Chaire de Physique Mésoscopique
Michel Devoret
Année 2009, 12 mai - 23 juin

CIRCUITS ET SIGNAUX QUANTIQUES (II)
QUANTUM SIGNALS AND CIRCUITS (II)

Première leçon / First Lecture

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PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"
CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expéri-
mentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR ON MAY 26!

CONTENT OF THIS YEAR’S LECTURES

OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year’s course
2. Audit of information processing machines
3. Readout of qubits
4. Amplifying quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Can Bloch oscillations be observed?
7. Defying the fine structure constant: Fluxonium qubit

NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS
LECTURE I: INTRODUCTION, THE AUDIT OF INFORMATION PROCESSING MACHINES

1. Limitations of information processing machines
2. Review of quantum circuits
3. Non-linearity of Josephson junctions
4. Summary of questions addressed by this course
**Modular architecture**
- small number of basic building blocks,
- large number of combinations into useful networks

**Parallel fabrication**
- reliable assembly of networks with large number of elements (10⁹ transistors/chip)
- uniformity of like elements
- miniaturization

**Classical analysis**
- dynamics of information carrying signals like voltages and currents usually described by classical equations (however, quantum mechanics enters at lower level in characteristics of single elements such as transistors, tunnel diodes, etc.)

► This course deals with **quantum** electrical circuits, i.e. circuits in which information carrying signals must be treated quantum-mechanically.

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**ELECTROMAGNETIC SPECTRUM**

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$\lambda$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^0$</td>
<td>$10^9$</td>
</tr>
<tr>
<td>$10^3$</td>
<td>$10^6$</td>
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<tr>
<td>$10^6$</td>
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<td>$10^9$</td>
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<tr>
<td>$10^{12}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>$10^{20}$</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

- AUDIO
- VIDEO
- RF / $\mu$W
- IR
- UV
- RX
- $\gamma$

**ELECTRICITY** ➔ **OPTICS**
1 microwave signal with 0/1 photon → 1 bit?

10 GHz ~ 0.5K
INFORMATION PROCESSING CIRCUITS ARE NOW SMALL AND FAST, BUT ARE THEY EFFICIENT?

Landauer, Bennett, Feynman

clock speed \times 10^6

volume \ / \ 10^{15}

power \ / \ 10^4

"Google is fast — a typical search returns results in less than 0.2 seconds. Queries vary in degree of difficulty, but for the average query, the servers it touches each work on it for just a few thousandths of a second. Together with other work performed before your search even starts (such as building the search index) this amounts to 0.0003 kWh of energy per search, or 1 kJ. For comparison, the average adult needs about 8000 kJ a day of energy from food, so a Google search uses just about the same amount of energy that your body burns in ten seconds."  

OFFICIAL GOOGLE BLOG

1 GOOGLE QUERY = 1kJ!
THE PROBLEM OF HEAT ENGINES EFFICIENCY GAVE BIRTH TO THERMODYNAMICS

\[ \eta < \eta_{\text{THEOR.}} = \frac{T_{\text{HOT}} - T_{\text{COLD}}}{T_{\text{HOT}}} \]

Order of magnitude \( \eta_{\text{I.C.E.}} \sim 0.3 \)

"Réflexions sur la puissance motrice du feu" (1824)

JUST AS A THERMAL ENGINES CONVERT SOURCE HEAT INTO MECHANICAL WORK AND WASTE HEAT, A COMPUTER CONVERTS FREE ENERGY INTO MATHEMATICAL WORK AND WASTE HEAT.

"The thermodynamics of computation", C. Bennett, 1982

HOW MUCH ENERGY IS NEEDED PER ELEMENTARY OPERATION? WHAT IS ITS MINIMUM DURATION?
IN PRINCIPLE, COMPUTATION CAN BE REVERSIBLE. THEN COST IS ONLY THE ERASURE OF INFORMATION IN OUTPUT REGISTER, i.e. $k_B T \ln 2$ PER BIT

1 0 1 1 0 0 1 0 1 0 0 1 1 1 0

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

$k_B T \ln 2 \sim 10^{-20} \text{J} @ 300K$

MAXWELL'S DEMON
MAXWELL'S DEMON

DEMON'S MEMORY DURING HIS WORK:

already used     remaining “fresh zeroes”

end of memory

THE ULTIMATE COST OF COMMUNICATION

maximum channel capacity for signal power $P$:

$$C \leq \sqrt{\frac{\pi \frac{P}{3}}{\hbar}}$$

Gordon (1964), Lebedev and Levitin (1966)

$100\mu W \sim 10^{15}$ bits/s!
1. Limitations of information processing machines

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QUANTUM INFORMATION PROCESSING

# of atoms

- OPTICAL PHOTONS
- NUCLEAR SPINS
- IONS
- ATOMS
- MOLECULES
- QUANTUM DOTS
- SUPERCONDUCTING JOSEPHSON TUNNEL CIRCUITS

coupling with env't.

courtesy of J. Martinis, 2009
from Metcalf et al., 2007
A RF CIRCUIT IS A NETWORK OF ELECTRICAL ELEMENTS

TWO DYNAMICAL VARIABLES CHARACTERIZE THE STATE OF EACH DIPOLE ELEMENT AT EVERY INSTANT:

Voltage across the element: \[ V_\beta (t) = \int_{n}^{p} \vec{E} \cdot d\vec{l} \]

Current through the element: \[ I_\beta (t) = \int \int \vec{j} \cdot d\sigma_{np} \]

CONSTITUTIVE RELATIONS OF ELEMENTS

EACH ELEMENT IS TAKEN FROM A FINITE SET OF ELEMENT TYPES

EACH ELEMENT TYPE IS CHARACTERIZED BY A RELATION BETWEEN VOLTAGE AND CURRENT

Examples:

inductance: \[ V - L \frac{dI}{dt} = 0 \]

capacitance: \[ I - C \frac{dV}{dt} = 0 \]
CONSTITUTIVE RELATIONS OF ELEMENTS

EACH ELEMENT IS TAKEN FROM A FINITE SET OF ELEMENT TYPES

EACH ELEMENT TYPE IS CHARACTERIZED BY A RELATION BETWEEN VOLTAGE AND CURRENT

Examples:

inductance: \[ V - L \frac{dI}{dt} = 0 \]

capacitance: \[ I - C \frac{dV}{dt} = 0 \]

Caveat:

resistance: \[ V - R I = 0 \]

tend to kill quantum coherence

not the correct constitutive relation of a resistance anyway (see next lecture)

JOSEPHSON JUNCTION PROVIDES A NON-LINEAR INDUCTOR

1nm

S

L

S

superconductor-insulator-superconductor

tunnel junction

\[ L_J \]

\[ C_j \]
JOSEPHSON JUNCTION PROVIDES A NON-LINEAR INDUCTOR

\[ I = \frac{\phi}{L_J} \]

\[ \phi = \int_{-\infty}^{t'} V(t') dt' \]

JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION

\[ L_J = \frac{\phi_0}{I_0} \]

\[ I = I_0 \sin \left( \frac{\phi}{\phi_0} \right) \]

\[ \phi_0 = \frac{\hbar}{2e} \]
JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION

\[ L_J = \frac{\phi_0^2}{E_J} \]

LOW-LYING EXCITATIONS OF A JUNCTION: CHARGE STATES

\[ U = -E_J \cos(\phi / \phi_0) \]

\[ \phi = \int_{-\infty}^{t} V(t') dt' \]

bare Josephson potential

\[ \phi_0 = \frac{\hbar}{2e} \]
QUANTUM TREATMENT OF CIRCUITS

Need to take branch flux and branch charge as basic variables:

\[
\phi_\beta(t) = \int_{-\infty}^{t} V_\beta(t') dt' \\
Q_\beta(t) = \int_{-\infty}^{t} I_\beta(t') dt'
\]

For every branch \( \beta \) in the circuit:

\[
\left[ \hat{\phi}_\beta, \hat{Q}_\beta \right] = i\hbar
\]

LC CIRCUIT AS A QUANTUM HARMONIC OSCILLATOR

\[
\hat{a} = \frac{\hat{\phi}_{ZPF}}{\phi_{ZPF}} + i \frac{\hat{Q}_{ZPF}}{Q_{ZPF}}; \quad \hat{a}^\dagger = \frac{\hat{\phi}_{ZPF}}{\phi_{ZPF}} - i \frac{\hat{Q}_{ZPF}}{Q_{ZPF}}
\]

\[
\phi_{ZPF} = \sqrt{2\hbar\omega_0 L} \\
Q_{ZPF} = \sqrt{2\hbar\omega_0 C}
\]

\[
\hat{H} = \hbar\omega_0 \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)
\]

Annihilation and creation operators

\[
\left[ \hat{a}, \hat{a}^\dagger \right] = 1
\]

Trapped photons!
ALL TRANSITIONS BETWEEN QUANTUM LEVELS ARE DEGENERATE IN PURELY LINEAR CIRCUITS!

CANNOT STEER THE SYSTEM TO AN ARBITRARY STATE IF PERFECTLY LINEAR

NEED NON-LINEARITY TO FULLY REVEAL QUANTUM MECHANICS
1. Limitations of information processing machines
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**ENERGY SCALES OF THE JOSEPHSON JUNCTION "ATOM"**

![Energy Scales Diagram]

- **Hamiltonian:**
  \[ \hat{H}_J = 8E_C \left( \frac{\hat{N} - \hat{N}_{ext}}{2} \right)^2 - E_J \cos \hat{\phi} \]

- **Coulomb charging energy for 1e:**
  \[ E_C = \frac{e^2}{2C_j} \]

- **Josephson energy:**
  \[ E_J = \frac{1}{8} \mathcal{N} T \Delta \]

- **Reduced offset charge valid for opaque barrier:**
  \[ \hat{N}_{ext} = \frac{q_{ext}}{2e} \]

- **Barrier transparency valid for # condensed channels:**
  \[ E_J = \frac{1}{8} \mathcal{N} T \Delta \]

\[ [\hat{\phi}, \hat{N}] = i \]
3 TYPES OF BIASES

**Charge**

\[ U \]

\[ 2C_g \]

\[ C_j \]

\[ 2C_g \]

\[ L_j \]

\[ \frac{8E_c (\hat{N} - C_j U / 2)^2}{2} - E_j \cos \phi \]

"Cooper pair box"

\( \hat{\phi} \) lives on circle

\( \hat{N} \) integer

---

3 TYPES OF BIASES

**Flux**

\[ \Phi_b \]

\[ \frac{8E_c \hat{N}^2}{2} + E_L \left( \frac{\phi - 2e\Phi_b}{\hbar} \right)^2 - E_j \cos \phi \]

---

3 TYPES OF BIASES

**Charge**

\[ U \]

\[ 2C_g \]

\[ C_j \]

\[ 2C_g \]

\[ L_j \]

\[ \frac{8E_c (\hat{N} - C_j U / 2)^2}{2} - E_j \cos \phi \]

"Cooper pair box"

\( \hat{\phi} \) lives on circle

\( \hat{N} \) integer
3 TYPES OF BIASES

charge

flux

current

\[ \Phi_b \]

\[ J \]

\[ L_j \]

\[ C_j \]

\[ U \]

\[ 2C_g \]

\[ \frac{1}{2} \]

\[ 8E_c \left( \hat{N} - C_u U/2 \right)^2 - E_j \cos \phi \]

\[ 8E_c \hat{N}^2 - E_j \left( \cos \phi - \frac{I_b}{I_0} \phi \right) \]

"Cooper pair box"

\[ \phi \]

lives on circle

\[ \hat{N} \]

integer

\[ \frac{1}{2} \]

EFFECTIVE POTENTIAL

OF 3 MAIN BIAS SCHEMES

"charge" bias

\[ \frac{\phi}{2\pi} = \frac{2e\phi}{h} \]

CEA Saclay, NEC, Yale
Chalmers, JPL, ...

"flux" bias

 TU Delft, NEC, NTT, IBM,
MIT, UC Berkeley, SUNY,
JPhT Jena, ...

"phase" bias

 UC Berkeley, NIST, UCSB,
U. Maryland, I. Neel Grenoble...

see also proposals for
topologically protected
qubits, for example
Feigelman et al. PRL 92, 098301 (2004)

09-1-25
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IN PRINCIPLE, IT IS POSSIBLE TO ACCOUNT FOR EVERY PIECE OF ENERGY AND ENTROPY IN THE WORK OF A MACHINE PROCESSING QUANTUM INFORMATION.

THERE ARE PRINCIPLES GOVERNING THE ULTIMATE PERFORMANCE OF THESE MACHINES.

CAN WE SEE THESE PRINCIPLES IN ACTION IN THE PRACTICAL DESIGN OF QUANTUM CIRCUITS?

HOW DO WE TREAT QUANTUM-MECHANICALLY A DISSIPATIVE, NON-LINEAR, OUT-OF-EQUILIBRIUM ENGINEERED SYSTEM?
SELECTED BIBLIOGRAPHY

Books
Braginsky, V. B., and F. Y. Khalili, "Quantum Measurements" (Cambridge University Press, Cambridge, 1992)
Nielsen, M. and Chuang, I., "Quantum Information and Quantum Computation" (Cambridge, 2001)
Tinkham, M. "Introduction to Superconductivity" (2nd edition, Dover, New York, 2004)

Review articles and theses

END OF LECTURE