Fermi Statistics in Ballistic Conductors

in the light of quantum information





PARIS

Quantum information \rightarrow new way to study quantum systems

single particle interference (probed with current) \rightarrow wave nature variance of particle flux (quantum shot noise) \rightarrow particle nature

interference with few particles:

(noise correlation, coincidence measurements) \rightarrow probe entanglement, non-locality problems, ...

Quantum information → invent new experimental tools : to control initial sate of few particles to record the final state (statistical /single shot readout)

→ here: review some progresses and approaches using ballistic electrons

Goal of this talk :

show how Fermi statistics provides a *natural* mean to implement quantum correlations between ballistic electrons

present experimental approaches able to explore new quantum problems

single electron source (running experiment)

few electron sources (starting project) for Full counting statistics

single electron detection (project)

ballistic electron systems in 2D



III-V semi-conductor heterojunction GaAs/GaAlAs





ballistic electron systems in 2D



III-V semi-conductor heterojunction GaAs/GaAlAs







cyclotron motion



1D chiral ballistic electrons:

elementary quantum gates are realizable:

- beam splitter
- fabry-Pérot interferometer
- Mach-Zehnder interferometer





1D chiral ballistic electrons:

elementary quantum gates are realizable:

- beam splitter
- fabry-Pérot interferometer
- Mach-Zehnder interferometer (Ji et al. (Nature 2003))





(adapted from: P. Roche, P. Roulleau, F. Portier G. Faini D. Mailly)



1D chiral ballistic electrons:

elementary quantum gates are realizable:

- beam splitter
- fabry-Pérot interferometer
- Mach-Zehnder interferometer

finite coherence requires very low temperature :

``Direct Measurement of the Coherence Length of Edge States in the Integer Quantum Hall Regime ''

P. Roulleau, F. Portier, and P. Roche, A. Cavanna, G. Faini, U. Gennser, and D. Mailly PRL 100, 126802 (2008)

 $l_{\Phi} \approx \frac{22\,\mu\,m}{T_{[20\,mK]}}$

(P. Roche's talk last year)

... and also : F. Pierre's talk :

(energy relaxation of edge states)





OUTLINE

Magic properties of the Fermi sea

On-demand single electron source for flying qubits (running experiment)

N-electron source for Full Counting Statistics (new project)

Counting charges for FCS or quantum information (new project)

Conclusion

quantized conductance of a perfect conductor



quantized conductance of a perfect conductor



⇒ conductance quantization

(D. Warrham; B.J. van Wees 1988)



III-V semi-conductor GaA/GaAlAs heterojunction









perfect quantum wires are noiseless here: 1D quantum wires :



shot noise = electron partitioning



binomial statistics

Shot Noise < Schottky
(Poisson)
$$\propto D(1-D)$$

no noise for D = 1 !

$$\left< \Delta I^2 \right> = 2 e I (1 - D) \Delta f$$

G. Lesovik 89, M. Büttiker 91 Th. Martin, R. Landauer 92 Khlus (1987)











negative correlation for electrons

exchange effects in cross-correlation current noise

four terminal conductor :

experiments : (A), (B), then (A+B)

- (A) $V_1 = V$; $V_3 = 0$
- (B) $V_1 = 0$; $V_3 = V$

$$(A+B) V_1 = V ; V_3 = V$$

$$S_{I_{2}I_{4}}^{(A)} = -2\frac{e^{2}}{h}eV\left(s_{21}s_{21}^{*}s_{41}s_{41}^{*}\right)$$
$$S_{I_{2}I_{4}}^{(B)} = -2\frac{e^{2}}{h}eV\left(s_{23}s_{23}^{*}s_{43}s_{43}^{*}\right)$$
$$S_{I_{2}I_{4}}^{(A+B)} \neq S_{I_{2}I_{4}}^{(A)} + S_{I_{2}I_{4}}^{(B)}$$



factorizable as a product of transmissions

$$S_{I_{2}I_{4}}^{(A+B)} - S_{I_{2}I_{4}}^{(A)} - S_{I_{2}I_{4}}^{(B)} = -2\frac{e^{2}}{h}eV\left[s_{21}s_{23}^{*}s_{43}s_{41}^{*} + s_{23}s_{21}^{*}s_{41}s_{43}^{*}\right]$$

exchange terms : non separable

M. Büttiker, Phys. Rev B 46, 12 485, (1992).

natural entanglement in the Fermi sea

C. Beenakker (2003) :

VOLUME 91, NUMBER 14

``in contrast to bosons, fermions can be entangled by single-particle scattering even if the sources are in (local) thermal equilibrium "

 $|0\rangle \Rightarrow (1-D)|0\rangle + D|\uparrow\downarrow\rangle_{\mu}|\uparrow\downarrow\rangle_{\mu} + \dots$

$$\dots + \sqrt{2 D (1 - D)} 2^{-1/2} \left(\left| \uparrow \right\rangle_{h} \left| \uparrow \right\rangle_{e} + \left| \downarrow \right\rangle_{h} \left| \downarrow \right\rangle_{e} \right)$$

entanglement production rate $\approx D \frac{eV}{m}$ (2 GHz for 100 μ V, D=0.1) week ending 3 OCTOBER 200 PHYSICAL REVIEW LETTERS

Proposal for Production and Detection of Entangled Electron-Hole Pairs in a Degenerate Electron Gas

(Intel[®] : beware!)

eV

0

0

C.W.J. Beenakker, C. Emary, M. Kindermann, and J.L. van Velsen Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands (Received 6 May 2003; published 1 October 2003)

tunnel junction transmission τ

(D < < 1)

two-electron interference / entanglement

(P. Samuelsson, E.V. Sukhorukov, M. Buttiker) Phys. Rev. Lett. 92, 026805 (2004)



two-electron interference / entanglement

(P. Samuelsson, E.V. Sukhorukov, M. Buttiker) Phys. Rev. Lett. 92, 026805 (2004)



need to develop more controlled approaches

• biased contact = continuous source of electrons at rate eV/h \rightarrow more than 2 e in a MZI \rightarrow interaction may spoil coherence



• new approach:



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On-demand single electron source for flying qubits (running experiment)

(flying electrons above the Fermi sea)

N-electron source for Full Counting Statistics (new project)

Counting charges for FCS or quantum information (new project)

Conclusion









the single electron gun

differences with known (dc) single electron source

- (ac electron source)
- energy ε of emitted electron well defined
- energy width $h\Gamma$ = fundamental emission rate
- \Rightarrow suitable for coherent manipulation of electron
 - ... while for a metallic electron box :
- energy ε of electron not well defined
- ⇒ energy averaging will smear coherent interference effects

 $0 \le \varepsilon \le \delta E$





practical realization



2D electron gaz in GaAs/Ga(Al)As heterojunction

1 300 nm

capacitor: C ~ 1 f F energy level spacing $\Delta = 2.5 \text{ K} >> e^2/C$

Photo. Y. Jin, LPN CNRS Marcoussis)









energy level spacing of the mesoscopic capacitor

coherent regime : $D > k_B T/\Delta$ $\frac{1}{R_q} = \frac{2e^2}{h}$ NEW !

thermally incoherent regime : $D < k_B T / \Delta$

$$\frac{1}{R_q} = \frac{De^2}{h} \frac{\Delta}{4k_BT} \cdot \frac{1}{\cosh^2\left(\frac{\varepsilon_F - \varepsilon_n}{2k_BT}\right)} \quad \text{(known)}$$

Linear response : the quantum RC circuit



same parameters : C, C_{μ} , transmission $D(V_G)$ for all data wide frequency range


periodic injection of single electrons and single holes

- 2) ⇒ time-domain measurement of the averaged current spikes
 fast acquistion and signal averaging
- 3) ⇒ phase resolved harmonic response to a square excitation
 In phase and in quadrature (I-Q) detection of first harmonic
 ⇒ quantized ac current I = 2 e f

time-domain measurements



time-domain measurements



time-domain measurements



modeling :

Gwendal Fève's Thesis

G. Fève *et al.*, Science **316**, *1169 (2 007 in S.O.M.)*

for simplicity :
$$C \rightarrow \infty$$
 $\frac{e^2}{C_{\mu}} \rightarrow \Delta$ Gwendal
G. Fève *et a*
 $1169 (2)$
 R_q^{nl} C_q^{nl}
 R_q^{nl} C_q^{nl}
 M_q^{nl} M_q^{nl} C_q^{nl}
 M_q^{nl} M

 $q = V_{exc} C_q^{nl}$ $\tau = R_q^{nl} C_q^{nl}$

$$C_q^{nl} = e^2 \int d\varepsilon \ N(\varepsilon) \times \frac{f(\varepsilon - eV_{exc}) - f(\varepsilon)}{eV_{exc}}$$

$$R_q^{nl} = \frac{h}{2e^2} \frac{\int d\varepsilon \ N^2(\varepsilon) \times \frac{f(\varepsilon - eV_{exc}) - f(\varepsilon)}{eV_{exc}}}{\left[\int d\varepsilon \ N(\varepsilon) \times \frac{f(\varepsilon - eV_{exc}) - f(\varepsilon)}{eV_{exc}}\right]^2}$$

$$\rightarrow \langle I(t) \rangle = \frac{q}{\tau} e^{-t/\tau}$$

for :
$$eV_{exc} = \Delta$$

 $C_q^{nl} = \frac{e^2}{\Delta}$, $q = e$
 $D << 1$, $R_q^{nl} = \frac{h}{e^2 D}$
 $\tau = \frac{h}{D\Delta}$



escape (or emission) time



MESURED ESCAPE TIME AGREE WITH : $\tau = h/D\Delta$ (no adjustable parameter)

 \Rightarrow suitable for coherent manipulation of single electrons

G. Fève et al., Science 316, 1169 (2 007)

single electron (hole) partitioning





 $\langle \Delta I_1 . \Delta I_2 \rangle \propto -2e D (1-D)(2ef)$

should provide unambiguous electron anticorrelation

Two- electron (hole) partitioning (probe of Fermi statistics)



Two- electron (hole) partitioning (probe of Fermi statistics)



should provide unambiguous electron anti-bunching (analog Hong-Ou-Mandel)

- measure of electron coherence length with HOM type correlation with electrons
- deviations from Fermi statistics should provide information on e-e interactions
- on-demand S.E.S first step toward implementation of quantum information : (statistical measurements of flying qubit)

• SES has stimulated many theoretical predictions :

Shot Noise of a Mesoscopic Two-Particle Collider Ol'khovskaya, Phys. Rev. Lett. 101, 166802 (2008)

Coherent Particle Transfer in an On-Demand Single-Electron Source, J. Keeling, A.V. Shytov, and L. S. Levitov, PRL 101, 196404 (2008)
 Quantized Dynamics of a Coherent Capacitor, M. Moskalets P. Samuelsson and M. Büttiker, PRL 100, 086601 (2008)
 Electron counting with a two-particle emitter, Janine Splettstoesser, Sveta Ol'khovskaya, Michael Moskalets and Markus Büttiker PRB 78, 205110 (2008)

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Electron Full Counting Statistics

• $P_{N_0}(n)$: probability of n- transmitted electrons for N_0 emitted electrons



Electron Full Counting Statistics

• $P_{N_0}(n)$: probability of n- transmitted electrons for N_0 emitted electrons



Present approaches to Full Counting Statistics

• Statistics of flux (current), not of particles.

Measure the pth-moment of current fluctuations

p=1 conductance
p=2 shot noise
p=3 third moment

$$G \propto \sum_{n} D_{n}$$

$$S_{I} \propto \sum_{n} D_{n} (1 - D_{n})$$

$$S_{I}^{(3)} \propto \sum_{n} D_{n} (1 - D_{n})(1 - 2D_{n})$$
...

• non-unique definition of n-moment current fluctuations at high frequency

 $S_{I}^{(p)}(\omega_{1},\omega_{2},...,\omega_{p-1})$

 measurements of the low frequency third moment have been done in tunnel junctions (Reulet et al 2003, Reznikov 2005), LeMasne (2009)) and in a QPC (Reznikov 2008).

Present approaches to Full Counting Statistics



Also : S. Gustavson, Phys. Rev. Lett. **96**, 076605 (2006) (E.T.H. Zürich)

non-coherent regime : slow sequential events

quantum statistics of few electrons



Full Counting Statistics generalised to multi-terminal conductors

n-electron statistics: binomial \rightarrow multi-nomial

invent new tools : ► e e - electrons n_2 detected (contact 2) n_1 - electrons injected (contact 1) n_3 - electrons > detected phase coherent multi-terminal conductor (contact 3) (few) electron launcher е electron playground electron bunch detector

the n-electron source (injecting n electrons) requirement : as simple as possible



THE MARSHMALLOW SHOOTER^{**}. (n = 20)

This clever pump-action device shoots sweet, edible miniature marshmallows over 30', and it even has an LED sight that projects a safe beam of red light to help locate a target for pinpoint accuracy. The easy-to-refill magazine holds 20 marshmallows (or foam pellets—not included) for fast, nonstop action. Barrel and magazine are top rack dishwasher safe, and the back of the box includes a target for practice. Ages 6 and up. 4" H x 17%" L. (1% lbs.) 71405G \$24.95

the n-electron source

requirement : as simple as possible ... and reliable



THE MARSHMALLOW SHOOTER".

This clever pump-action device shoots sweet, edible miniature marshmallows over 30°, and it even has an LED sight that projects a safe beam of red light to help locate a target for pinpoint accuracy. The easy-to-refill magazine holds 20 marshmallows (or foam pellets—not included) for fast, nonstop action. Barrel and magazine are top rack dishwasher safe, and the back of the box includes a target for practice. Ages 6 and up. 4° H x 17%° L. (1½ lbs.) 71405G \$24.95



received it. Great idea for a child but it needs to be better made. Maybe charge a little more but it should be made with better quality products."

Gender: Female Age: 36-40

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available single electron source

known remarkable electron source :

 voltage biased contact : continuous generation of electrons entangled in a giant Slater determinant at eV/h pace ! But : no time control



 single electron pumps : controlled injection of single charge but : sequential electron injection (incoherent: not a Slater determinant)





• recent single electron gun:

perfect source for flying qubit realisation coherent single electron source opens new field of quantum experiments But: difficult to operate for n-electrons





the n-electron source

electrons in a shake !

gentle shake of Fermi sea using voltage pulse

$$\int eV(t)\,dt = n\,h$$



the n-electron source

gentle shake of Fermi sea using voltage pulse





$$\int eV(t)\,dt = n\,h$$

 $n.\frac{h}{eV}$

V(t)

V

0

non trivial th. result: minimal excitation pulses must have Lorentzian time dependence







 \rightarrow generation of single electrons with voltage pulse

- reveal fundamental properties of the Fermi sea
- allows to investigate few electron FCS

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Counting charges for FCS or quantum information (new project)

(cutting the Fermi sea)

Conclusion

detecting single electrons arriving in contacts :



detecting single electrons arriving in contacts :

possible strategy catching electrons



- 1) isolate electrons in contacts
- 2) take necessary time for single charge accuracy
- 1 e⁻ over 100fF contact \rightarrow 1.6 μ V \rightarrow easily detectable in 1 μ s

new tools : electron detector

possible strategies : ... or catching electrons







cutting the Fermi sea



FIG. 2 (color online). Noise power (17) in a QPC driven by a pulse train vs the pulse width. Parameters used: driving frequency $\nu = 500$ MHz, short-time cutoff $\tau = 20$ ps. The noise as well as the entropy production are symmetric under $w \rightarrow T - w$. Note that at a narrow pulsewidth $w \ll T$, the dependence (17) reproduces the $\frac{1}{3} \log L$ [2] behavior of the entropy.



Quantum Noise as an Entanglement Meter



the quantum switch





CONCLUSION

the Fermi sea provides :

continuous single electron source natural entanglement

quantum information with ballistics electrons requires new tools:

On-demand single electron source

for H.O.M. experiments with electrons, flying qubits

Levitov's n-electron source

new Physics of the Fermi sea, allows to study FCS with few electrons

counting charges for FCS or quantum information

rises new fundamental problems and physics of the Fermi sea

very rich physics although very few groups are working on these topics !

Weizmann Inst., GNE Saclay, Meso. Phys. Group LPA Paris, Un. Regensburg, Un. Basel and less related : ETH Zürich, NTT Atsugi, Harvard, Cambridge(UK),...

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