Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

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Probing Quantum objects with microwave signals

Santa Barbara 2010
see Andrew's talk at 3pm

Boulder 2011

Saclay 2011

Yale 2010

Berkeley 2011
Example: measuring the state of a Qbit

$$|0\rangle$$
$$|1\rangle$$
$$\alpha |1\rangle + \beta |0\rangle$$

$$\alpha |1\rangle \otimes |\alpha_1\rangle + \beta |0\rangle \otimes |\alpha_0\rangle$$

$$I \cos(\omega_{cav} t) + Q \sin(\omega_{cav} t)$$

[see Lecture V]
Why do we need good amplifiers?

\[ \alpha |1\rangle \otimes |\alpha_1\rangle + \beta |0\rangle \otimes |\alpha_0\rangle \]

\[ I \cos(\omega_{cav} t) + Q \sin(\omega_{cav} t) \]

|0\rangle

|1\rangle

\[ \alpha |1\rangle + \beta |0\rangle \]

not quantum limited
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\[ I \cos(\omega_{\text{cav}} t) + Q \sin(\omega_{\text{cav}} t) \]

Goal: evolution of the quantum object directly given by the measurement outcome
Two kinds of linear amplifiers

phase preserving

\[ \hat{a}_{\text{out}} = \sqrt{G} \hat{a}_{\text{in}} + \hat{N} \]

\[ [\hat{a}_{\text{out}}, \hat{a}_{\text{out}}^\dagger] = 1 \Rightarrow [\hat{N}^\dagger, \hat{N}] = G - 1 \]

\[ \Delta \hat{N}^2 = \frac{1}{2} \left\langle \left\{ \hat{N}, \hat{N}^\dagger \right\} \right\rangle \geq \frac{G - 1}{2} \]

[Caves, PRD (1982), Caltech HEMTs]

phase dependent

\[ \hat{a}_{\text{out}} = \frac{\sqrt{G}}{2} (\hat{a}_{\text{in}} + \hat{a}_{\text{in}}^\dagger) + \frac{1}{2\sqrt{G}} (\hat{a}_{\text{in}} - \hat{a}_{\text{in}}^\dagger) \]

\[ [\hat{a}_{\text{out}}, \hat{a}_{\text{out}}^\dagger] = 1 \]

\[ \Delta \hat{N}^2 \geq 0 \]

[Yurke et al., PRA (1989), Bell Labs]
[Castellanos-Beltran, Nat Phys. (2008), Boulder]
[Yamamoto et al., APL (2008), RIKEN]...
Two kinds of linear amplifiers

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[\text{Caves, PRD (1982), Caltech}]
Core of the amplifier: 3-wave mixing

\[ \hat{H}_{Mix} = \hbar g^{(3)} (a_S^\dagger a_I^\dagger a_P + a_S a_I a_P^\dagger) \]

[see Lecture III]

Basis of amplification
stimulated emission

\[ \hat{H}_{Mix} |1_S, 0_I, \alpha_P\rangle = \alpha_P \hbar g^{(3)} |2_S, 1_I, \alpha_P\rangle \]
Implementation in optics: non-linear crystal

\[ \hat{H}_{Mix} = \hbar g^{(3)} (a_S^\dagger a_I^\dagger a_P + a_S a_I a_P^\dagger) \]

\[ \hat{H}_{Mix} |0_S, 0_I, \alpha_P\rangle = \alpha_P \hbar g |1_S, 1_I, \alpha_P\rangle \]

spontaneous parametric down-conversion
How to reach the quantum limit for microwaves?

\[ \Delta \hat{N}^2 \approx \frac{G - 1}{2} \]

Need to minimize the number of information channels to 3

- **Superconducting circuits**: no dissipation
- **GHz signals**: no thermal photons at dilution fridge temperatures
- **Proper filtering**: no external electromagnetic noise
Cavities

3-wave mixing term needed

\[ \hat{H}_{Mix} = \hbar g^{(3)} (a_S^\dagger a_I^\dagger a_P + a_S a_I a_P^\dagger) \]
Non linear element: Josephson junction

\[ U = -\varphi_0 I_0 \cos(\varphi) \]
Non linear element: Josephson junction

\[ U = -\varphi_0 I_0 \cos(\varphi) - I_{\text{bias}} \varphi_0 \varphi \]

5 µm
Non linear element: Josephson junction

\[ U = -\varphi_0 I_0 \cos(\varphi) - I_{\text{bias}} \varphi_0 \varphi \]

\[ -\varphi_0 I_0 \]

\[ I_{\text{bias}} = 0.2 I_0 \]

\[ \propto \varphi^3 \]

To get \( a_S^\dagger a_I^\dagger a_P \), we use \( \varphi \varphi \varphi \)

Need to decompose \( \varphi^3 \mapsto \varphi \varphi \varphi \)
Josephson Parametric Converter (JPC)

spatial decomposition using a ring

\[ U = \alpha X Y Z + \mu (X^2 + Y^2 + Z^2) + O(\ldots^4) \]

symmetry forbids undesired terms

\[ XY, X^3, XY^2 \]

magnetic flux provides current bias

\[ \Phi \leftrightarrow I_{bias} \]

but phase slips possible!

[see Lecture III]
Josephson Parametric Converter (JPC)

\[ H \approx \alpha X Y Z + \mu (X^2 + Y^2 + Z^2) \]

\[ \omega_s \downarrow \]

\[ \omega_i \downarrow \]

\[ \Omega \uparrow \]

[Josephson et al., Nat. Phys. (2010)]
Josephson Parametric Converter (JPC)

\[ G = \left( \frac{1 + \rho^2}{1 - \rho^2} \right)^2 \]

\[ \rho = \frac{\sqrt{2} I_p}{4 I_0 \sqrt{P_s Q_s p_i Q_i}} \]

[Bergeal et al., Nat. Phys. (2010), Lecture III]
Realization

$I_0 \approx 5 \mu A$
Cabling of the dilution fridge
Resonance frequency as a function of field

35 mK

\[ Q_{\text{coupl}} = 35 \] idler

Pump OFF

\[ Q_{\text{coupl}} = 104 \] signal
Resonance frequency as a function of field $X$, $Y$, and $Z$.

$\text{arg}(r)(^\circ) = \frac{U}{\varphi_0 I_0}$

$\frac{X}{2\varphi_0} = \frac{Y}{2\varphi_0} = \frac{Z}{2\varphi_0}$

35 mK
3-wave mixing with the Josephson ring

\[
H \approx \alpha XYZ + \mu(X^2 + Y^2 + Z^2)
\]

\[
\Phi = 0
\]

best non-linearity
unstable

\[
\Phi = \Phi_0 / 2
\]

average non-linearity
stable
Gain as a function of pump power

\[ \Phi = \Phi_0 / 2 \]

35 mK

idler

signal

\[ \Delta f \approx 20 \text{ MHz for } G = 100 \]
How to improve the JPC?

- Magnetic flux provides current bias: $\Phi \sim I_{\text{bias}}$
- Phase slips possible!

- Frequency tunability with the flux cannot be tuned if stability required
- Robustness of the amplifier requires stability
How to improve the JPC?

Ideally, $\vec{B}_{\text{ext}} \circlearrowleft \varphi_{\text{ext}}$

$\Phi \Leftrightarrow I_{\text{bias}}$

Magnetic flux provides current bias.

Phase slips possible!

But phase slip because

$$L_J = \frac{\varphi_0}{I_0 \cos(\varphi_{\text{ext}}/4)}$$

goes negative when $\varphi_{\text{ext}}/4 > \frac{\pi}{2}$
How to improve the JPC?

- Magnetic flux provides current bias: $\Phi \Leftrightarrow I\text{bias}$
- Phase slips possible!

Solution: add inductances

$$U \rightarrow U + \frac{E_L}{4} \left(2X^2 + 2Y^2 + Z^2\right)$$

No phase slip if

$$L_J = \frac{\varphi_0}{I_0} > \frac{12}{5} L$$
New generation
Resonance frequency as a function of field

Pump OFF

\[ Q_{\text{coupl}} = 132 \]  
\[ \text{idler} \]

\[ Q_{\text{coupl}} = 220 \]  
\[ \text{signal} \]

No more hysteresis!
Gain as a function of magnetic field

maximal gain (dB)

35 mK

idler

signal

tunability!

\[
\frac{d\omega_s}{d\Phi} = \Phi_0 \frac{\omega_s^2 I_0 L}{2Z_0} \frac{L}{\varphi_0}
\]
Varying the critical current

small $I_0$

medium $I_0$

large $I_0$
Resonance frequency as a function of field

- $I_{\text{coil}}^{\text{meas}}$ (red circles)
- $I_{\text{coil}}^{\text{meas}}$ (black circles)

- Theory with $L = 0.06 - 0.07 \, \text{nH}$
- $L_{\text{series}} = 0.07 - 0.08 \, \text{nH}$
- Still OK with $\pm 20\%$

For $I_0 = 0.5 \, \mu\text{A}$:

For $I_0 = 1.2 \, \mu\text{A}$:

For $I_0 = 1.5 \, \mu\text{A}$:

Tunability with $l_0$
Gain as a function of pump power

$35 \text{ mK}$

$f_{\text{pump}} = 14.071 \text{ GHz}, I_{\text{coil}} = 3 \mu \text{A}$

- Amplification
- Lasing

Gain as a function of pump power

$P_{\text{dBm}}$ vs $f_{\text{GHz}}$

$\text{signal}$

$\text{signal gain (dB)}$ vs $f_{\text{GHz}}$
Gain as a function of pump power

\[ f_{\text{pump}} = 14.071 \text{ GHz}, \ I_{\text{coil}} = 3 \ \mu A \]
Gain as a function of pump power

$35 \text{ mK}$

\[ f_{pump} = 14.071 \text{ GHz}, \quad I_{\text{coil}} = 3 \mu A \]

![Graph showing gain as a function of pump power](image)
Normal metal tunnel junction: a good noise source
Noise measurement

\[ \hbar \omega \gg k_B T \Rightarrow R_t S_{II}(V, \omega_S) = \text{Max}(2|eV|, 2\hbar \omega_S) \]

\[ P_n(\omega_s) = \frac{Z_0 S_{II}(\omega_s)}{4} \Delta \omega \]

if \( R_t = Z_0 \) and \( R_t C_t \omega_S \ll 1 \)

perfect matching
Noise measurement

\[ S_{\text{tot}} = G_{\text{tot}} \left( S^{50} \Omega (\omega_I) + \frac{\omega_I}{\omega_S} S^{TJ} (\omega_S) \right) \]

\[ \hbar \omega_S, \hbar \omega_I \gg k_B T \]

\[ S^{50} \Omega (\omega_I) = \frac{\hbar \omega_I}{2} \]

\[ S^{TJ} (V, \omega_S) = \text{Max} \left( \frac{|eV|}{2}, \frac{\hbar \omega_S}{2} \right) \]
Noise measurement

\[ f_{\text{pump}} = 14.071 \text{ GHz}, \quad P_{\text{pump}} = -3.56 \text{ dBm}, \quad I_{\text{coil}} = 3 \mu \text{A} \]

slope change at \( eV_{\text{TJ}} = \hbar \omega_S \) even for \( S_{\text{tot}}(\omega_I) \)
Noise measurement

\[ f_{\text{pump}} = 14.071 \text{ GHz}, \ P_{\text{pump}} = -3.56 \text{ dBm}, \ I_{\text{coil}} = 3 \mu A \]

Can we use it to calibrate the added noise?

YES, but need to determine the impedance matching of the junction.
Noise measurement

\[ S_{tot} = G_{tot}(S^{50 \, \Omega}) + D \frac{\omega_I}{\omega_S} STJ(\omega_S) + (1 - D) \frac{\omega_I}{\omega_S} S^{50 \, \Omega} \]

\[ S^{50 \, \Omega}(\omega_I) = \frac{\hbar \omega_I}{2} \]

\[ D = \frac{4R_t \text{Re}[Z_{env}(\omega_S)]}{|R_t + Z_{env}(\omega_S)|^2} \approx 0.38 - 0.71 \]

\[ x \text{ insertion loss} \]

\[ D \approx 0.2 - 0.71 \]

\[ C_t \approx 0.5 - 1 \, \text{pF}, \quad R_t = 44 \, \Omega, \quad Z_0 = 50 \, \Omega, \quad \omega/2\pi = 8.6 \, \text{GHz} \]
\( f_{\text{pump}} = 14.071 \text{ GHz}, P_{\text{pump}} = -3.56 \text{ dBm}, I_{\text{coil}} = 3 \mu A \)

**Noise measurement**

\[
\frac{S_{\text{tot}}(\omega_S)}{\hbar\omega_S} \quad \frac{eV^{TJ}_S}{\hbar\omega_S} \\
\frac{S_{\text{tot}}(\omega_I)}{\hbar\omega_I} \quad \frac{eV^{TJ}_I}{\hbar\omega_I}
\]

**fit** \( D = 0.30, T^{TJ} = 40 \text{ mK} \)
Noise as a function of temperature

\[ R_t S_{II}(\omega) = (eV + \hbar \omega) \coth \left( \frac{eV + \hbar \omega}{2k_BT} \right) + (eV - \hbar \omega) \coth \left( \frac{eV - \hbar \omega}{2k_BT} \right) \]

40mK → 200mK

\[ \frac{S(\omega)}{\hbar \omega} \]

\[ V_{TJ}(\mu V) \]

note: other junction and amplifier
Conclusions

Ring of 4 Josephson junctions in cavity realizes a non-degenerate parametric amplifier for microwave photons

[Ring of 4 Josephson junctions in cavity diagram]

[Bergeal et al., Nat. Phys. (2010)]
[Bergeal et al., Nature (2010)]

Proper calibration of attenuation between noise source and amp needed to prove quantum limit is reached

Bandwidth tunability and stability achieved using additional inductances
Thanks !
Thanks!

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