Chaire de Physique Mésoscopique
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INTRODUCTION À LA PHYSIQUE MÉSOSCOPIQUE: ÉLECTRONS ET PHOTONS

INTRODUCTION TO MESOSCOPIC PHYSICS: ELECTRONS AND PHOTONS

Deuxième leçon / Second Lecture

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What do "electron" and "photon" mean in mesoscopic physics?

Purpose: provide groundwork for Landauer's approach of transport phenomena and quantum circuit theory.
The Landauer reservoir is to Fermi waves what a black-body is to Bose waves.
Mesoscopic wire: a collection of independent channels

- **reservoir**
- **mode** $m$
- **right-moving quasiparticles**
- **left-moving quasiparticles**

$E$ represents the energy level, $\mu_+$ and $\mu_-$ are the chemical potentials, and $eV$ represents the bias voltage.
Mesoscopic wire: a collection of independent channels

- Mode $m$
- Reservoir

- Right-moving quasiparticles
- Left-moving quasiparticles

- Energy $E$
- Chemical potential $\mu_+$
- Chemical potential $\mu_-$

- Fermi distribution $f_+$
- Fermi distribution $f_-$

- Energy difference $eV$
Mesoscopic wire: a collection of independent channels

- Mode $m$
- Reservoir

- Right-moving quasiparticles
- Left-moving quasiparticles

- Energy $E$
- Chemical potential $\mu_+$
- Chemical potential $\mu_-$

- Fermi distribution $f_+$
- Fermi distribution $f_-$

- Energy difference $eV$
THE LANDAUER-BÜTTIKER FORMULA
FOR THE AVERAGE CURRENT

\[ I = I_+ - I_- \]

\[ I_\pm = \frac{e}{h} \sum_m \int_{-\infty}^{+\infty} f_\pm(E) |t_m(E)|^2 \, dE \]

\[ f_\pm(E) = \frac{1}{1 + \exp \frac{E - \mu_\pm}{k_B T}} \]

\[ \mu_+ - \mu_- = eV \]

Electrons interact with the voltage source but not between themselves.
THE USUAL ELECTRON OF ATOMIC AND HIGH ENERGY PHYSICS

PARTICLE IDENTIFICATION CARD

Last Name: Electron     First name: Bare
Address: Vacuum          Genre: Fermion
Occupation: Wave packet   Lifetime: infinite
Average energy: $\hbar \omega$     Average momentum: $\hbar k$
Velocity: $v = \frac{d\omega}{dk}$     Mass: $\hbar k / dv = m_e$
Charge: $-e$
Spin: 1/2     Magnetic moment: $\mu_B$
An example of a Feynman diagram involving the usual electron and photon of atomic physics propagating in vacuum
### THE "ELECTRON" OF MESOSCOPICS

<table>
<thead>
<tr>
<th>PARTICLE IDENTIFICATION CARD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Last Name:</strong> Electron</td>
</tr>
<tr>
<td><strong>Address:</strong> Metal</td>
</tr>
<tr>
<td><strong>Occupation:</strong> Wave packet</td>
</tr>
<tr>
<td><strong>Average energy:</strong> $\hbar \omega$</td>
</tr>
<tr>
<td><strong>Velocity:</strong> $v = d\omega/dk$</td>
</tr>
<tr>
<td><strong>Transverse charge:</strong> -e</td>
</tr>
<tr>
<td><strong>Spin:</strong> 1/2</td>
</tr>
<tr>
<td><strong>First name:</strong> Quasi</td>
</tr>
<tr>
<td><strong>Genre:</strong> Fermion</td>
</tr>
<tr>
<td><strong>Lifetime:</strong> finite, except @ $k_F$</td>
</tr>
<tr>
<td><strong>Average momentum:</strong> $\hbar k$</td>
</tr>
<tr>
<td><strong>Mass:</strong> $\hbar dk/dv = m_{\text{eff}}(k)$</td>
</tr>
<tr>
<td><strong>Longitudinal charge:</strong> 0 (q→0)</td>
</tr>
<tr>
<td><strong>Magnetic moment:</strong> $g \mu_B$</td>
</tr>
</tbody>
</table>

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Definition of the longitudinal and transverse part of a field:

\[ \vec{F} = \vec{F}_l + \vec{F}_t \]
\[ \nabla \cdot \vec{F}_t = 0 \]
\[ \nabla \times \vec{F}_l = 0 \]

The longitudinal and transverse charges are the sources of the longitudinal and transverse parts of the electrical field, respectively.
A METAL AT LOW ENERGY: FERMI QUASIPARTICLES + BOSONIC PLASMONS

cannot solve the full many body problem, but….

low-lying excitations of strongly interacting bare electrons

nearly free quasielectrons and holes

bosonic plasma modes

photons
positive background \( n_0 e \) (jellium)

negatively charged fluid \(-ne = -(n_0 + \delta n)e\)

current density \(\vec{j} = -en\vec{\nu}\)

charge density \(\rho = -e\delta n; \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0\)

constitutive equation \(m \left[ \frac{\partial n\vec{\nu}}{\partial t} + (\vec{\nu} \cdot \nabla) n\vec{\nu} \right] = -en \left( \vec{E} + \vec{v} \times \vec{B} \right) - \nabla P\)

approximation \(\frac{\delta n}{n_0}\) small; \(\frac{\nu B}{E}\) small

Quantum Mechanics enter in internal pressure
Box volume $V$

Number of fermions $N$

Total energy $E_K$

Length scale $a_0$

Energy scale $Ry$

\[
N = n = \frac{4\pi}{3} \frac{k_F^2}{(2\pi)^2} = \frac{1}{4\pi} \frac{k^2}{a^3}; \quad r_s = \frac{a}{a_0}; \quad k_F = \frac{1.92}{r_s a_0}
\]

\[
\frac{E_K}{N} = \frac{3}{5} \left( \frac{\hbar k_F}{2m_e} \right)^2 = \frac{3}{5} E_F = \frac{2.22}{r_s^2} Ry
\]

\[
v_F = \frac{\hbar}{m_e} k_F = v_g \bigg|_{E_F}
\]

\[
\frac{\partial E_K}{\partial V} = \text{Fermi pressure}
\]

\[
c_0 = \sqrt{\frac{\partial \left( \frac{\partial E}{\partial V} \right)_N}{m \partial n}} = \frac{1}{3} v_F
\]
ARRIVE AT LINEARIZED EQUATIONS FOR FIELDS AND ELECTRON FLUID

\( \frac{\partial \vec{j}}{\partial t} + v_s^2 \nabla \rho = \omega_p^2 \varepsilon_0 \vec{E} \)

\( \nabla \cdot \vec{E}_l = \frac{\rho}{\varepsilon_0} \)

\( \nabla \cdot \vec{j}_l + \frac{\partial \rho}{\partial t} = 0 \)

\( \nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}_t}{\partial t} = \mu_0 \vec{j}_t \)

\( \nabla \times \vec{E}_t + \frac{\partial \vec{B}}{\partial t} = 0 \)

\( \nabla \cdot \vec{B} = 0 \)

\( \omega_p = \sqrt{\frac{e^2 n_0}{m \varepsilon_0}} \)  
plasma frequency

\( v_s = \sqrt{\frac{1}{mn_0} \left( \frac{\partial P}{\partial n} \right)_0} \)  
sound velocity

\( \vec{E} = \vec{E}_l + \vec{E}_t \)

\( \vec{j} = \vec{j}_l + \vec{j}_t \)

\( \{ \vec{E}_l; \vec{j}_l; \rho \} \)  
longitudinal part

\( \{ \vec{E}_t; \vec{j}_t; \vec{B} \} \)  
transverse part

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BOUNDARY CONDITIONS:
WIRE ABOVE A GROUND PLANE
BOUNDARY CONDITIONS:
WIRE ABOVE A GROUND PLANE

Field lines from wire end on ground plane

$h << \lambda$
LONGITUDINAL MODE CHARGES

TRANSVERSE MODE CHARGES

\( \lambda \)
DISPERSION RELATION OF ELECTRODYNAMIC MODES OF WIRE

\[ \omega = P_s k v \]

PLASMA MODE:
- LONGITUDINAL = "SOUND"
- TRANSVERSE = GUIDED "LIGHT"

SLOPE \( \sim c_{eff} \)

\[ k_s = \frac{\omega_p}{v_s} \]
RESPONSE : SCREENING

\[
\left[ \frac{1}{\omega_P^2} \frac{\partial^2}{\partial t^2} + 1 - \ell_s^2 \nabla^2 \right] \rho = -\rho_{\text{ext}} \quad \ell_s = \frac{v_s}{\omega_P} \sim a_0 \\
\text{for 3D metals}
\]

plane wave dispersion relation:

\[
\omega^2 = \omega_P^2 + v_s^2 k^2
\]

dielectric response function:

\[
\epsilon_r(k, \omega) = 1 + \frac{1}{\ell_s^2 k^2 - \frac{\omega^2}{\omega_P^2}}
\]

screened potential:

\[
V_{\text{eff}}(r, \omega \to 0) = \frac{e}{\epsilon_0} \frac{e}{r} \frac{r}{\ell_s}
\]

07-II-16
NEUTRAL METALLIC SPHERE

density of ions

density of electrons

jellium

total charge density

electroneutrality
CHARGED METALLIC SPHERE

density of ions

density of electrons

total charge density

electroneutrality

07-II-17bis
SELF-CONSISTENT PICTURE OF ELECTRON STATES

\[ \rho_{\text{total}} \]

\[ E \]

\[ U \]

\[ W \]

\[ E_F \]

\[ \rho_{\text{elec}} \]
WHAT IS $\mu$?

- Add charge stays same to a very good approximation.

$$W_{0} + \delta W(\mu)$$

$U$
IDEAL METALLIC TOROIDAL WIRE

ions

electrons
LOW ENERGY ELECTRODYNAMIC EXCITATIONS
LOW ENERGY ELECTRODYNAMIC EXCITATIONS
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LOW ENERGY ELECTRODYNAMIC EXCITATIONS

BOSONIC EXCITATIONS "PHOTONS"
ELECTROSTATIC EXCITATION

Example: torus in parallel plate capacitor
OTHER QUASI-STATIC MACROSCOPIC EXCITATION OF ELECTRONS IN TORUS: ELECTRICAL CURRENT

Example: flux through torus increases linearly with time

Electrons move bodily with respect to ions. No surface charge.
In other words, just heat!
QUASIPARTICLE EXCITATIONS

GROUND STATE

ONE "ELECTRON"

ONE "HOLE"
FINITE LIFETIME OF QUASIPARTICLES

ONE "ELECTRON"

TWO "ELECTRONS"
+ ONE "HOLE"
DISPERSION RELATION
OF ELEMENTARY EXCITATIONS
IN A METAL

\[ \hbar \omega_p \]

\[ \hbar k_F \]

velocity \( \sim c \)

velocity \( = v_F \)

electrodynamic modes

holes

electrons

0
DISSIPATION CORRESPONDS TO CREATION OF ELECTRON-HOLE PAIRS FROM ELECTRODYNAMIC EXCITATIONS

\[ \hbar \omega_p = Fk \]

\[ \hbar \omega_{RF} = \varepsilon_h + \varepsilon_e \]

IMPURITIES NO LONGER CONSERVED

momentum

energy

electro-dynamic modes

holes

electrons
The reverse process corresponds to Johnson noise.

\[ \hbar \omega = F k \]

Electrodynamic modes:

\[ \hbar \omega_{RF} = \varepsilon_h + \varepsilon_e \]

Holes and electrons:

\[ \hbar k_F \]
JOHNSON NOISE IS EQUIVALENT TO BLACK-BODY RADIATION

Power per unit frequency:

\[ P(\nu, T) = \frac{2h\nu}{e^{\frac{k_B T}{h\nu}} - 1} \]

1-D version of Planck's radiation law

\[ I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{k_B T}{h\nu}} - 1} \]

Emission of "photons" by excited quasielectron-hole pairs analogous to emission of photons by black-body atoms.
DISPERSION RELATION
OF ELEMENTARY EXCITATIONS
IN A SUPERCONDUCTING METAL

(caveat: S-wave, with gap)
"Electrons" and "photons" in mesoscopic physics are "dressed" particles with properties which can greatly differ from their counterparts in free space.

These properties can be designed. We can construct a quantum Lego set, explore its various combinations and "invent" new quantum effects.
COMPARISON BETWEEN QUANTUM OPTICS AND QUANTUM TRANSPORT EXPERIMENTS

sources

atom

detectors

DC

mesoscopic device

V

I
COMPARISON BETWEEN QUANTUM OPTICS AND RF QUANTUM TRANSPORT EXPERIMENTS
<table>
<thead>
<tr>
<th>QUANTUM OPTICS</th>
<th>QUANTUM CRYOLECTRONICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>atoms, molecules</td>
<td>tunnel devices, semic. dots</td>
</tr>
<tr>
<td>light beams, fibers</td>
<td>coax. transmission lines</td>
</tr>
<tr>
<td>mirrors, beam splitters, etc</td>
<td>filters, couplers, circulators</td>
</tr>
<tr>
<td>light sources : lasers</td>
<td>microwave generators</td>
</tr>
<tr>
<td>photodetectors, photomultipliers</td>
<td>cryogenic amplifiers</td>
</tr>
<tr>
<td>$T_{\text{background}} = 300\text{K}$</td>
<td>$T_{\text{background}} = 30\text{mK}$</td>
</tr>
<tr>
<td>cavity</td>
<td>resonator, oscillator</td>
</tr>
<tr>
<td>weak atom-field coupling</td>
<td>strong artificial atom – field coupling</td>
</tr>
<tr>
<td>photon loss and dispersion</td>
<td>resistance and reactance</td>
</tr>
</tbody>
</table>

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SOME KEY IDEAS

Rolf Landauer

"think conductance, not conductivity!"

David Thouless

\[ E_{\text{Thouless}} = \frac{\hbar D}{L^2} \]

Joe Imry

"(for quasi-electrons) what is important is loss of quantum information: decoherence"

Tony Leggett

dissipative non-linear circuits: to what extent do they obey quantum mechanics?

07-II-33
How do we treat a macroscopic circuit quantum-mechanically?

How do we describe non-linear elements like tunnel junctions, both normal and superconducting?

What are the properties of quantum noise? How does it limit the processing of signals?
LE COURS DE L'AN PROCHAIN:
"CIRCUITS ET SIGNAUX QUANTIQUES"

Début: 13 mai 2008

Comment traiter quantiquement un circuit électronique macroscopique?

Comment décrire les composants non-linéaires comme les jonctions tunnel?

Quels sont les propriétés du buit quantique? Quel est son influence sur le traitement du signal?
Acknowledgements: D. Esteve, H. Pothier, D. Stone and C. Urbina