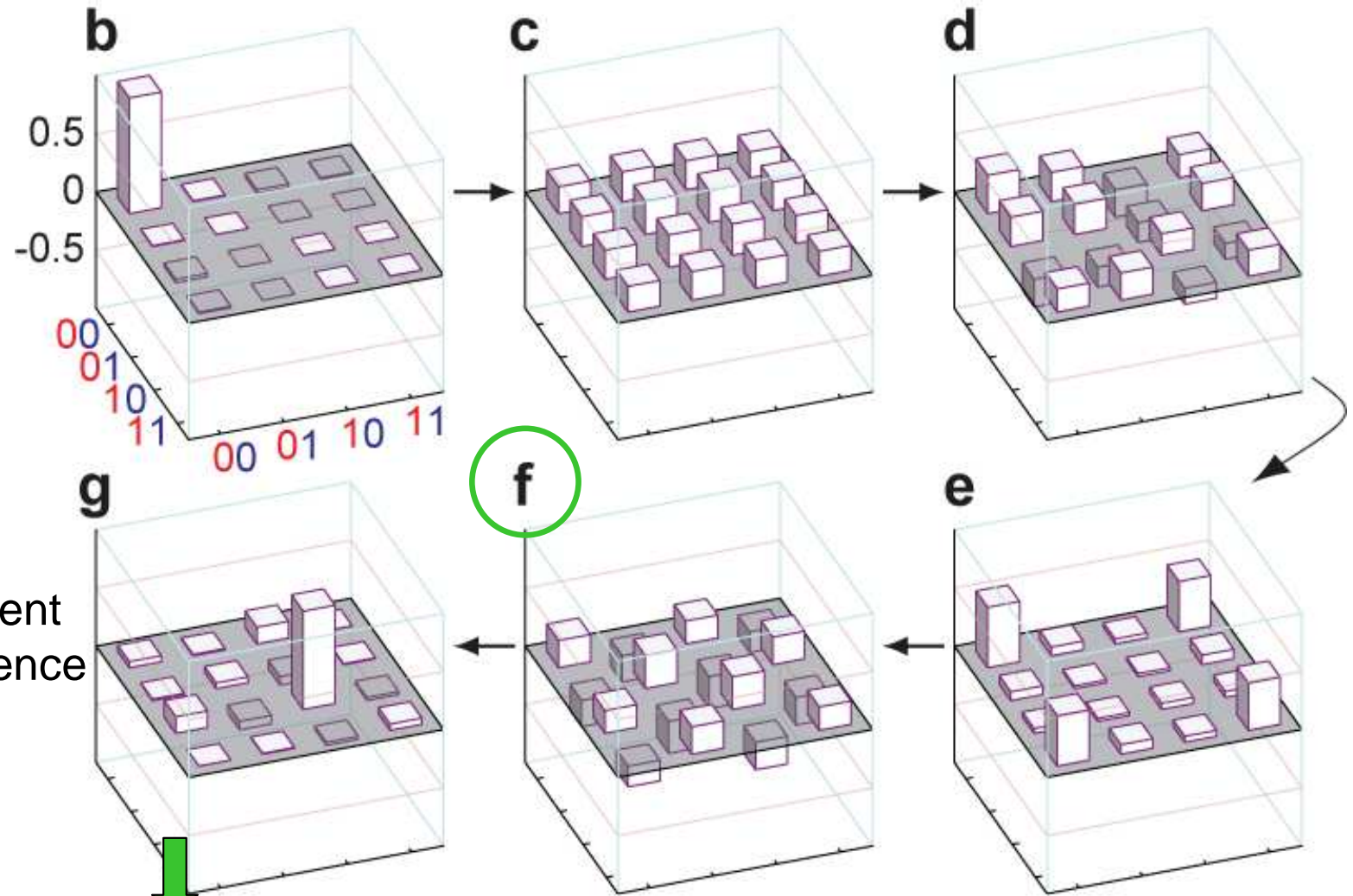
Some more 1-qubit rotations...

Now we arrive in one of the four Bell states

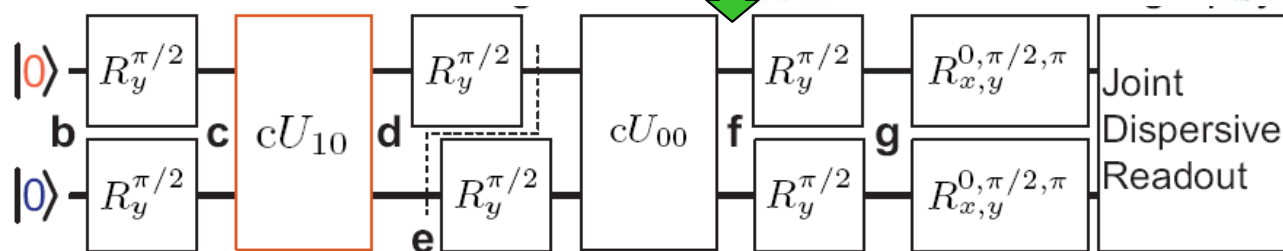


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

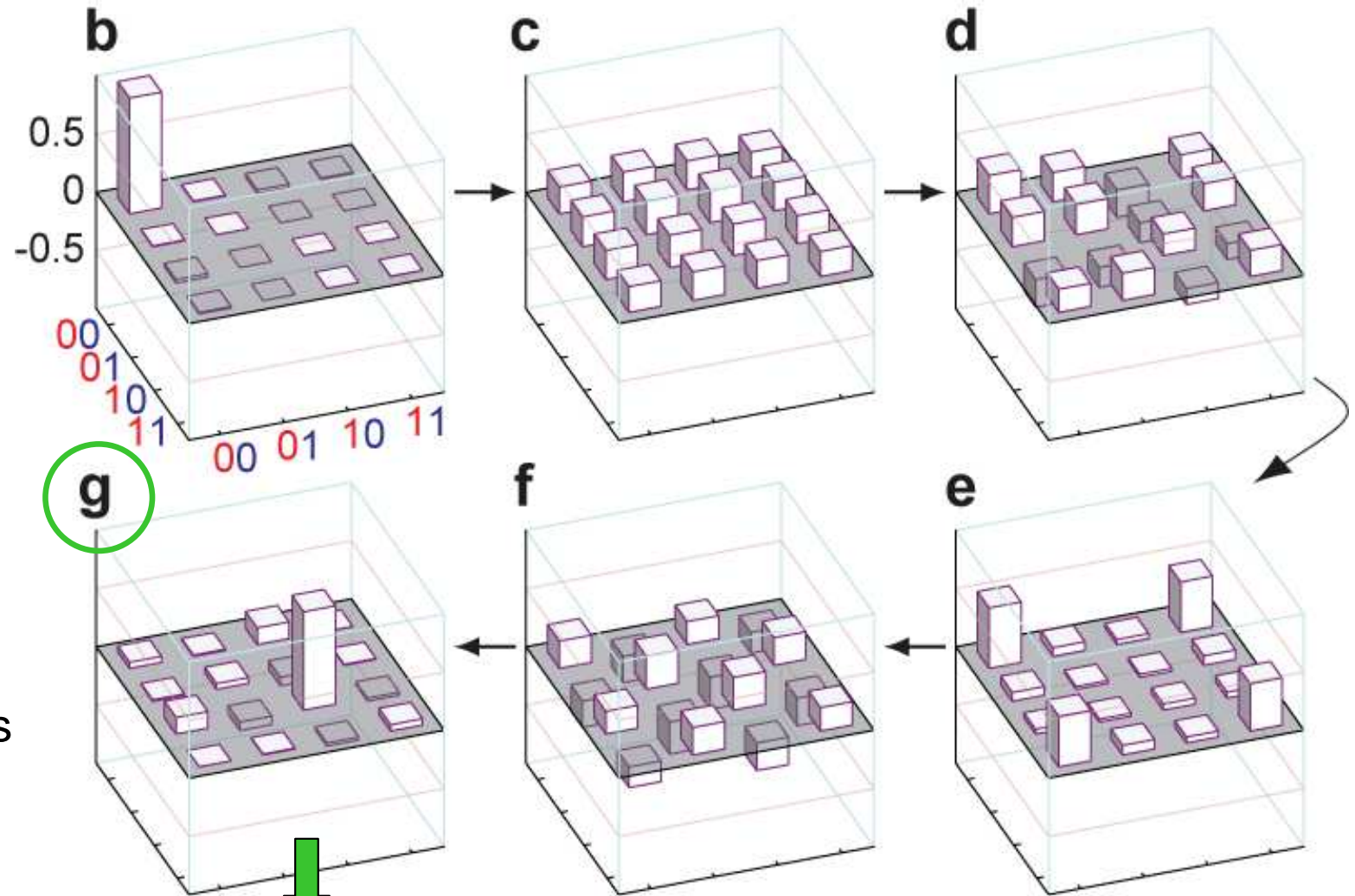


Another (but known) 2-qubit operation now undoes the entanglement and makes an interference pattern that holds the answer!



Grover Step-by-Step

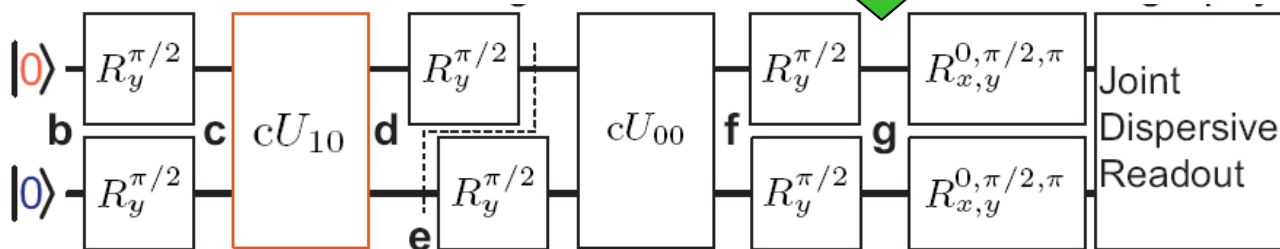
$$|\psi_{\text{ideal}}\rangle = |10\rangle$$



Final 1-qubit rotations reveal the answer:

The binary representation of “2”!

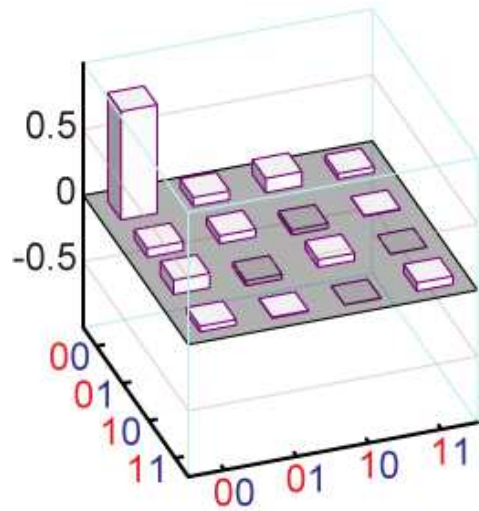
The correct answer is found **>80%** of the time!



Grover with Other Oracles

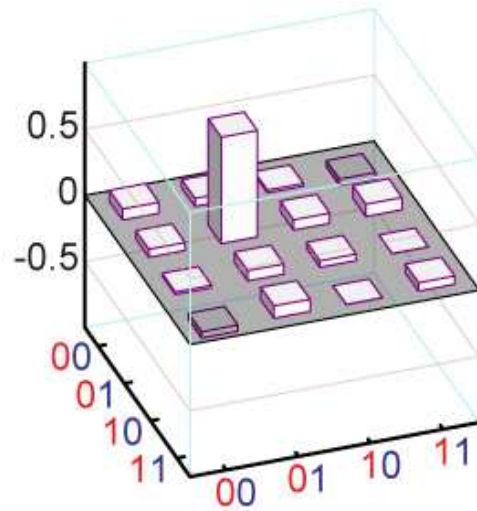
Oracle

$$\hat{O} = cU_{00}$$



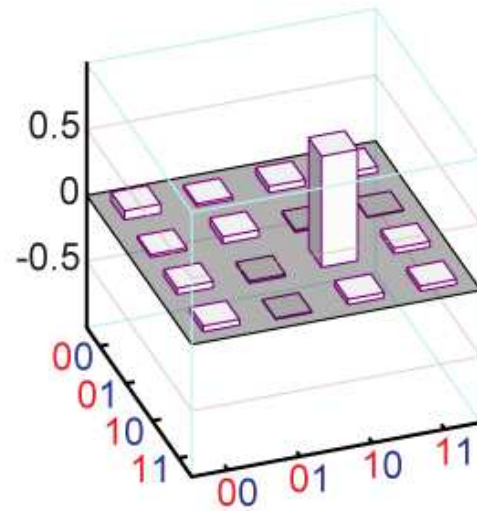
$$\overline{F} = 81\%$$

$$cU_{01}$$



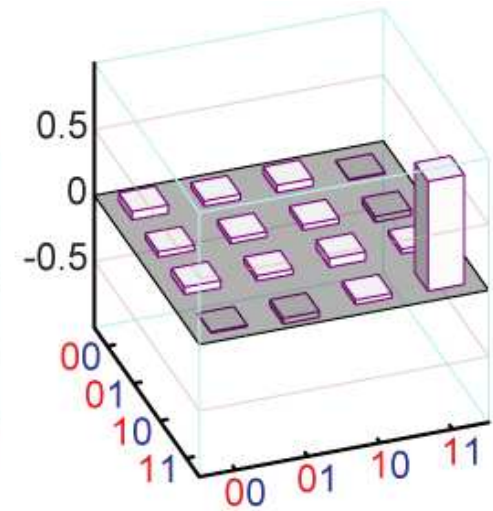
$$80\%$$

$$cU_{10}$$



$$82\%$$

$$cU_{11}$$

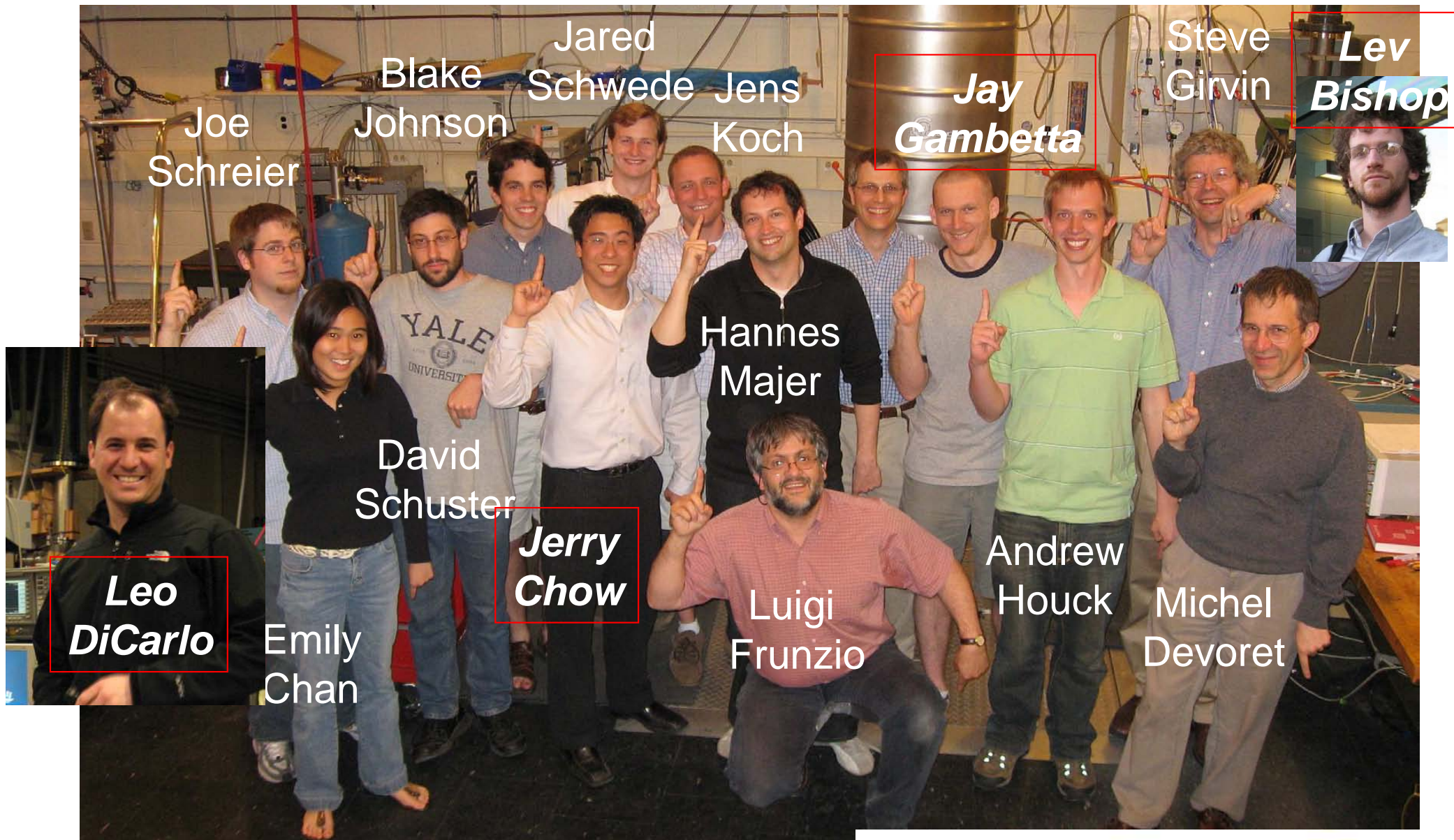


$$81\%$$

Fidelity $F = \langle \psi_{\text{ideal}} | \rho | \psi_{\text{ideal}} \rangle$ to ideal output

(average over 10 repetitions)

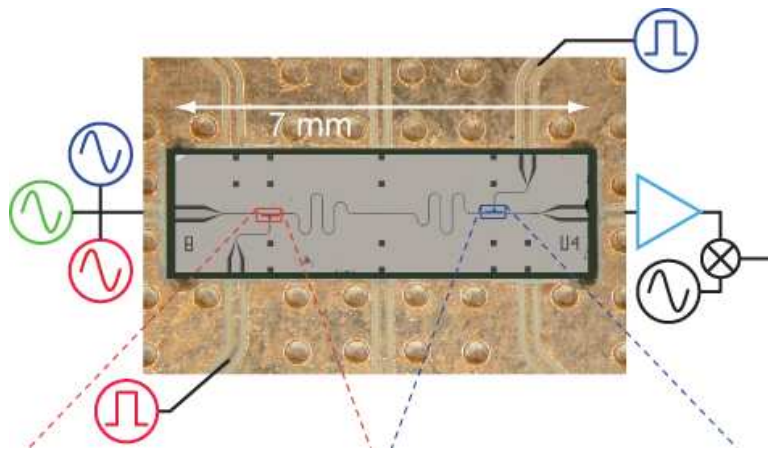
Circuit QED Team Members



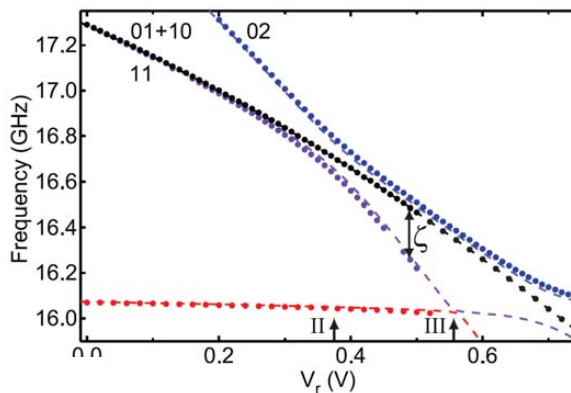
Funding:



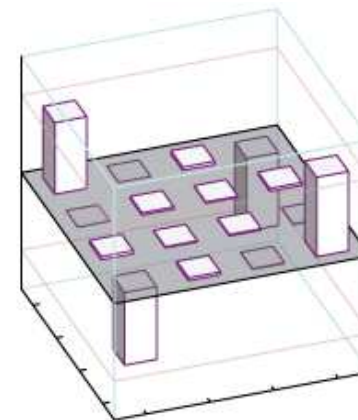
Summary



Rudimentary two-qubit processor



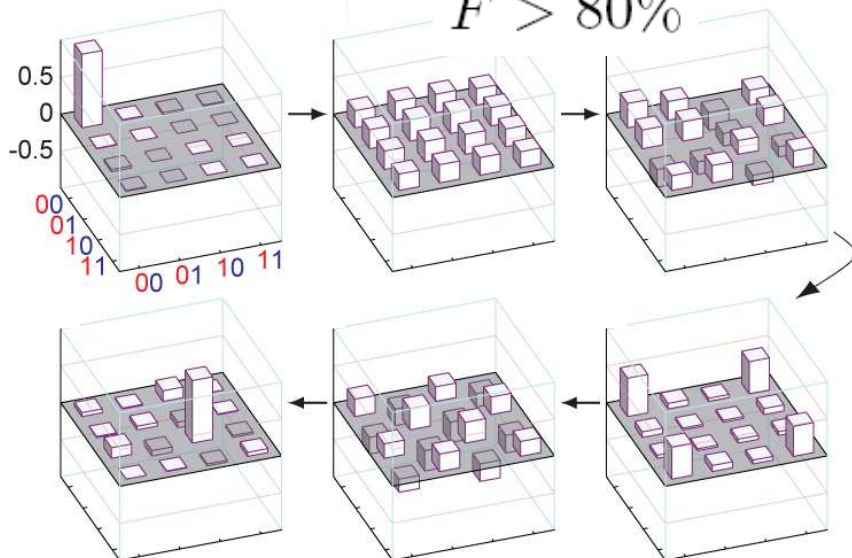
Adiabatic C-phase gate



Entanglement on demand

- Grover algorithm with Fidelity

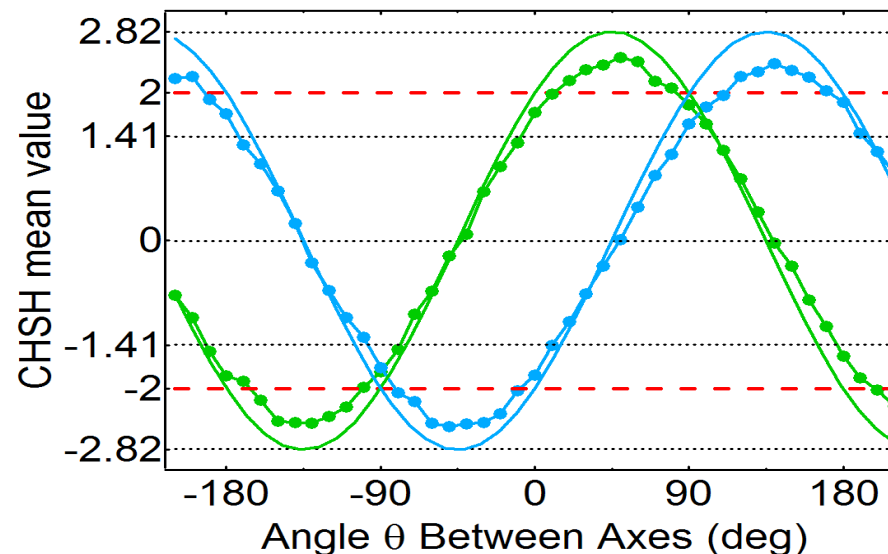
$$F > 80\%$$



$$F = 87-94\%$$

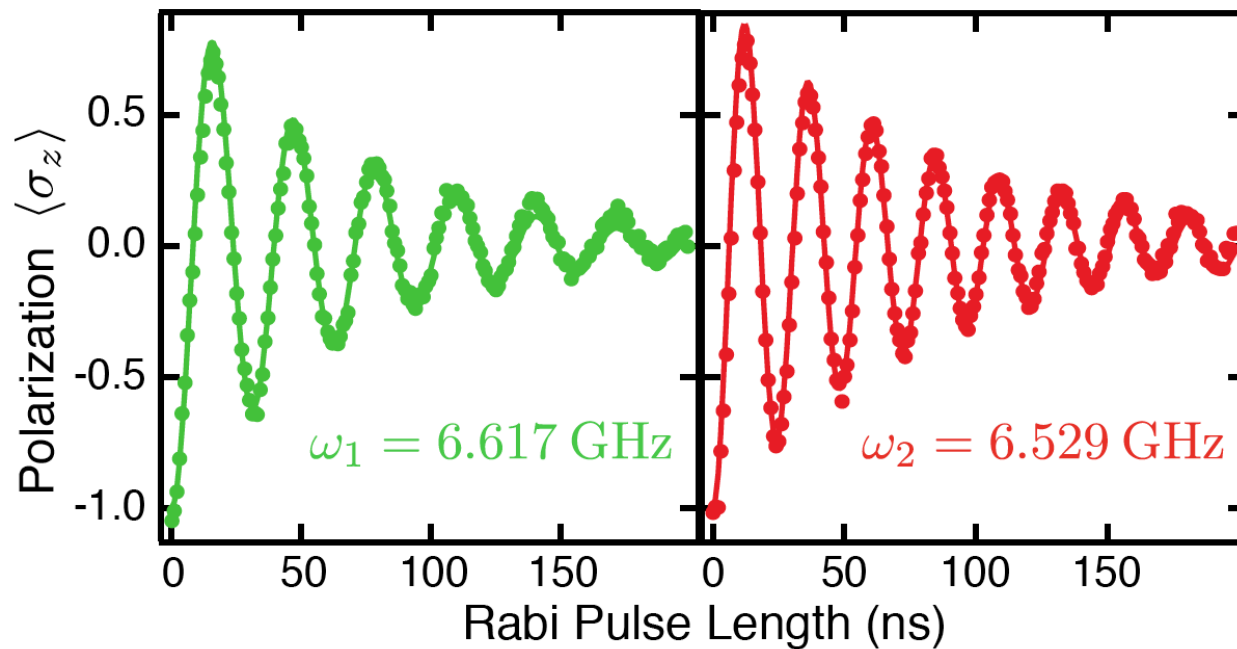
$$C = 81-94\%$$

CHSH as entanglement witness

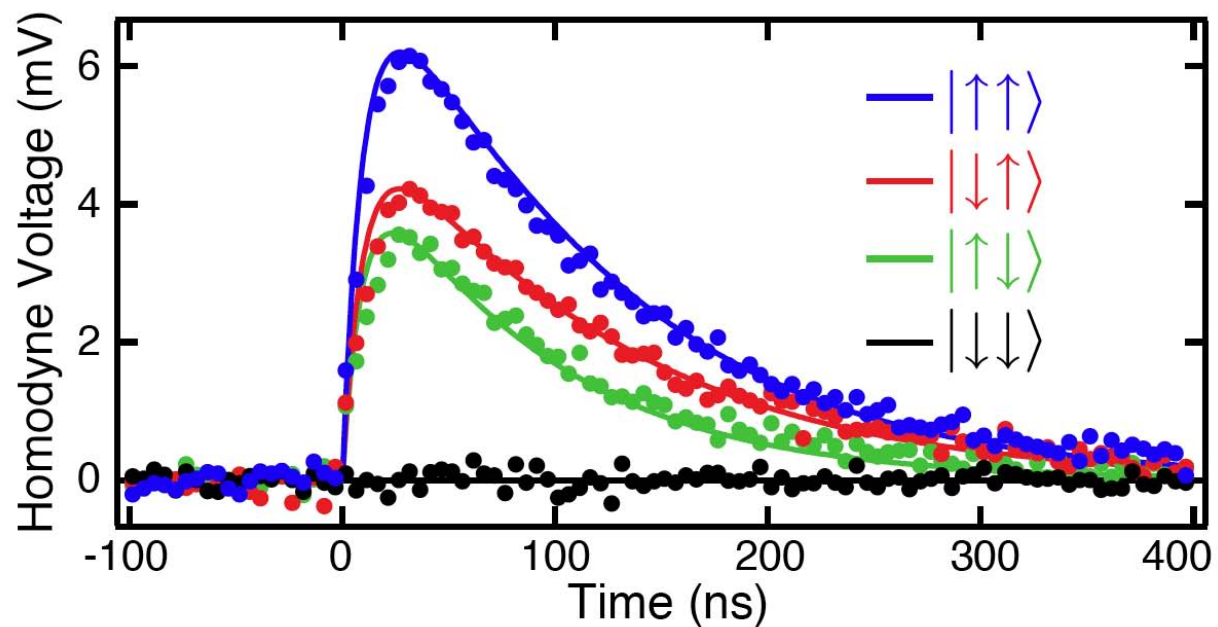
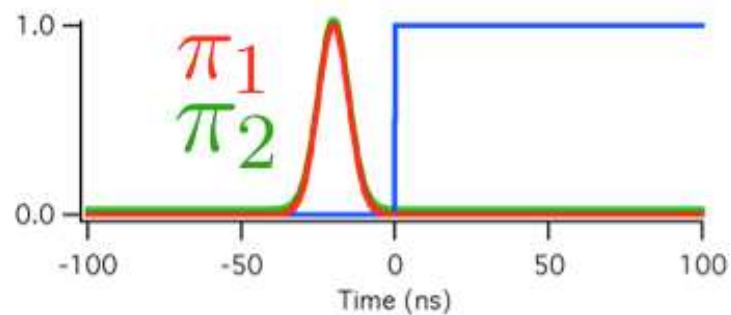


Additional Slides Follow

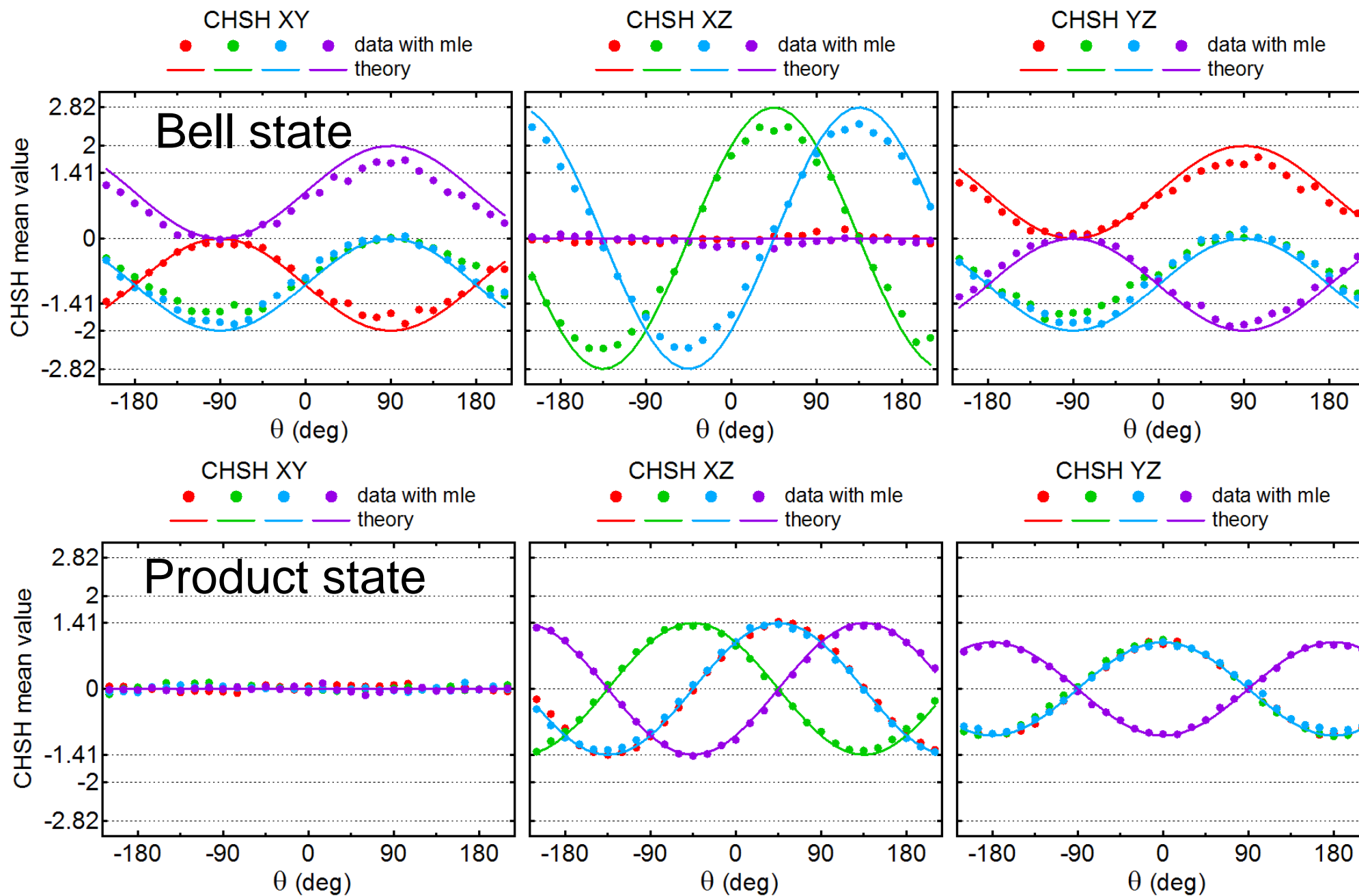
Multiplexed Qubit Control and Read-Out



Single
Oscillation

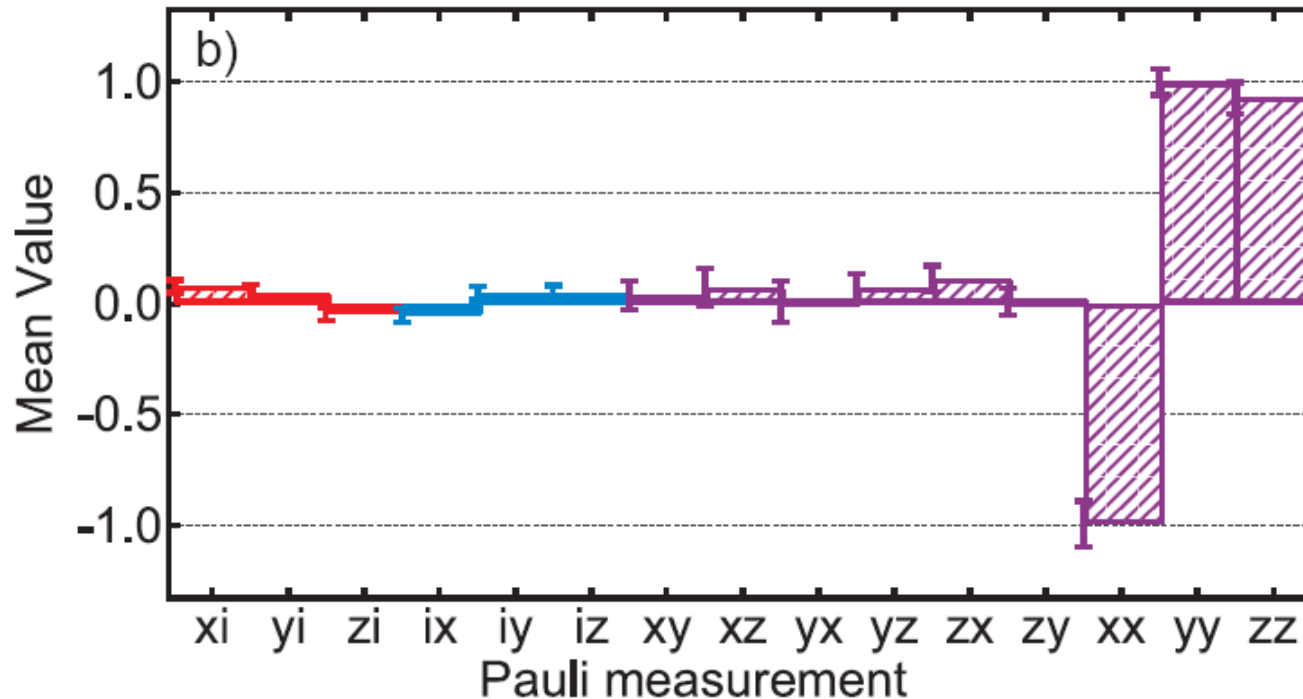


Witnessing Entanglement



Measuring the Two-Qubit State

Now apply a two-qubit gate to entangle the qubits



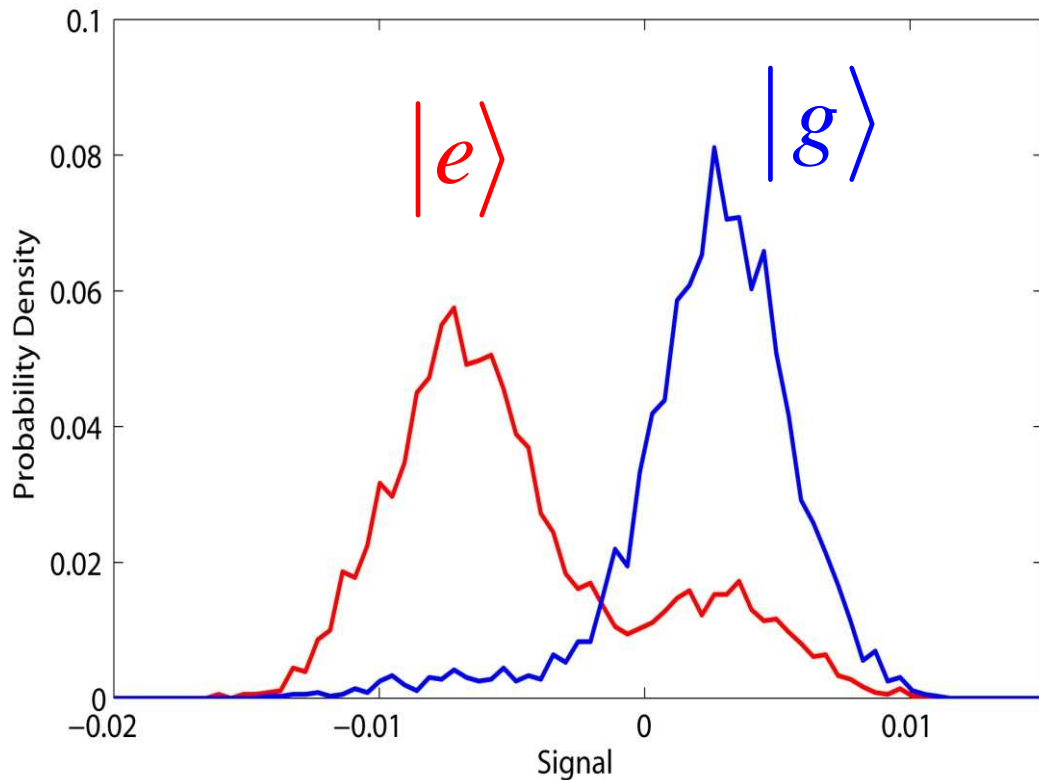
Concurrence **directly**:
(for pure states)

$$C = \sqrt{\frac{Q-1}{2}}$$

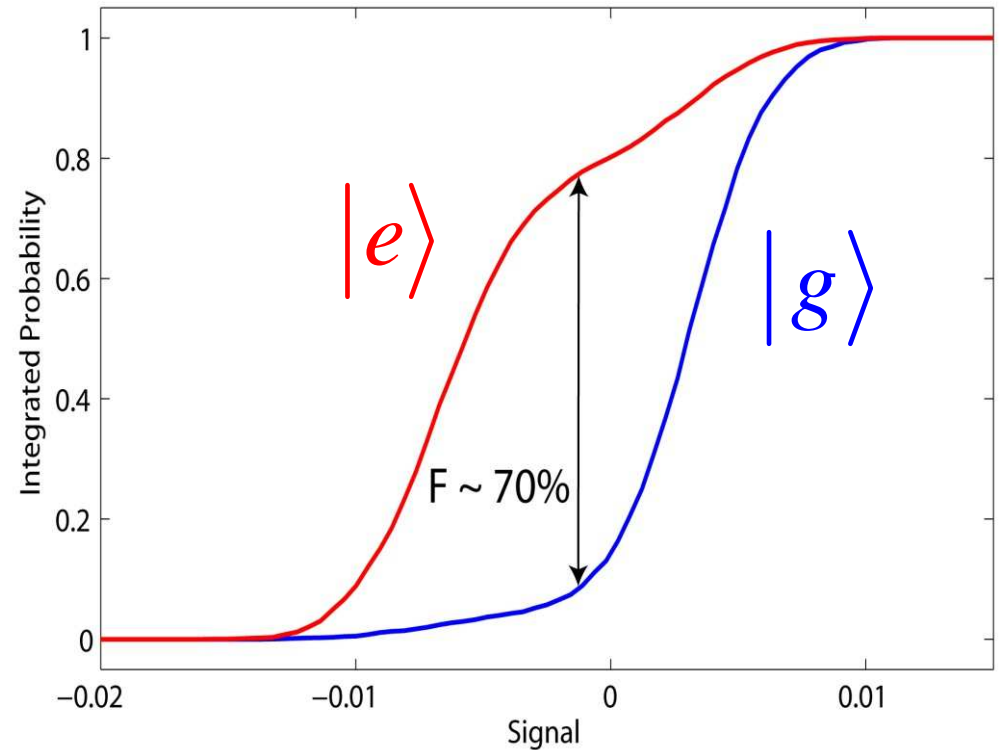
$$Q = \langle XX \rangle^2 + \langle XY \rangle^2 + \langle XZ \rangle^2 + \langle YX \rangle^2 + \langle YY \rangle^2 + \dots + \langle ZZ \rangle^2$$

Single shot readout fidelity

Histograms of single shot msmts.



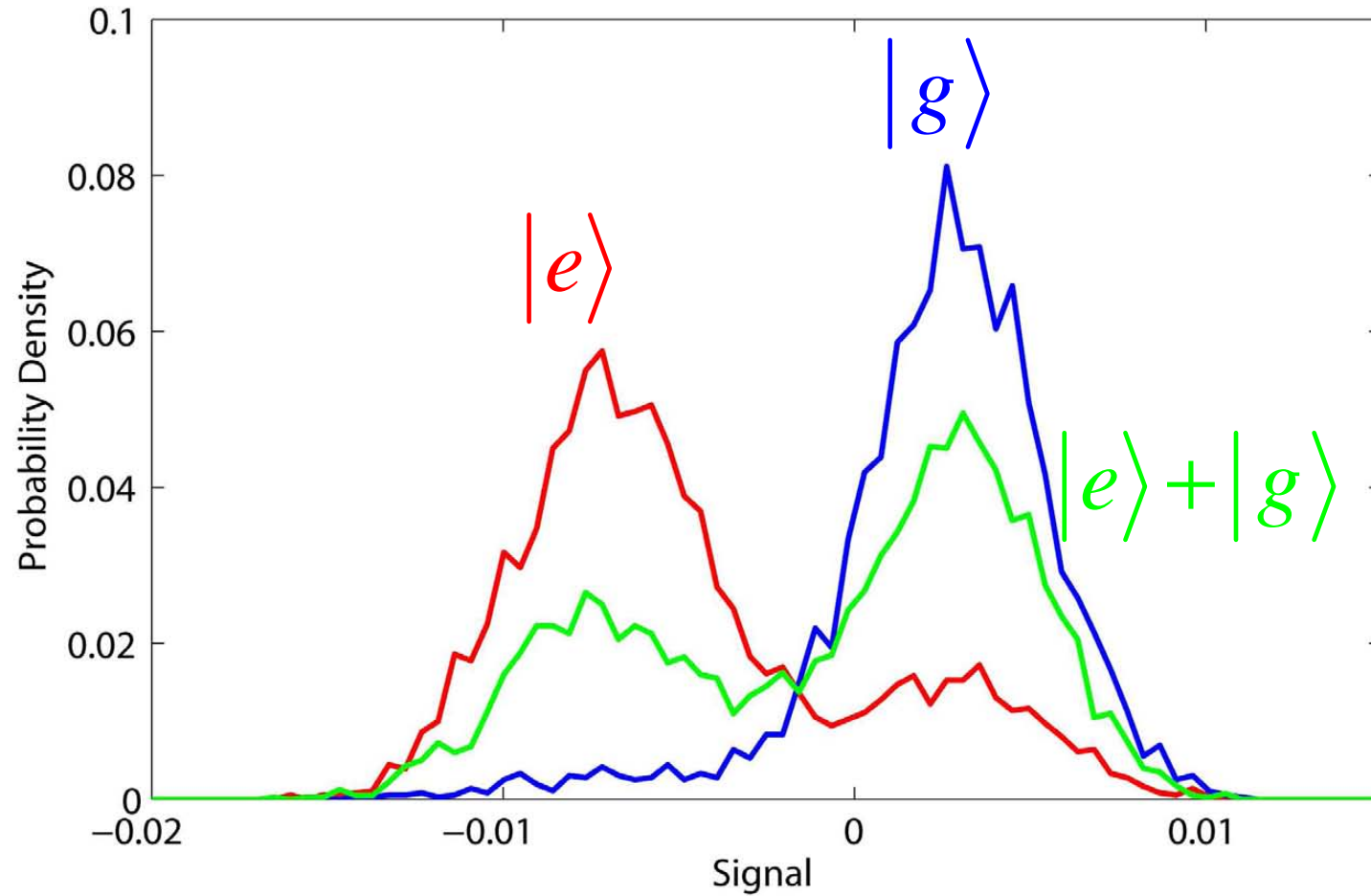
Integrated probabilities



Measurement with ~ 5 photons in cavity;
SNR ~ 4 in one qubit lifetime (T_1)

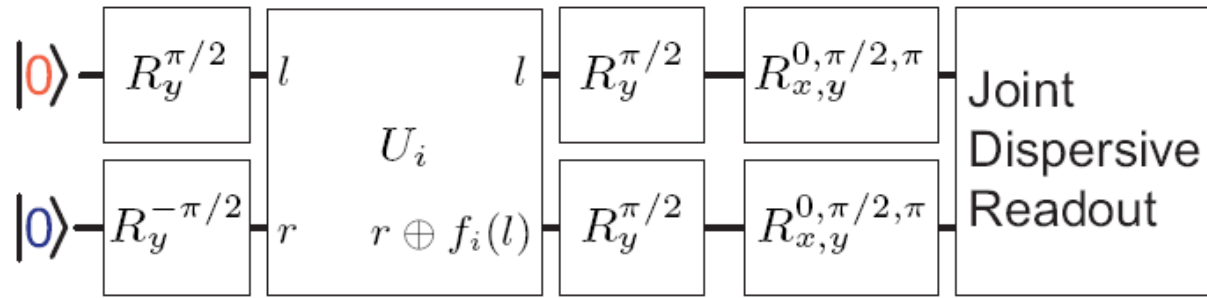
$T_1 \sim 300$ ns, low Q cavity on sapphire

Projective measurement

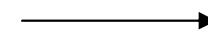
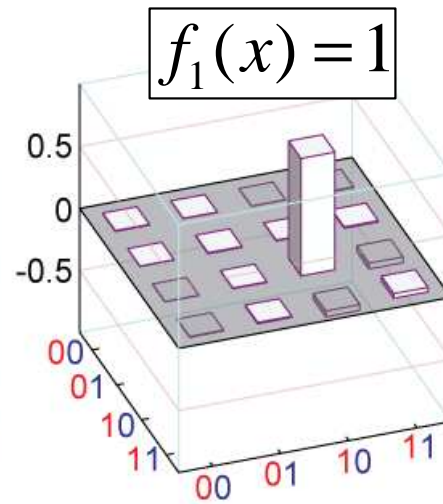
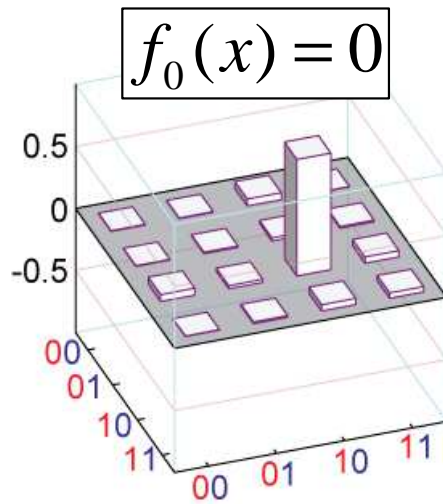
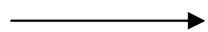


- Measurement after $\pi/2$ pulse bimodal, halfway between

Deutsch-Jozsa Algorithm



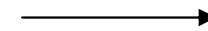
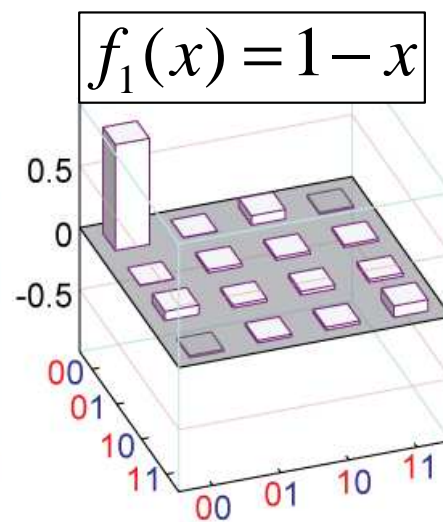
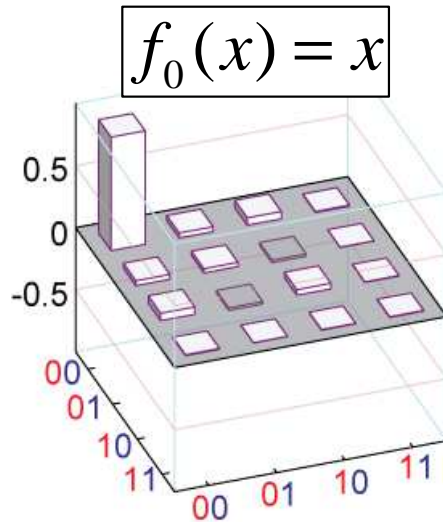
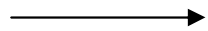
Constant functions



$|10\rangle$

Answer is encoded in the state of left qubit

Balanced functions



$|00\rangle$

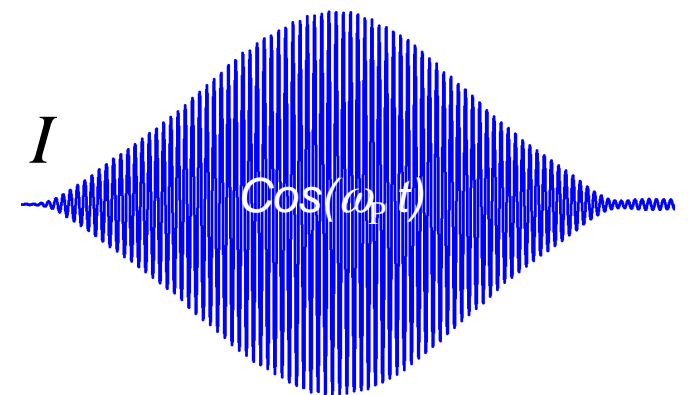
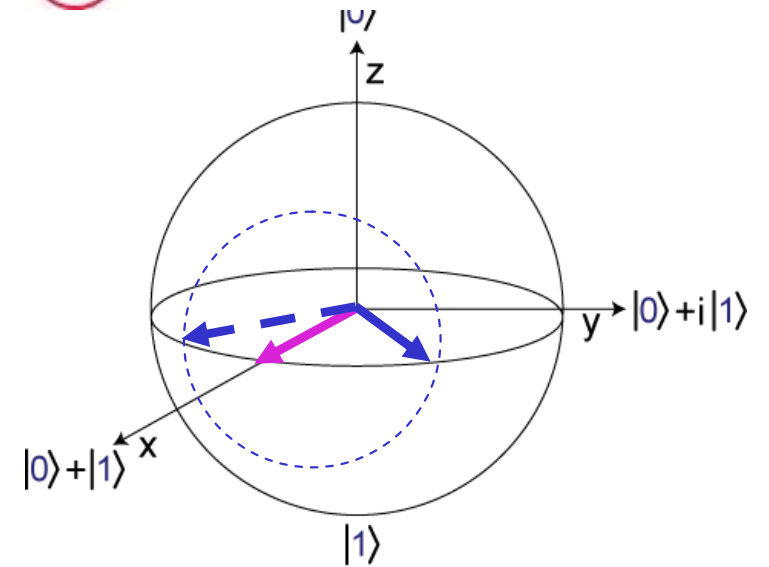
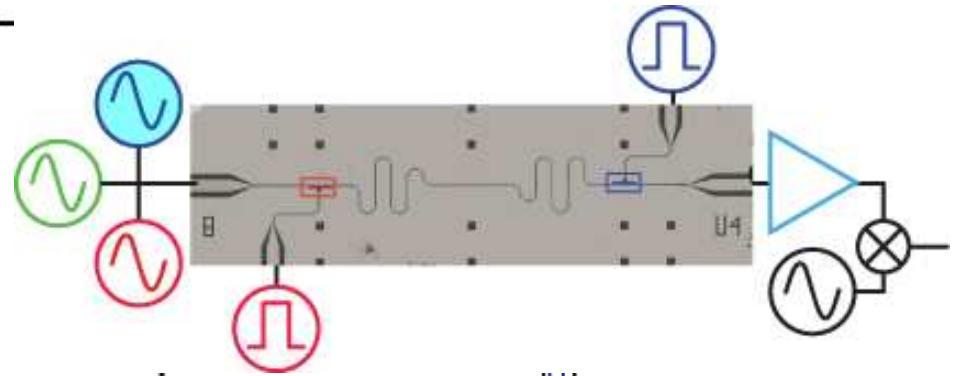
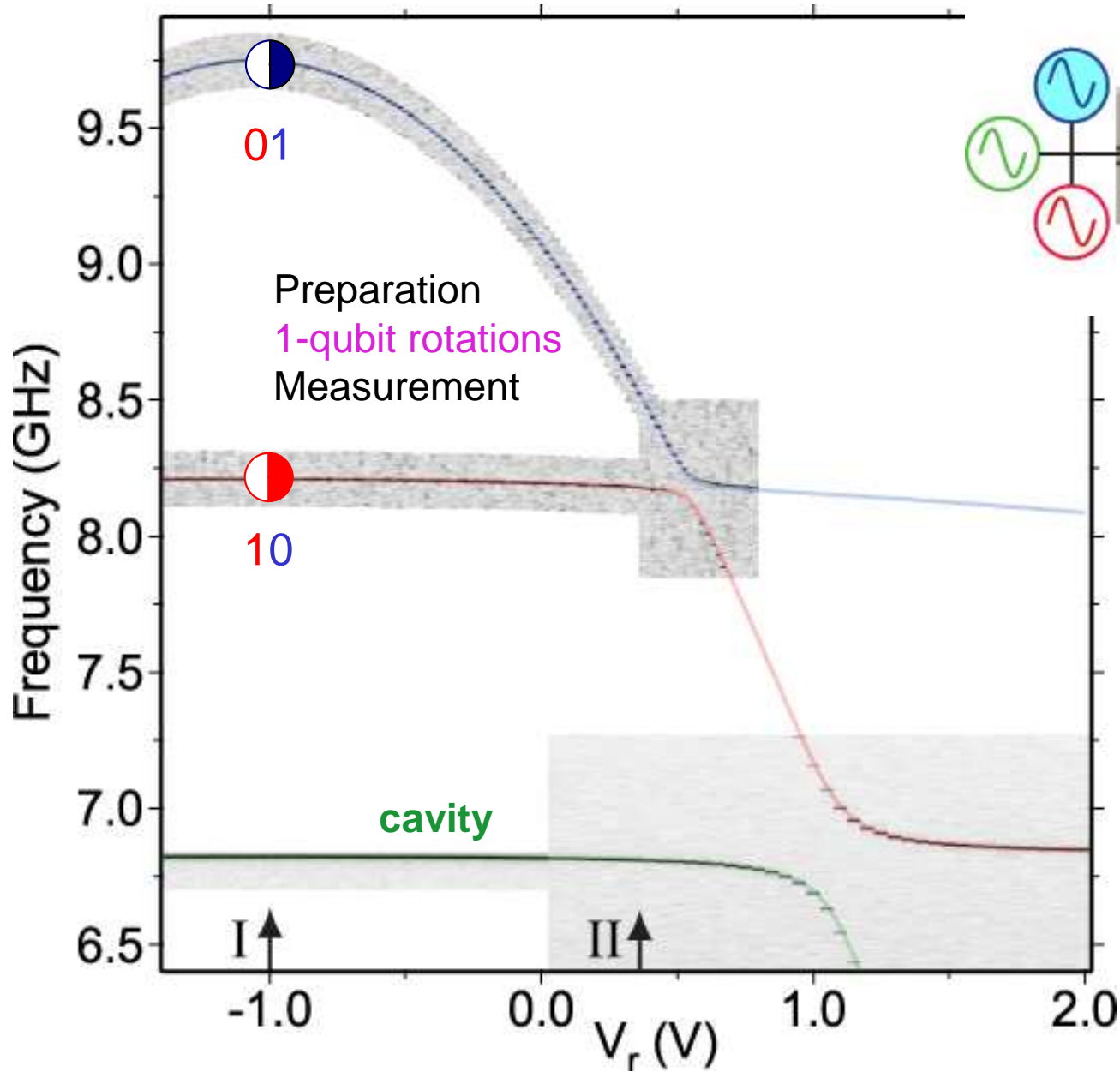
The correct answer is found **>84%** of the time.



The cost of entanglement

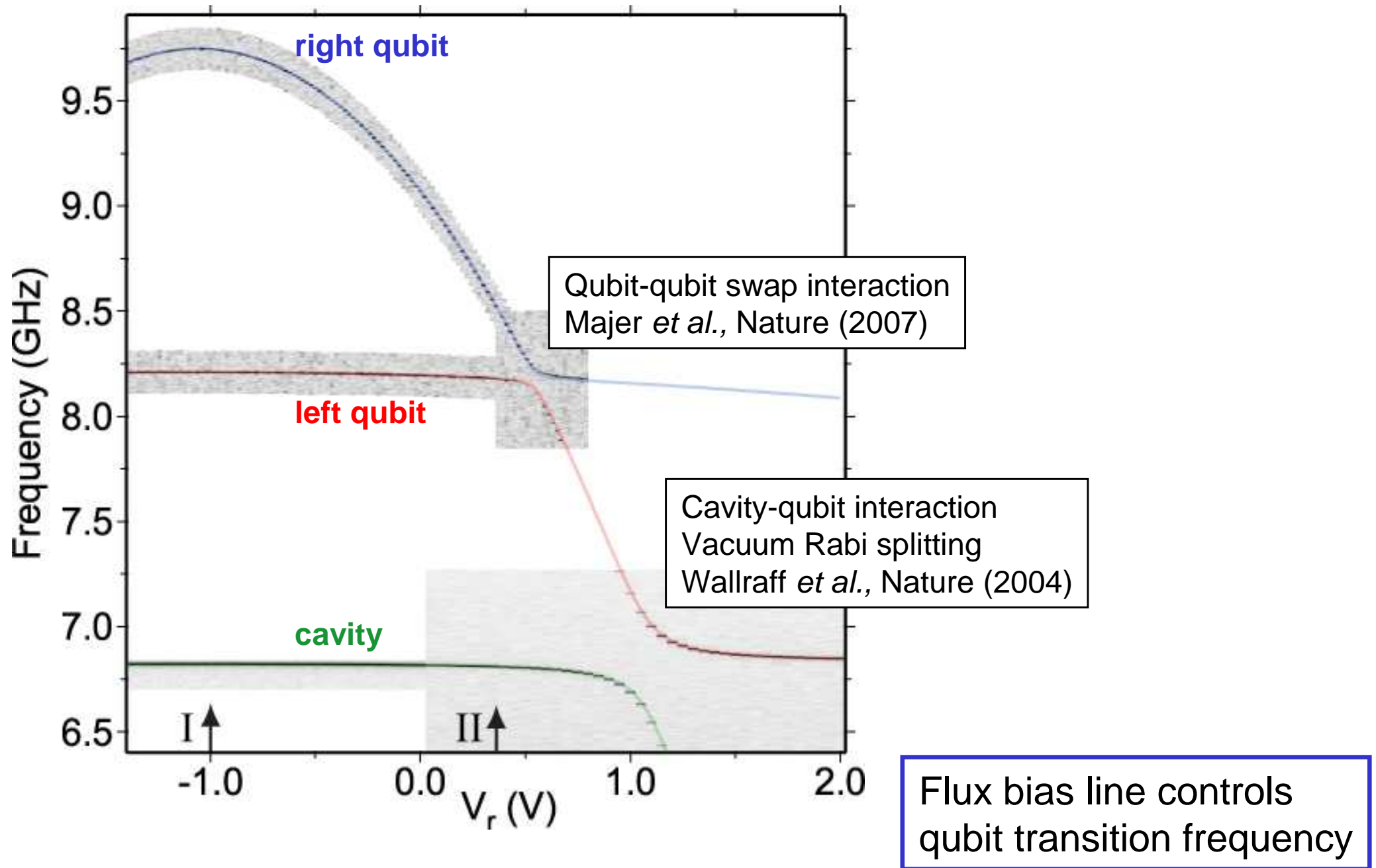
- 1 Cryogenic HEMT amp
- 2 Room Temp Amps
- 1 Two-channel digitizer
- 1 Two-channel AWG
- 1 Four-channel AWG
- 2 Scalar signal generators
- 2 Vector signal generators
- 1 Low-frequency generator
- 1 Rubidium frequency standard
- 2 Yokogawa DC sources
- 1 DC power supply
- 1 Amp biasing servo
- 1 Computer
- 10^3 Coffee pods

One-Qubit Gates

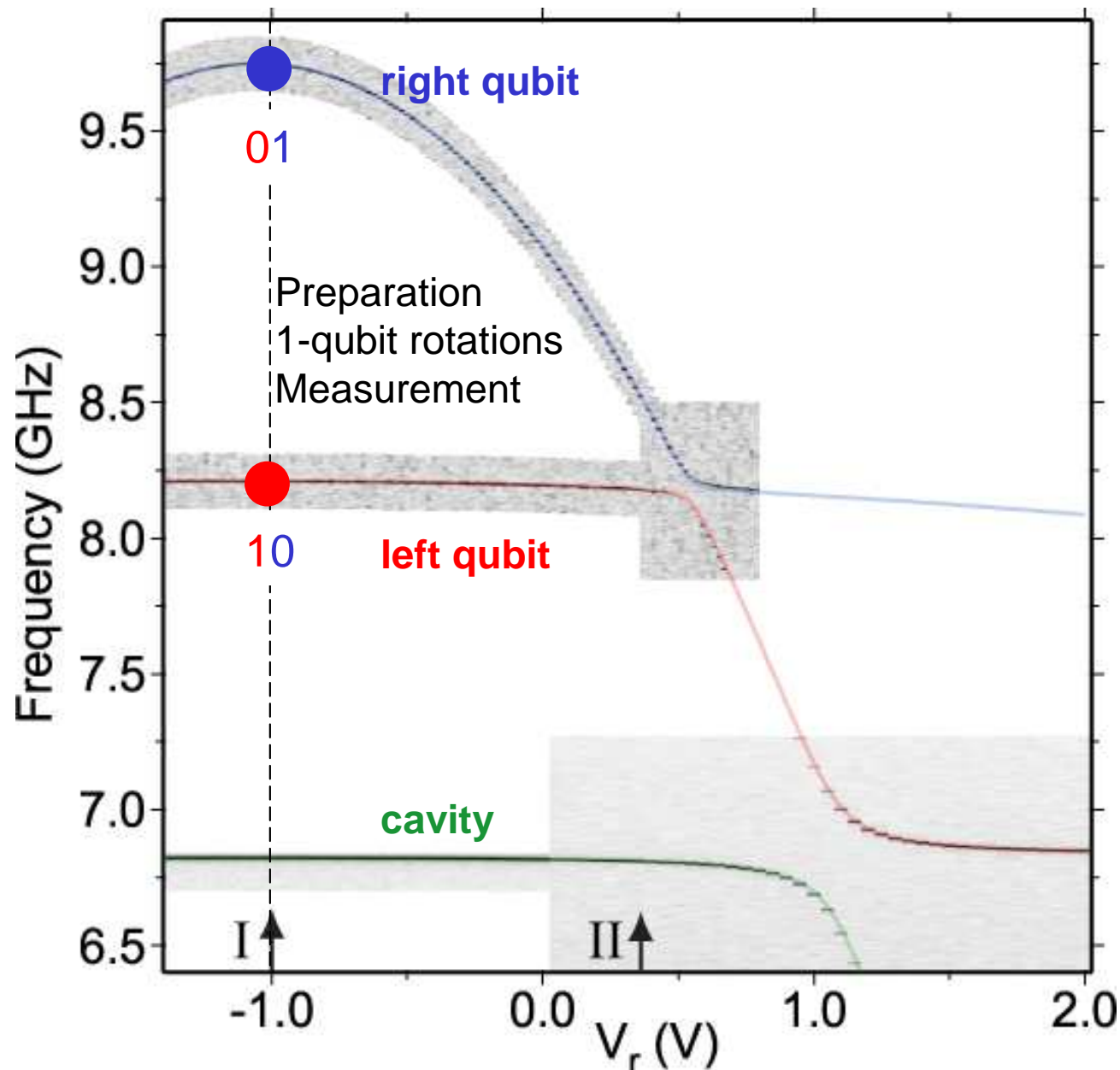


Apply microwave pulse resonant with qubit

Spectroscopy of Qubits Interacting with Cavity



Spectroscopy of Qubits Interacting with Cavity



Qubits mostly separated
and non-interacting
due to frequency difference

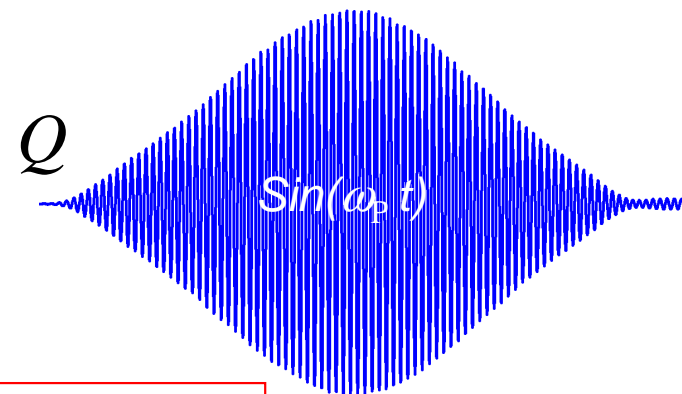
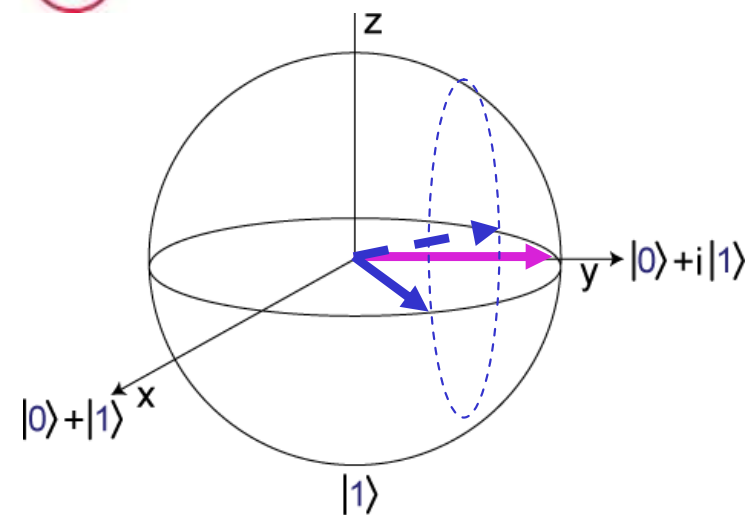
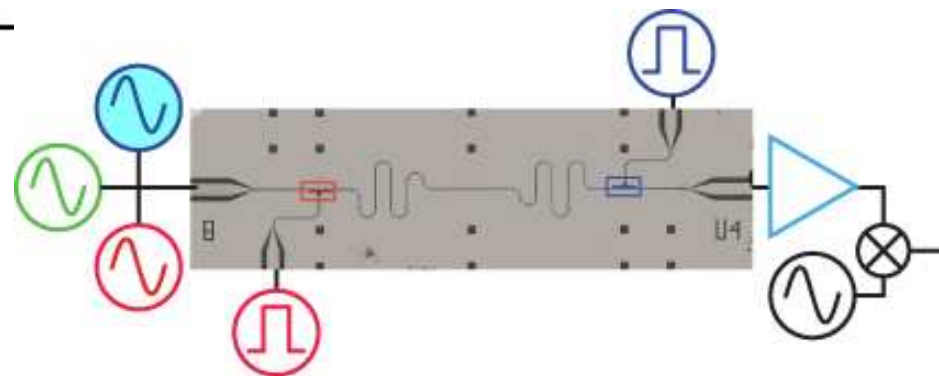
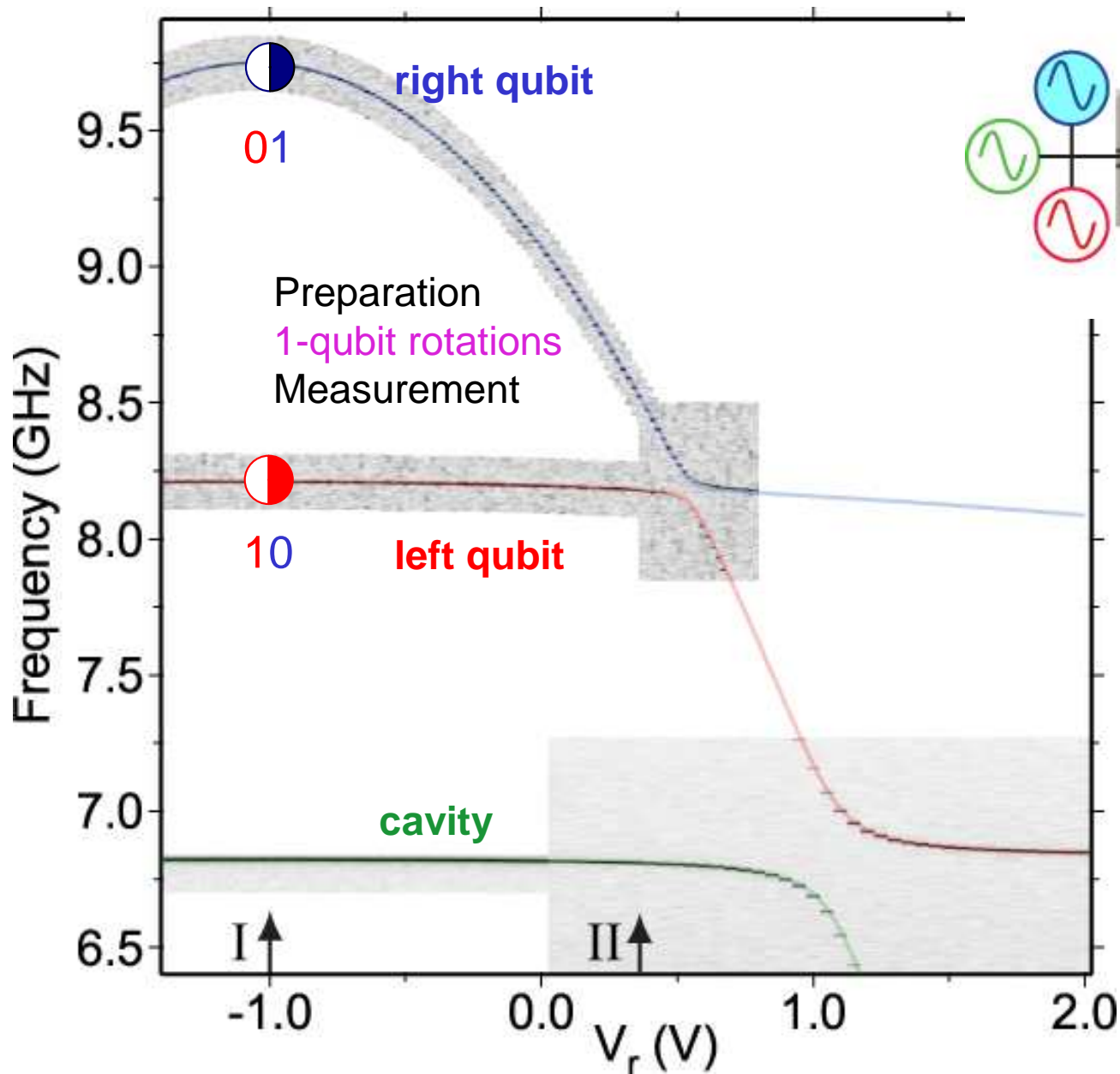
$$T_{1,r} = 0.79 \mu\text{s}$$

$$T_{2,r}^* = 1.15 \mu\text{s}$$

$$T_{1,l} = 1.3 \mu\text{s}$$

$$T_{2,l}^* = 1.8 \mu\text{s}$$

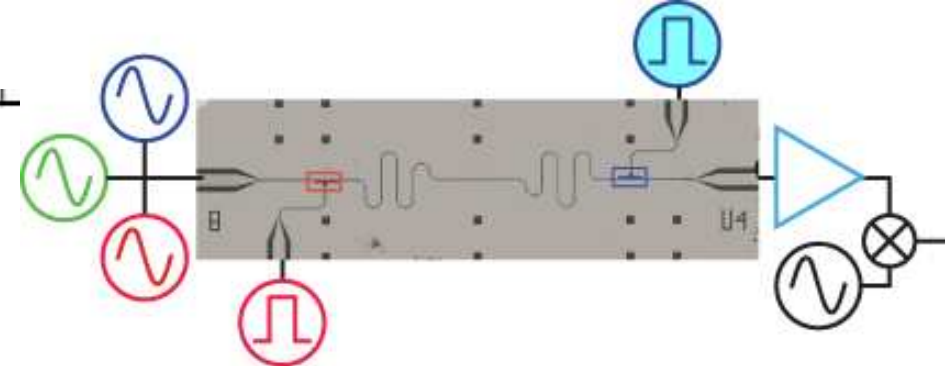
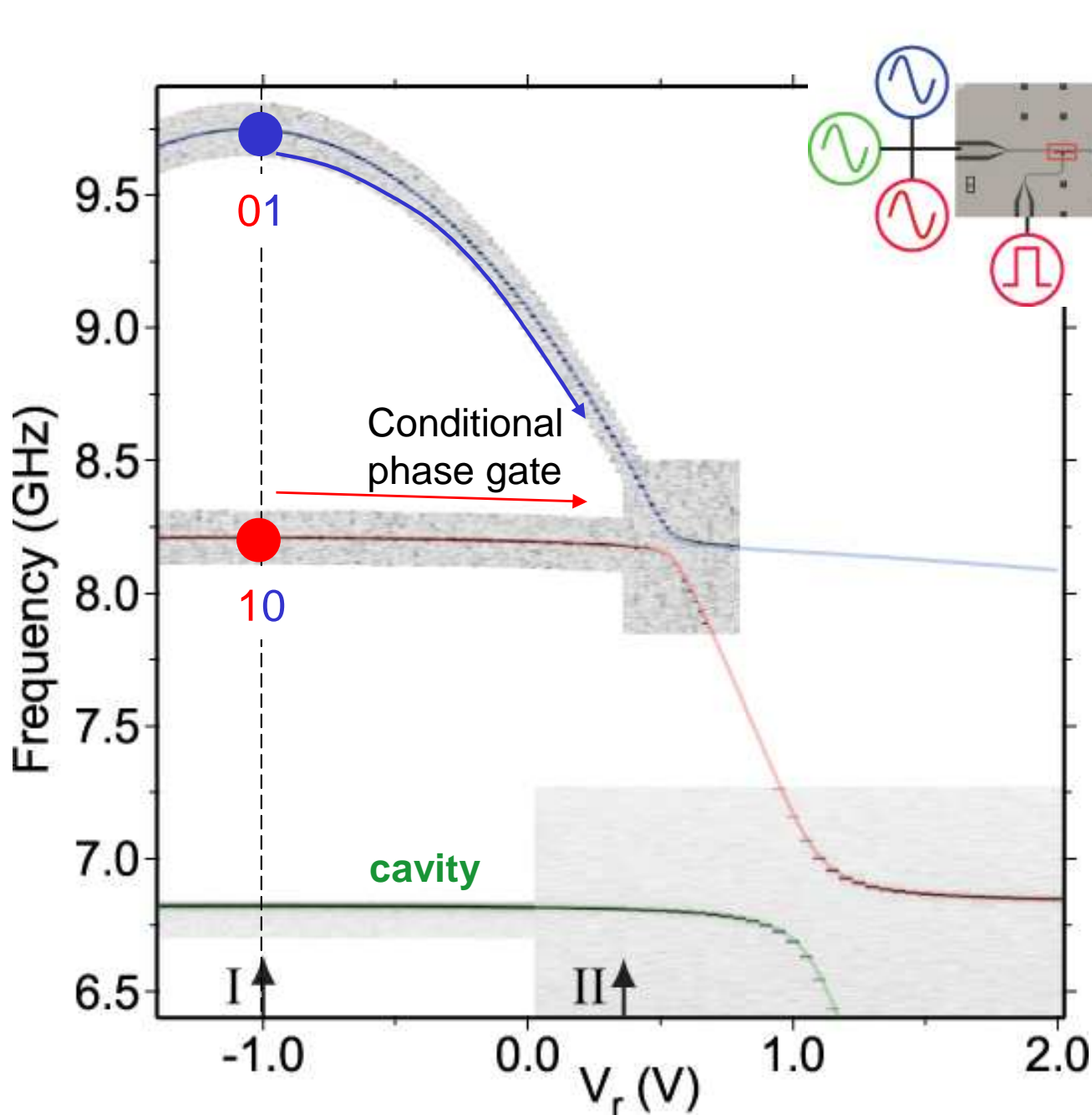
One-Qubit Gates



J. Chow *et al.*, PRL (2009):

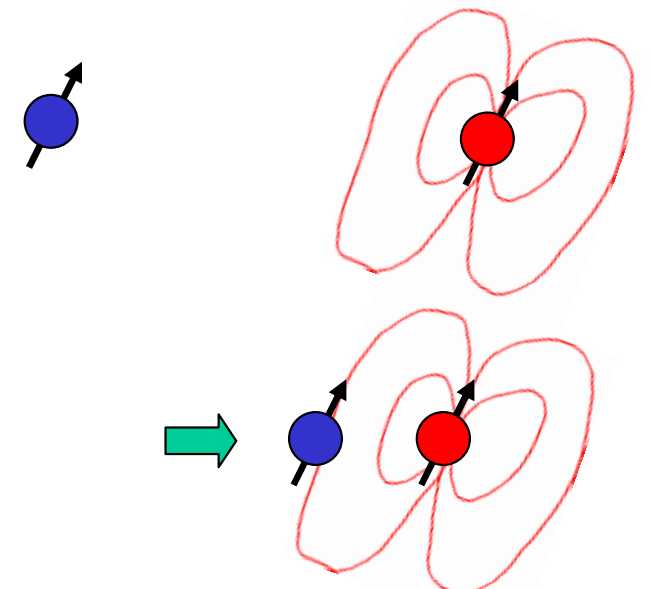
Fidelity = 99%

Two-Qubit Gate: Turn On Interactions

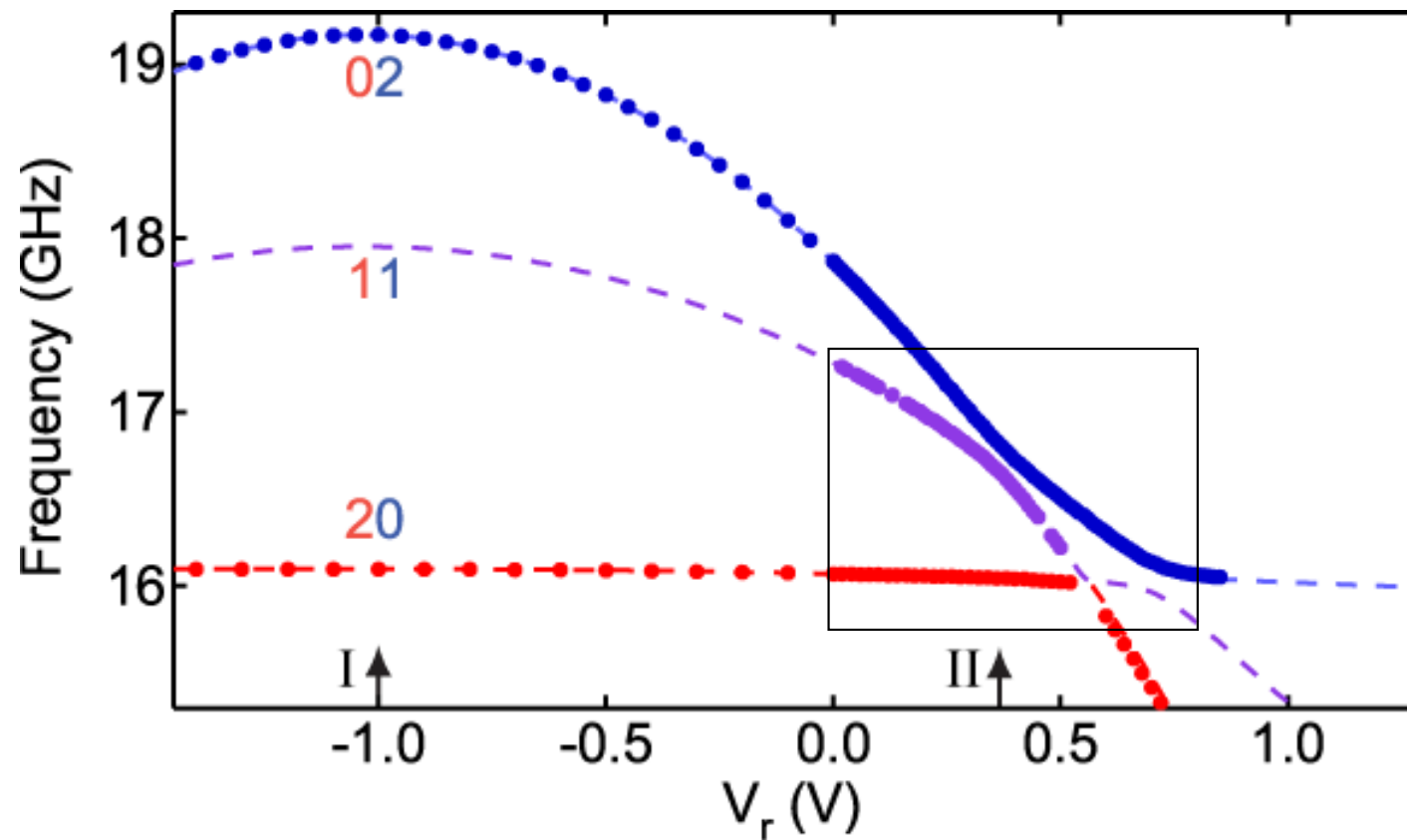


Use control lines to push qubits near a resonance:

→ A controlled z-z interaction also ala NMR

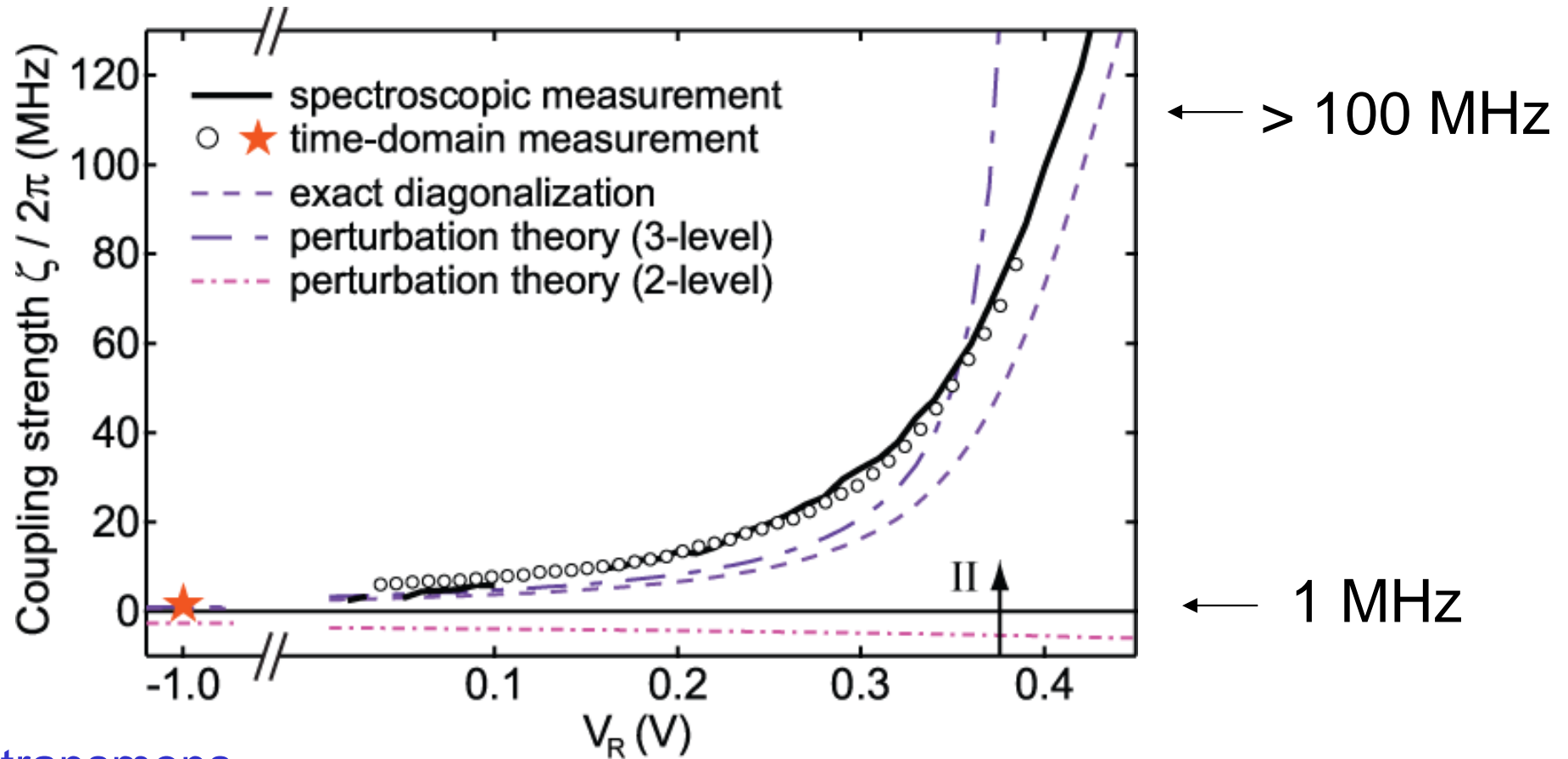


Two-Excitation Manifold of System



“Qubits” and cavity both have multiple levels...

On/Off Ratio for Two-Qubit Coupling



3-level transmons

$$\zeta = -2g_L^2 g_R^2 \left(\frac{1}{(\omega_{01}^L - \omega_C)(\omega_{01}^R - \omega_C)^2} + \frac{1}{(\omega_{01}^R - \omega_C)(\omega_{01}^L - \omega_C)^2} + \frac{1}{(\omega_{01}^R - \omega_{12}^L)(\omega_{01}^L - \omega_C)^2} + \frac{1}{(\omega_{01}^L - \omega_{12}^R)(\omega_{01}^R - \omega_C)^2} \right)$$

↑
Diverges at Point II

4th-order in qubit-cavity coupling!

State Tomography

$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$

Combine joint readout with one-qubit “analysis” rotations

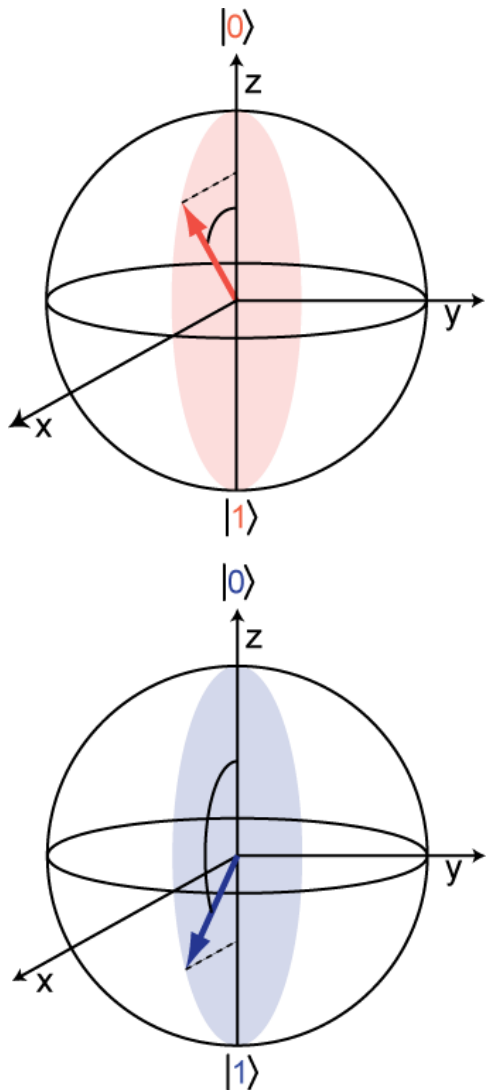
$$\langle \sigma_z^L \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^R) \leftarrow \pi\text{-pulse on right}$$

$$\langle \sigma_z^R \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^L) \leftarrow \pi\text{-pulse on left}$$

$$\langle \sigma_z^L \sigma_z^R \rangle \sim V_H(\text{Ident.}) + V_H(Y_\pi^R, Y_\pi^L) \leftarrow \pi \text{ on both}$$

Possible to acquire correlation info.,
even with single, ensemble averaged msmt.!

See similar from Zurich group: Phillip et al., PRL **102**, 200402 (2009).



Measuring the Two-Qubit State

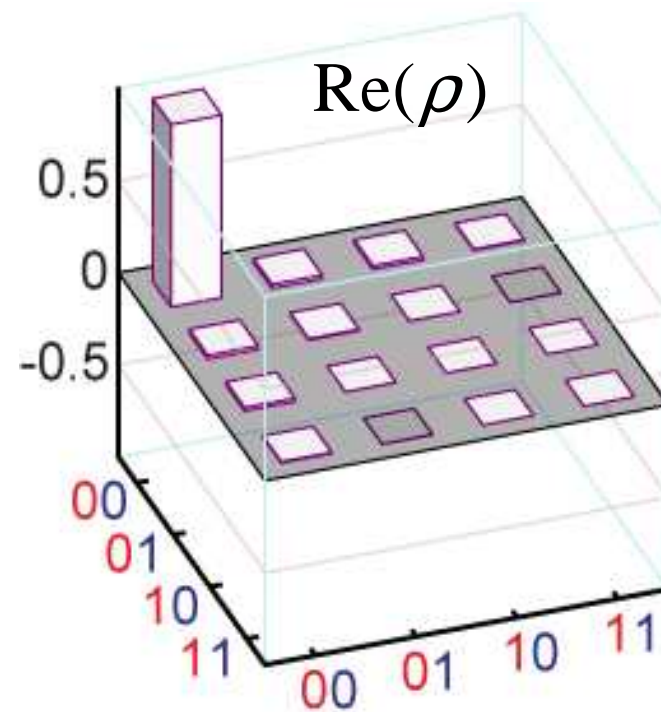
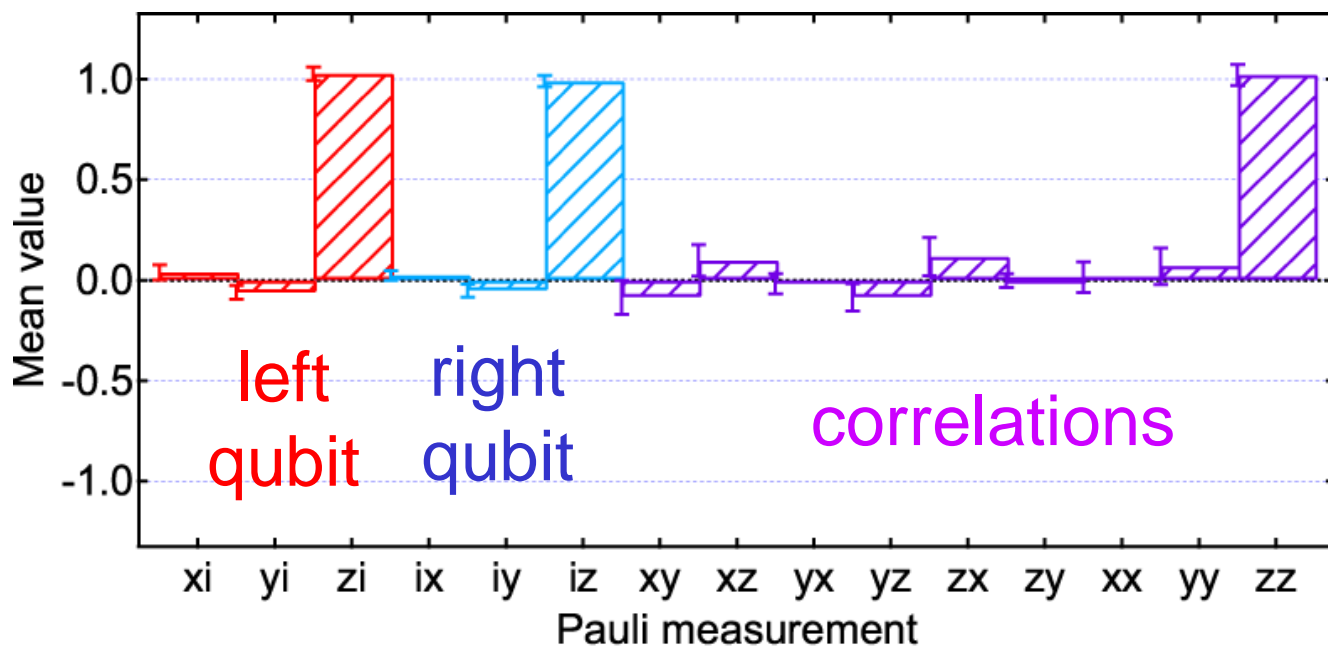
Total of 16 msmts.: $I, Y_{\pi}^L, X_{\pi/2}^L, Y_{\pi/2}^L$

and combinations

$I, Y_{\pi}^R, X_{\pi/2}^R, Y_{\pi/2}^R$

max. likelihood
(nonlinear!)

(almost) raw data

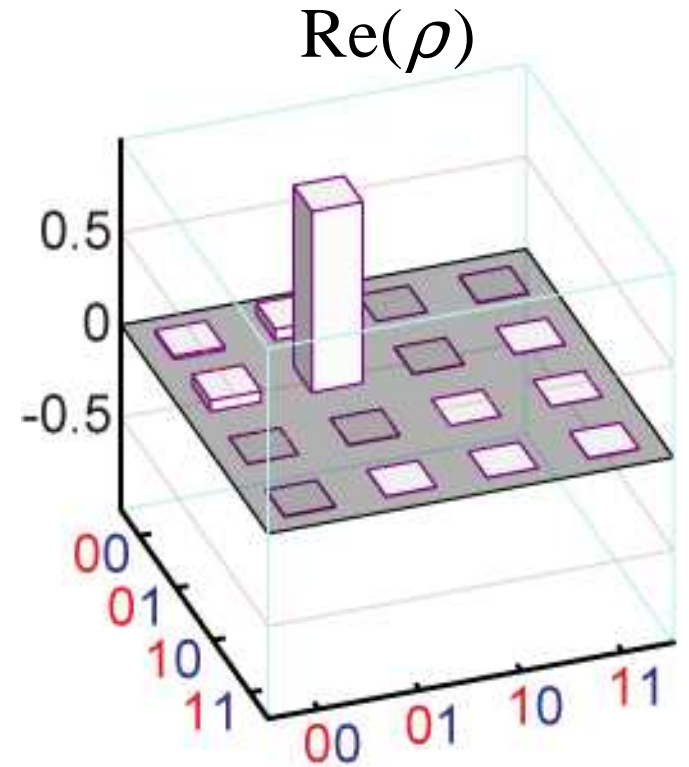
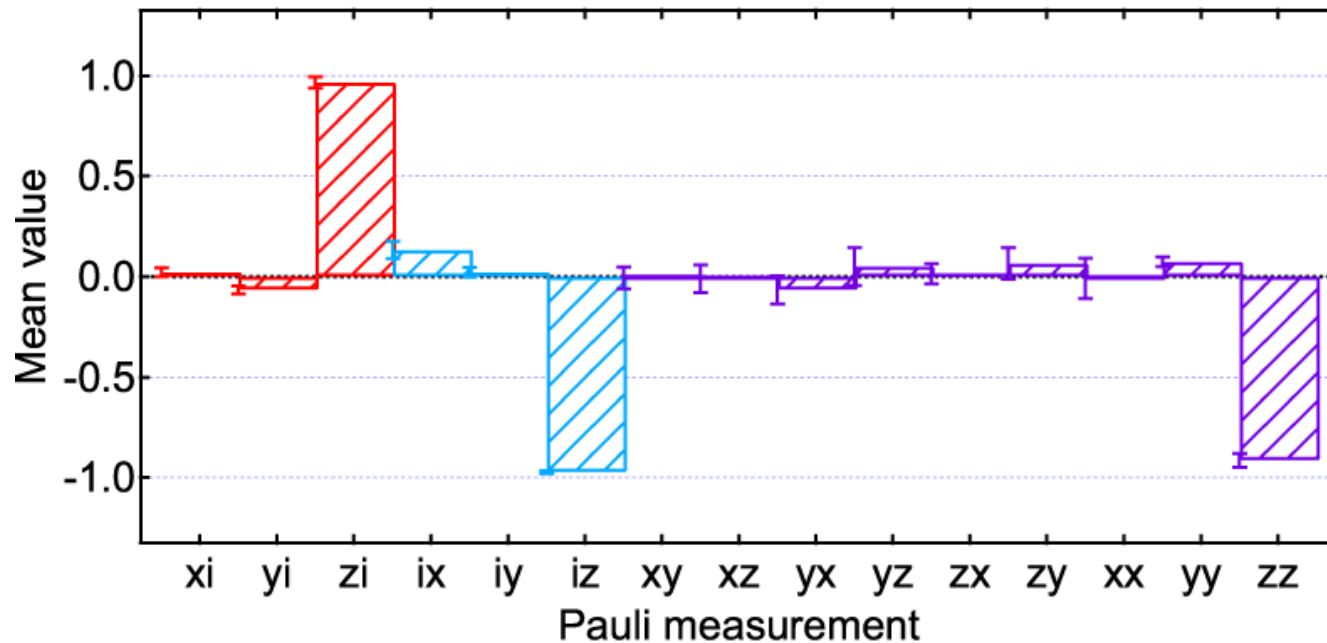


Ground state: $|\psi\rangle = |00\rangle$

Density matrix

Measuring the Two-Qubit State

Apply π -pulse to invert state of **right** qubit



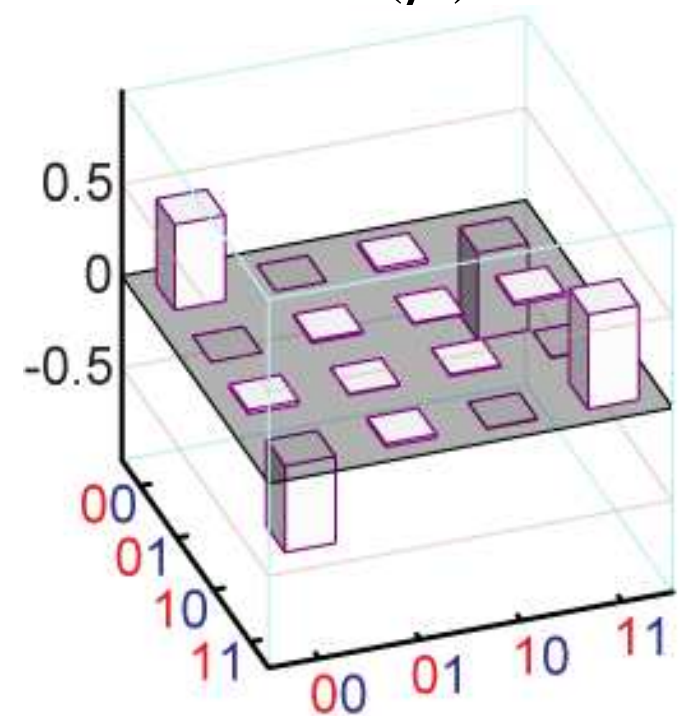
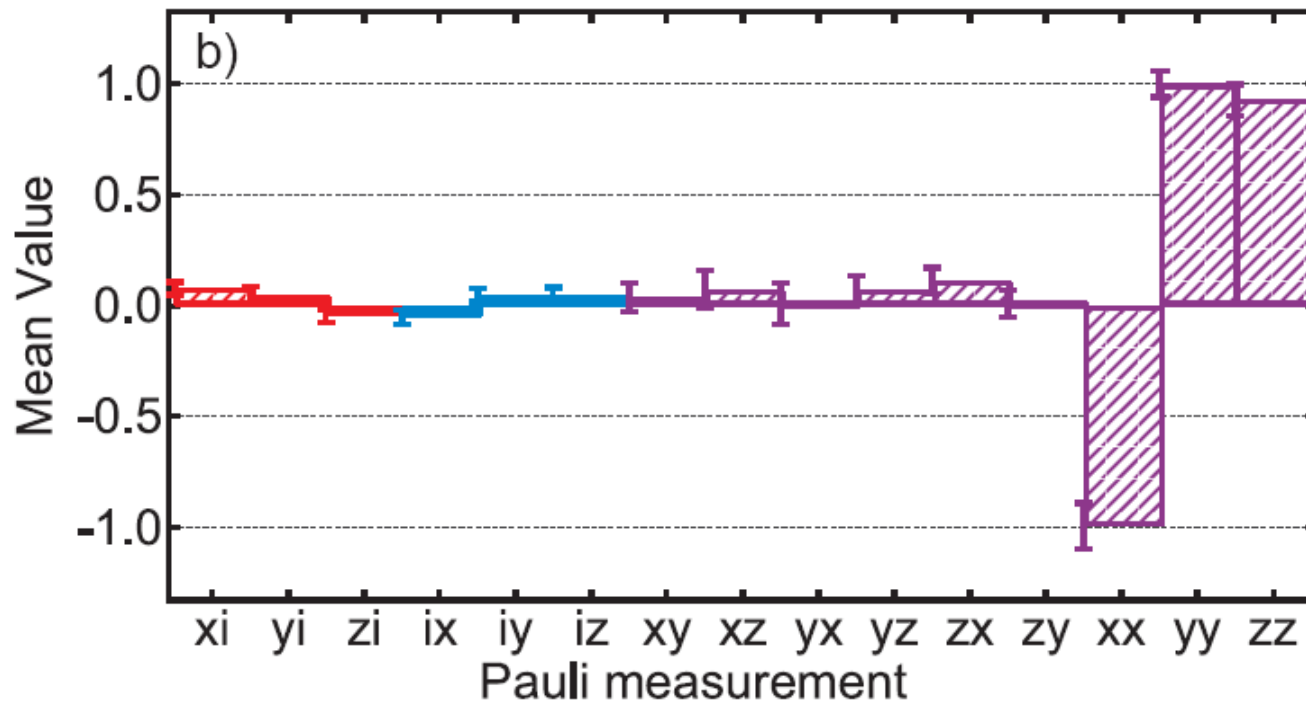
One qubit excited: $|\psi\rangle = |01\rangle$

Measuring the Two-Qubit State

Now apply a two-qubit gate to *entangle* the qubits

Entangled state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

Re(ρ)



$C = 0.94 \pm ??$

What's the entanglement metric?

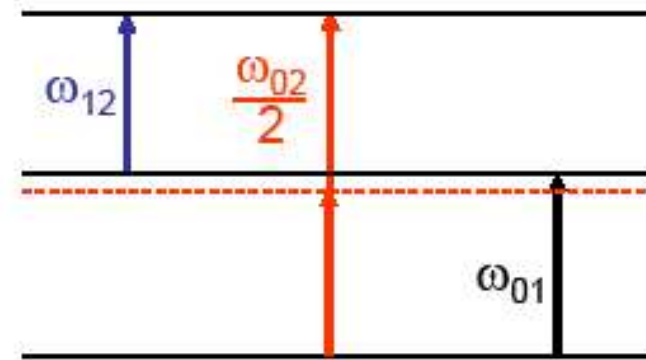
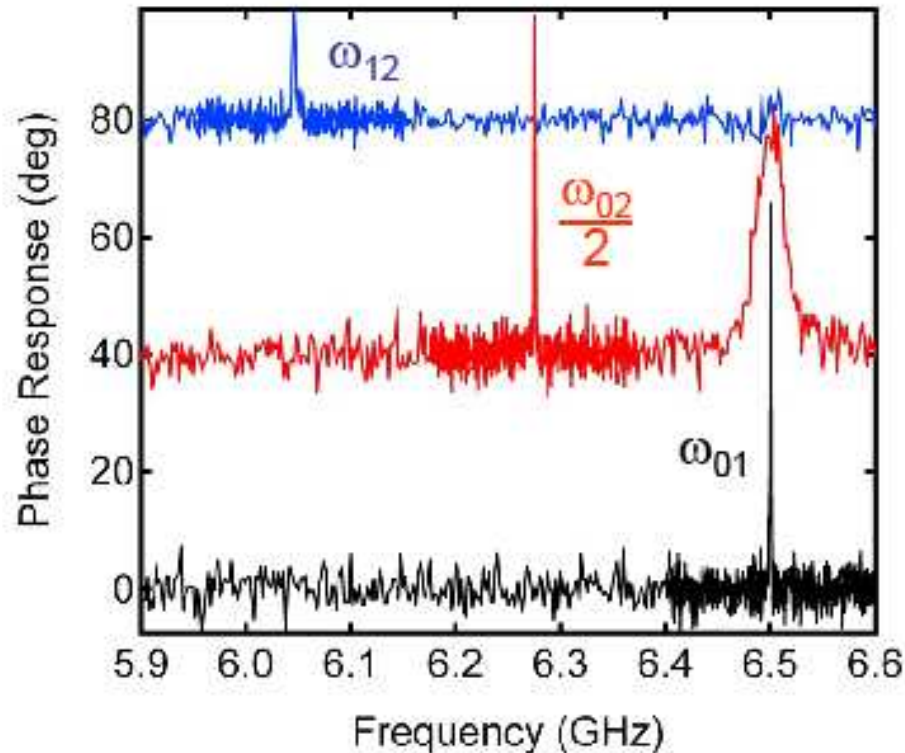
“Concurrence”:

$$C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ are e-values of $\sqrt{\rho\tilde{\rho}}$

SPECTROSCOPY OF A JOSEPHSON ATOM

J. Schreier et al., Phys. Rev. B '08



Microwave pulses at ω_{01} can be used for single qubit rotations.

Anharmonicity:

$$\omega_{01} - \omega_{12} = 455\text{MHz} \simeq E_C$$

Sufficient to control the artificial atom as a two level system: Qubit