COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2009, 12 mai - 23 juin

CIRCUITS ET SIGNAUX QUANTIQUES (II)

QUANTUM SIGNALS AND CIRCUITS (II)

Quatrième Leçon / *Fourth Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow Enseignement > Sciences Physiques > Physique Mésoscopique > ....

or http://www.physinfo.fr/lectures.html ⟸ NEW!

PDF FILES OF ALL LECTURES ARE POSTED ON THESE WEBSITES

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expéri-mentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

09-IV-3

CONTENT OF THIS YEAR'S LECTURES

OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Non-linearity of Josephson tunnel junctions
3. Readout of qubits
4. Amplification of quantum signals and quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Defying the fine structure constant: Fluxonium qubit and the prospect of the observation of Bloch oscillations.

NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS

09-IV-4

LECTURE IV : READOUT OF SUPER-CONDUCTING ARTIFICIAL ATOMS

1. Readout resonators and circuits
2. Principle of dispersive readout
3. Calculation of cavity pull for transmon
4. Escaping from the Purcell effect

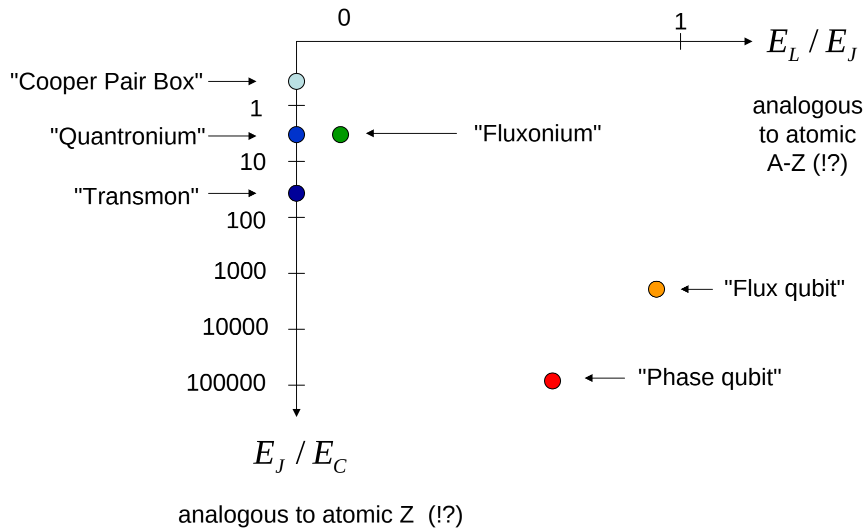
09-IV-5

OUTLINE

1. Readout resonators and circuits
2. Principle of dispersive readout
3. Calculation of cavity pull for transmon
4. Escaping from the Purcell effect

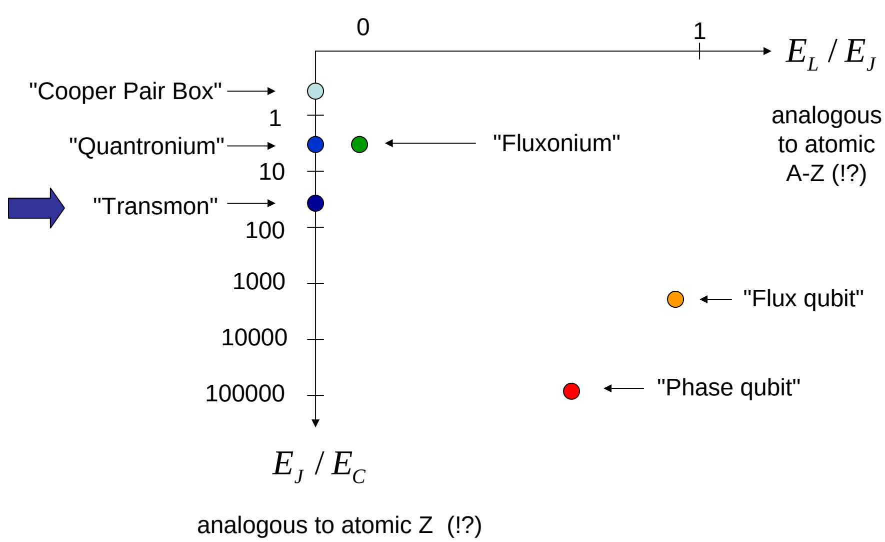
09-III-5a

THE CONTINUOUS "TABLE" OF SUPERCONDUCTING ARTIFICIAL ATOMS



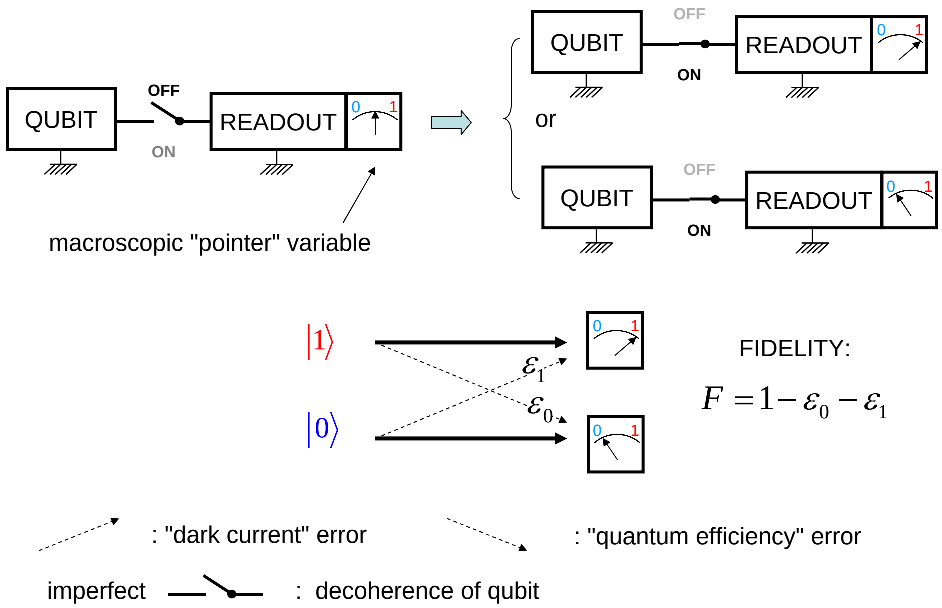
09-IV-6

THE CONTINUOUS "TABLE" OF SUPERCONDUCTING ARTIFICIAL ATOMS



09-IV-6a

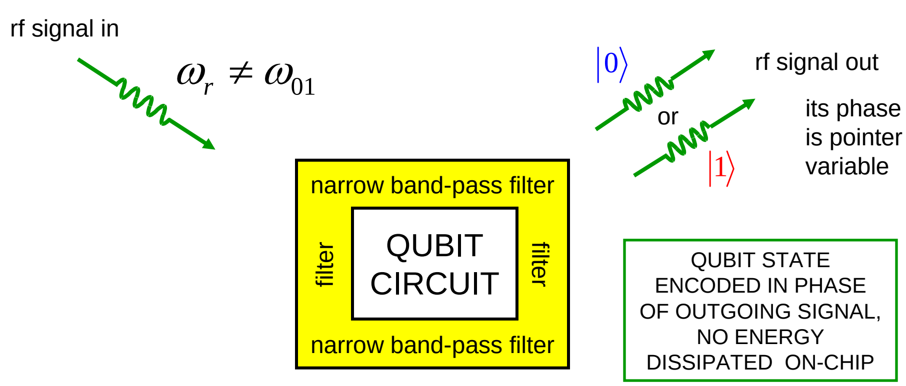
THE QUBIT MEMORY READOUT PROBLEM



09-IV-7

DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004

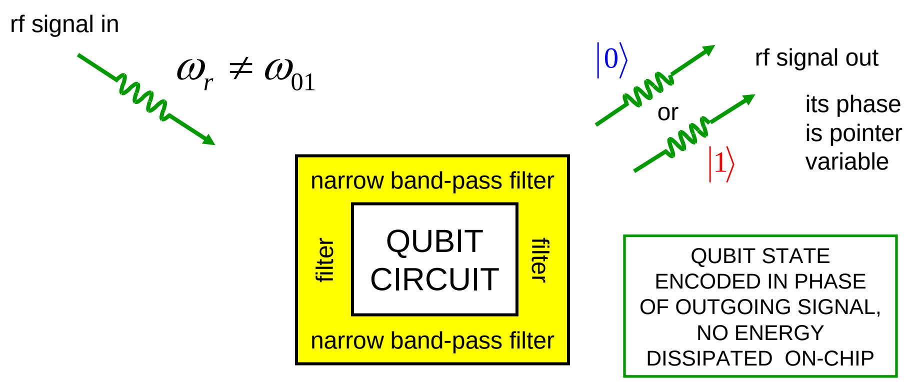


A) SHELTER QUBIT FROM ALL RADIATION EXCEPT READOUT RF @ ω_r
B) AMPLIFY OUTGOING SIGNAL WITH LOWEST ADDED NOISE
C) SEND ENOUGH PHOTONS TO BEAT ADDED NOISE

09-IV-8

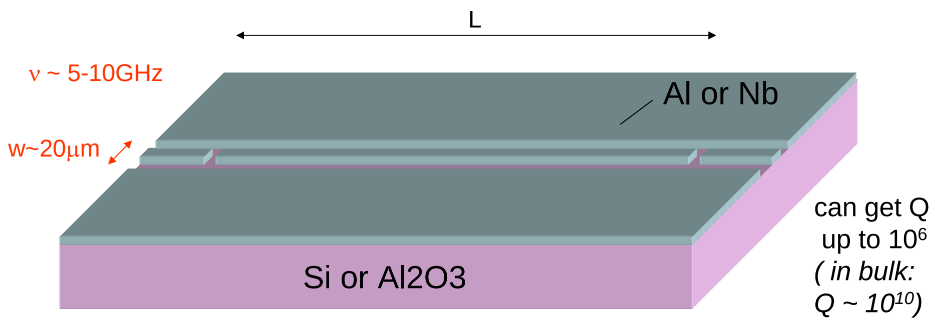
DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004



UNLIKE NATURAL ATOMS, ARTIFICIAL SUPERCONDUCTING ATOMS INTERACT ALMOST TOO MUCH WITH ELECTROMAGNETIC RADIATION

SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY



length L ~ 1cm, but photon decay length ~ 10km!

SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY

place transmon here
at a voltage antinode

(Walraff et al., 2004)

resonator acts as a filter rejecting
all frequencies which are not
a multiple of $v_p/(2L)$

09-IV-9a

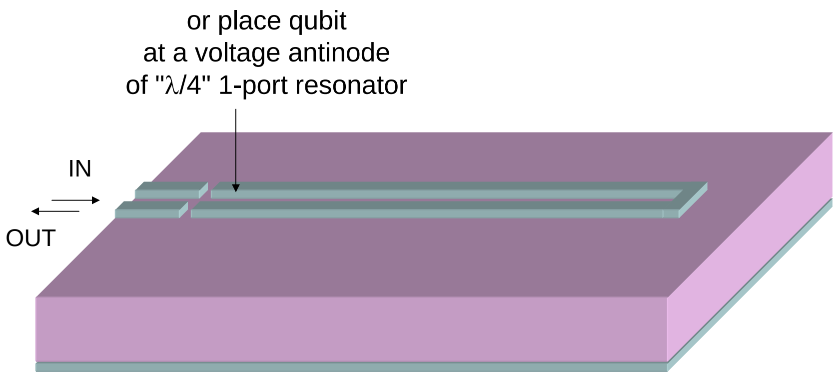
OTHER TYPE OF COUPLING

or place transmon here
at a voltage antinode of
a 1-port resonator

(Bertet et al., 2009)

09-IV-10

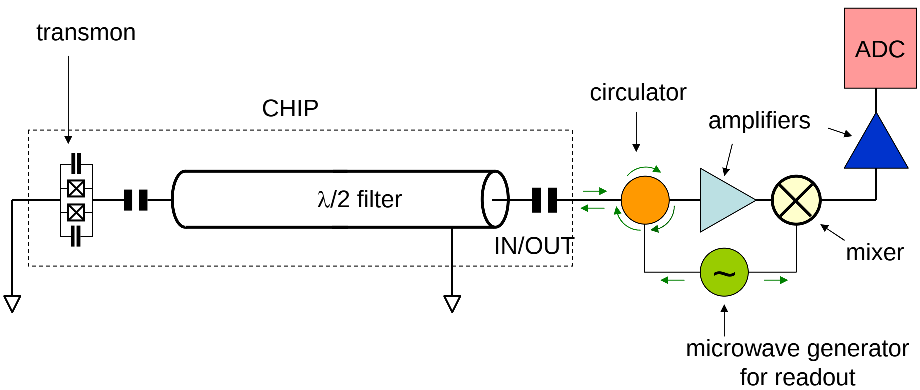
COUPLED MICROSTRIP RESONATOR



(Manucharyan et al., arXiv:0906.0831; 2009)

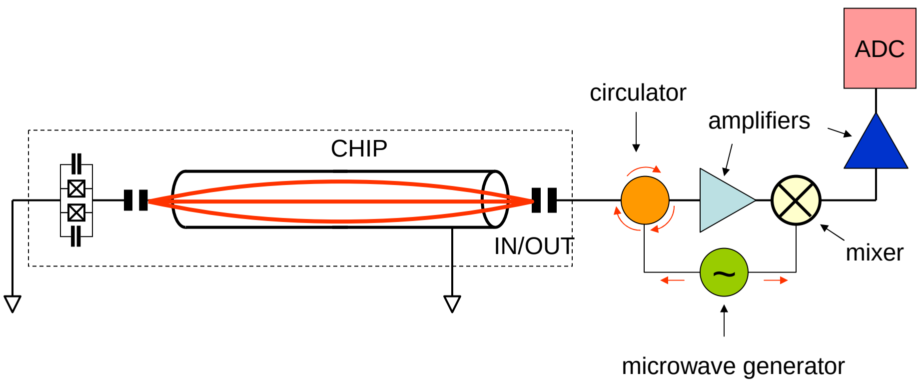
09-IV-11

TRANSMON WITH DISPERSIVE READOUT



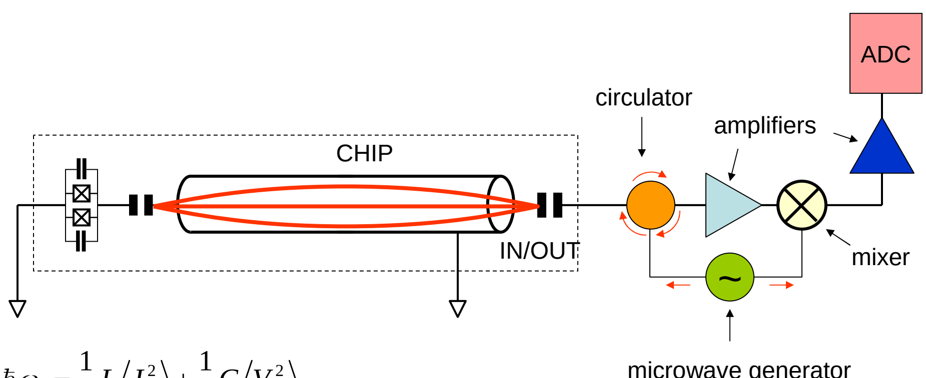
09-IV-12

HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?



09-IV-12a

HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?



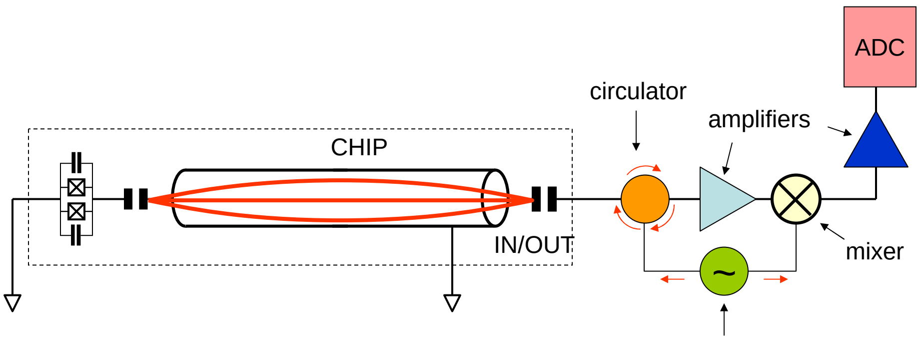
$$\hbar\omega_r = \frac{1}{2}L\left\langle I^2\right\rangle + \frac{1}{2}C\left\langle V^2\right\rangle$$

$$\frac{1}{\sqrt{LC}} \simeq \omega_r$$

$$\sqrt{\frac{L}{C}} \simeq Z_c$$

09-IV-12b

HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?



$$\hbar\omega_r = \frac{1}{2}L\langle I^2\rangle + \frac{1}{2}C\langle V^2\rangle$$

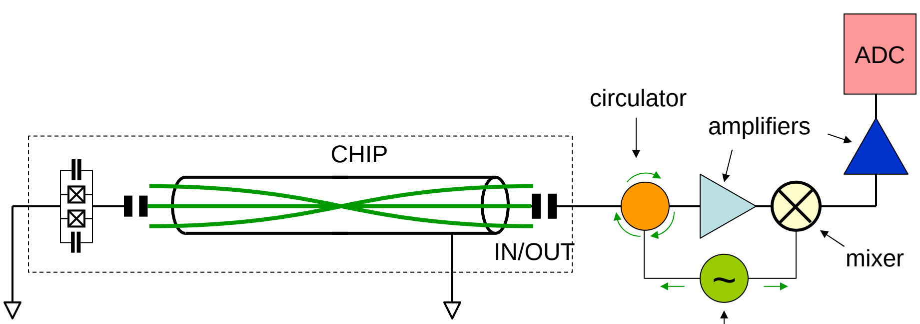
$$\frac{1}{\sqrt{LC}} \simeq \omega_r \qquad \langle I^2\rangle \simeq \hbar\omega_r^2 / Z_c$$

1 photon @ 10GHz: ~100nA

$$\sqrt{\frac{L}{C}} \simeq Z_c \qquad \langle V^2\rangle \simeq \hbar\omega_r^2 Z_c$$

09-IV-12c

HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?



$$\hbar\omega_r = \frac{1}{2}L\langle I^2\rangle + \frac{1}{2}C\langle V^2\rangle$$

$$\frac{1}{\sqrt{LC}} \simeq \omega_r \qquad \langle I^2\rangle \simeq \hbar\omega_r^2 / Z_c$$

1 photon @ 10GHz: ~100nA
~ 5μV

$$\sqrt{\frac{L}{C}} \simeq Z_c \qquad \langle V^2\rangle \simeq \hbar\omega_r^2 Z_c$$

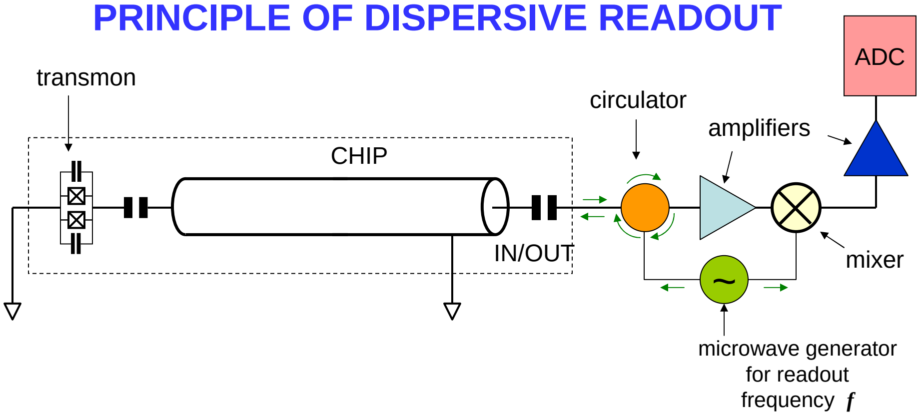
09-IV-12d

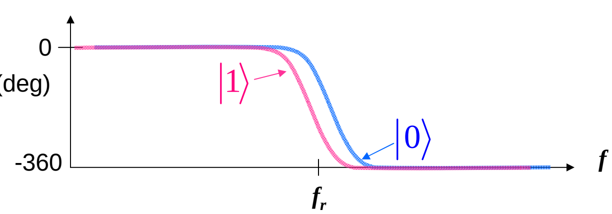
OUTLINE

1. Readout resonators and circuits
2. Principle of dispersive readout
3. Calculation of cavity pull for transmon
4. Escaping from the Purcell effect

09-III-5b

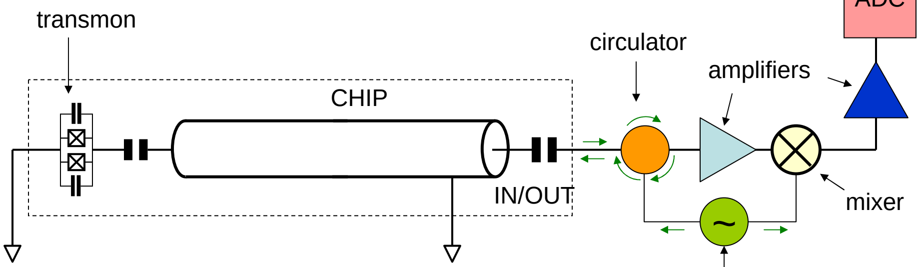
PRINCIPLE OF DISPERSIVE READOUT



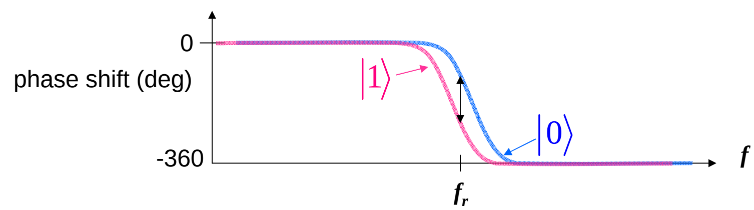


09-IV-13

PRINCIPLE OF DISPERSIVE READOUT

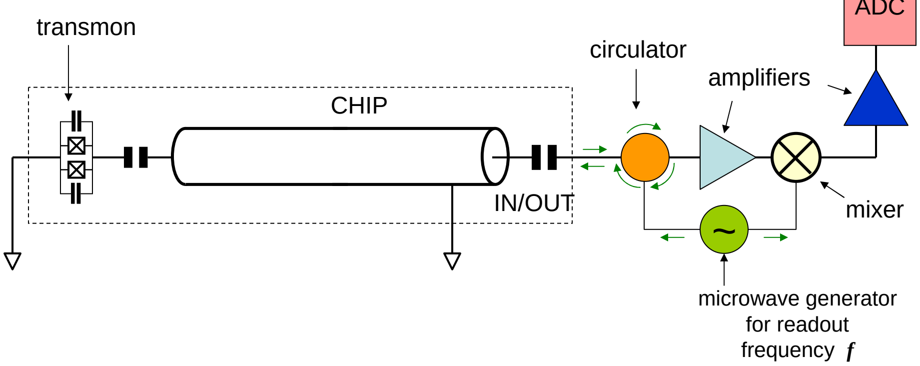


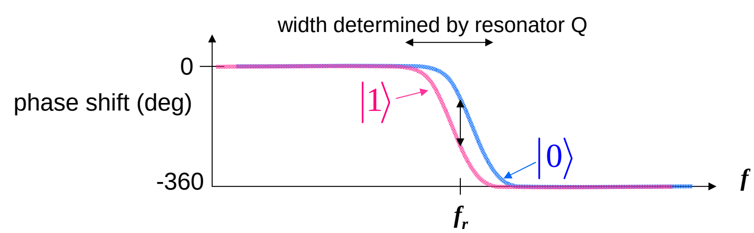
How well can we discriminate the qubit states?



09-IV-13a

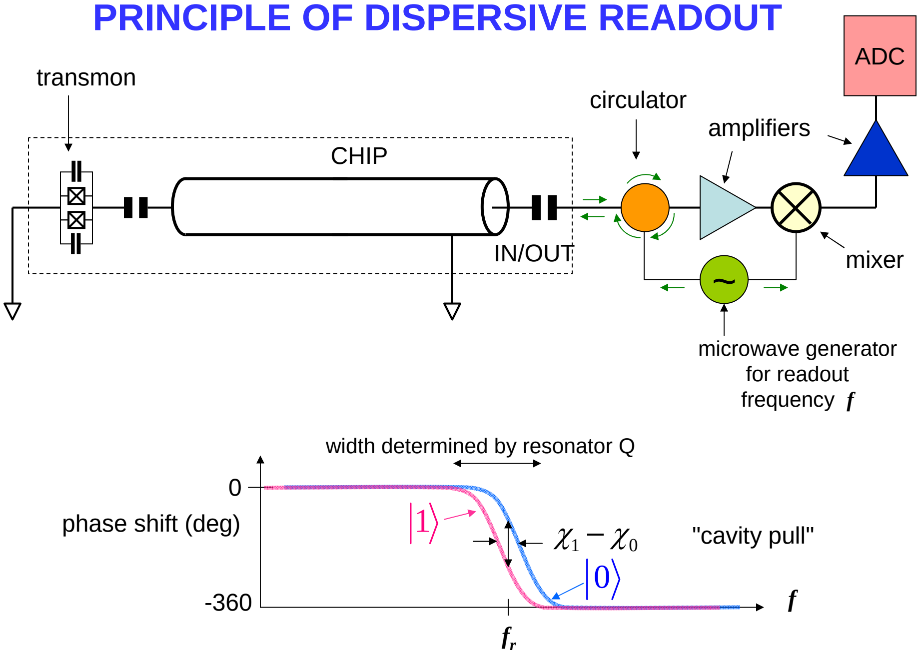
PRINCIPLE OF DISPERSIVE READOUT





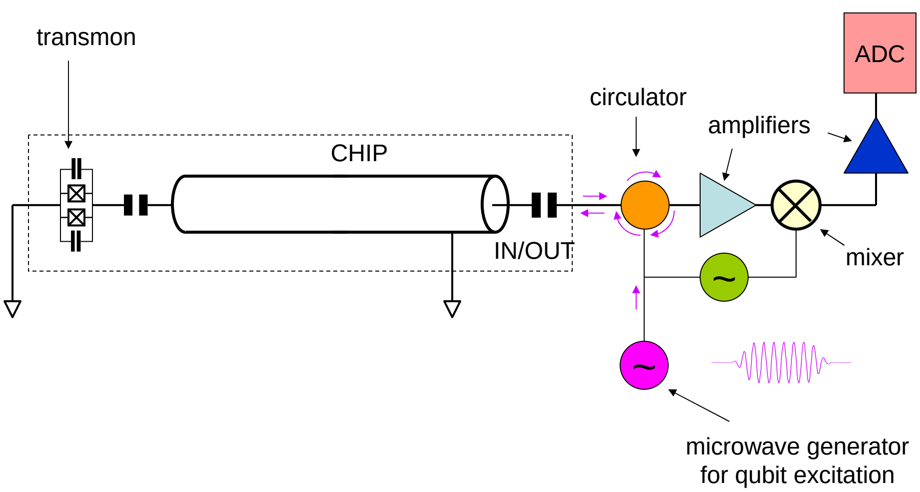
09-IV-13b

PRINCIPLE OF DISPERSIVE READOUT



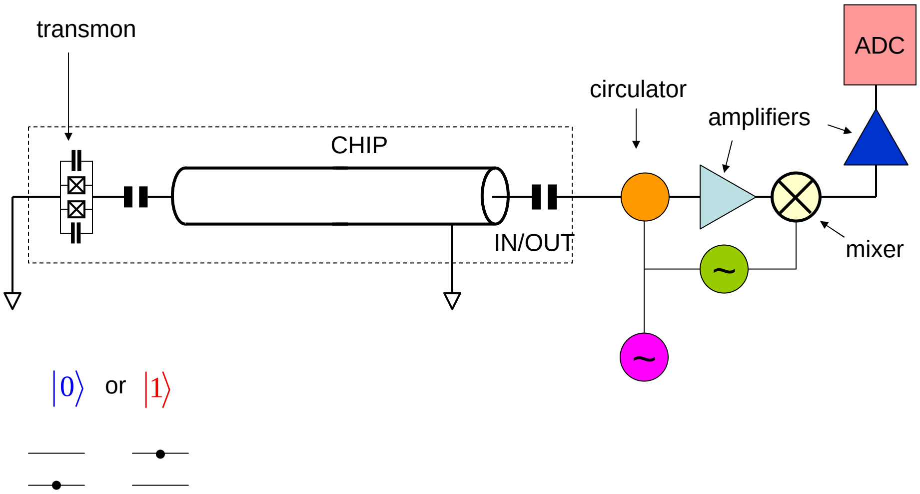
09-IV-13c

READOUT PROTOCOL: EXCITATION OF QUBIT



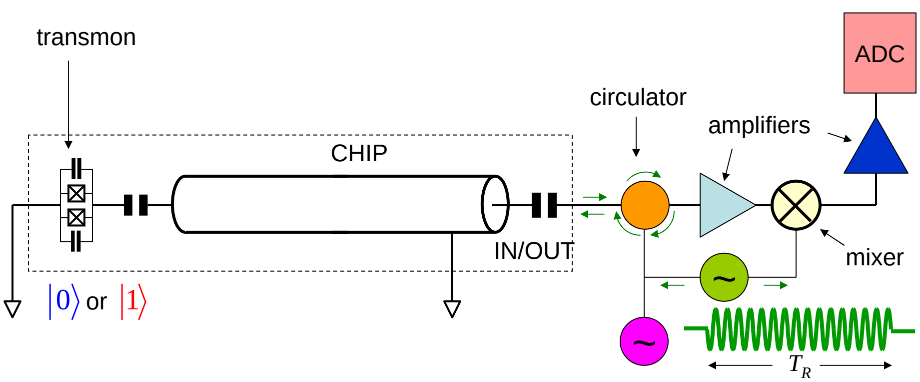
09-IV-14

READOUT PROTOCOL: EXCITATION OF QUBIT



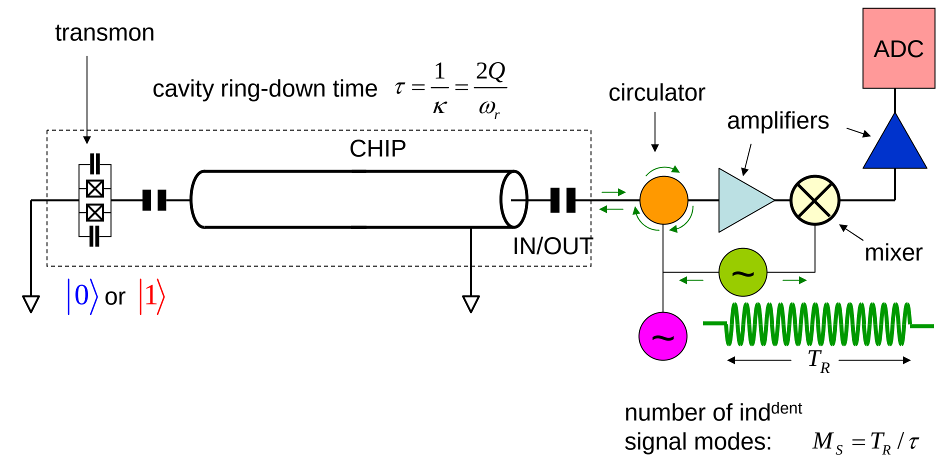
09-IV-14a

READOUT PROTOCOL: SEND READOUT CW TONE



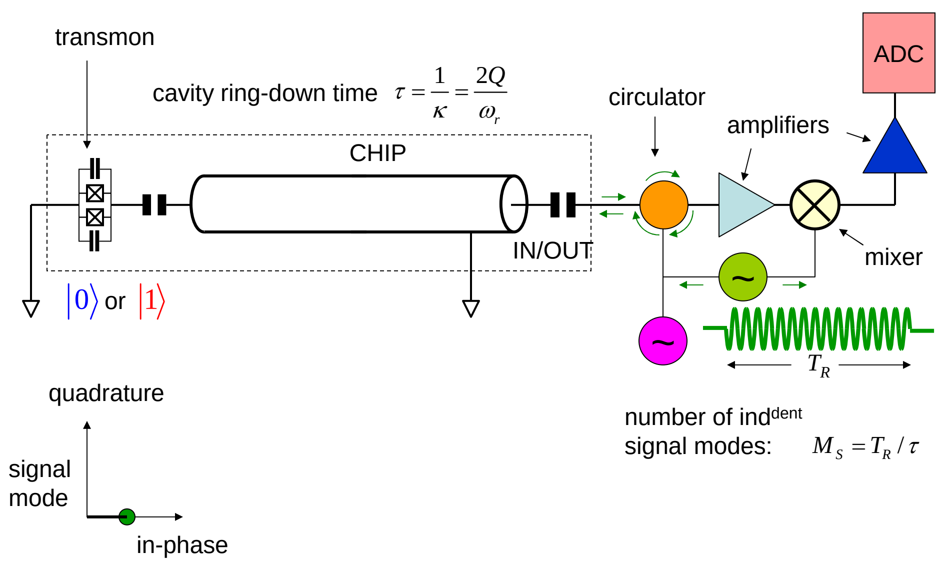
09-IV-15

READOUT PROTOCOL: SEND READOUT CW TONE



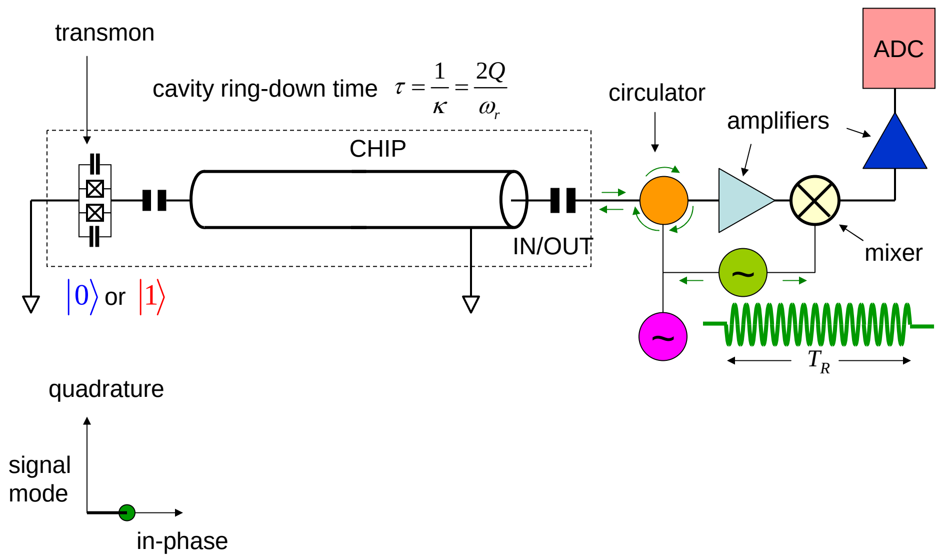
09-IV-15a

READOUT PROTOCOL: SEND READOUT CW TONE



09-IV-15b

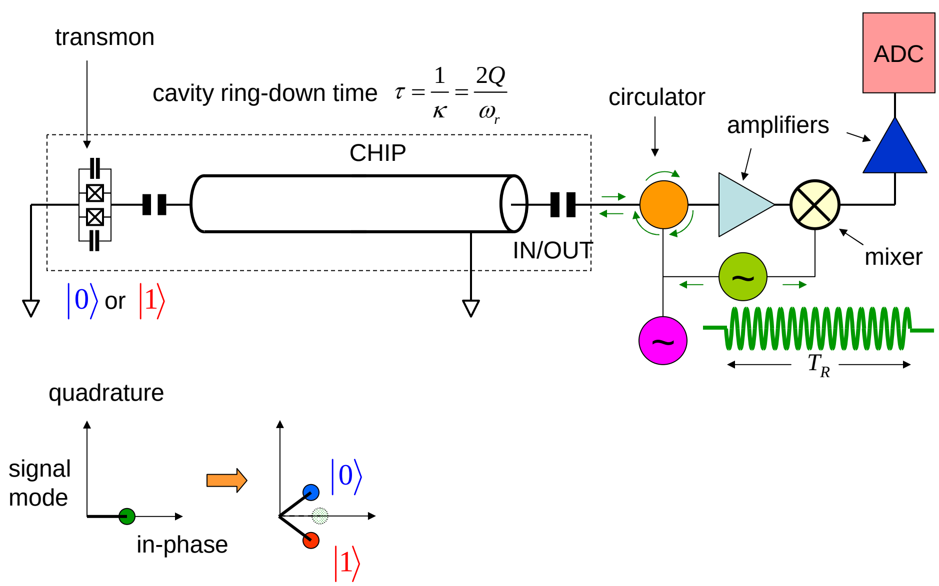
READOUT PROTOCOL: SEND READOUT CW TONE



number of ind$^{\text{dent}}$
signal modes: $M_S = T_R / \tau$

09-IV-15c

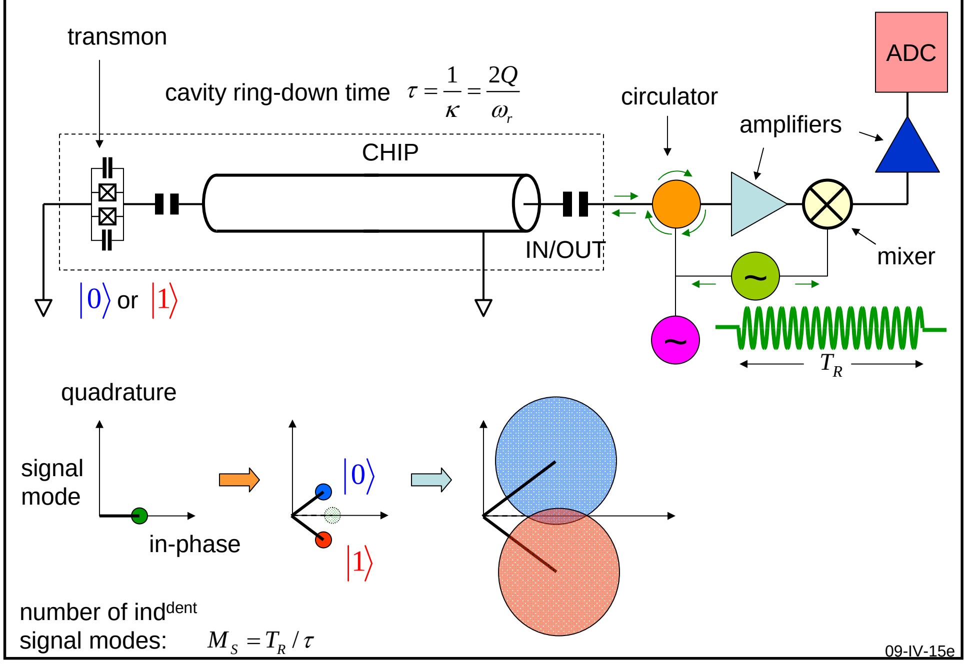
READOUT PROTOCOL: SEND READOUT CW TONE



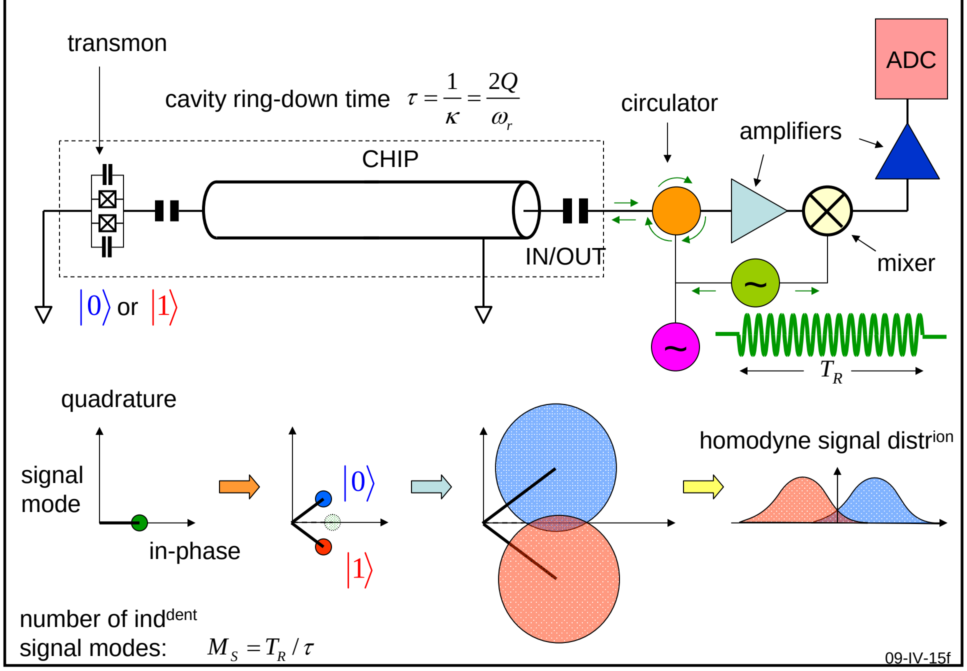
number of ind$^{\text{dent}}$
signal modes: $M_S = T_R / \tau$

09-IV-15d

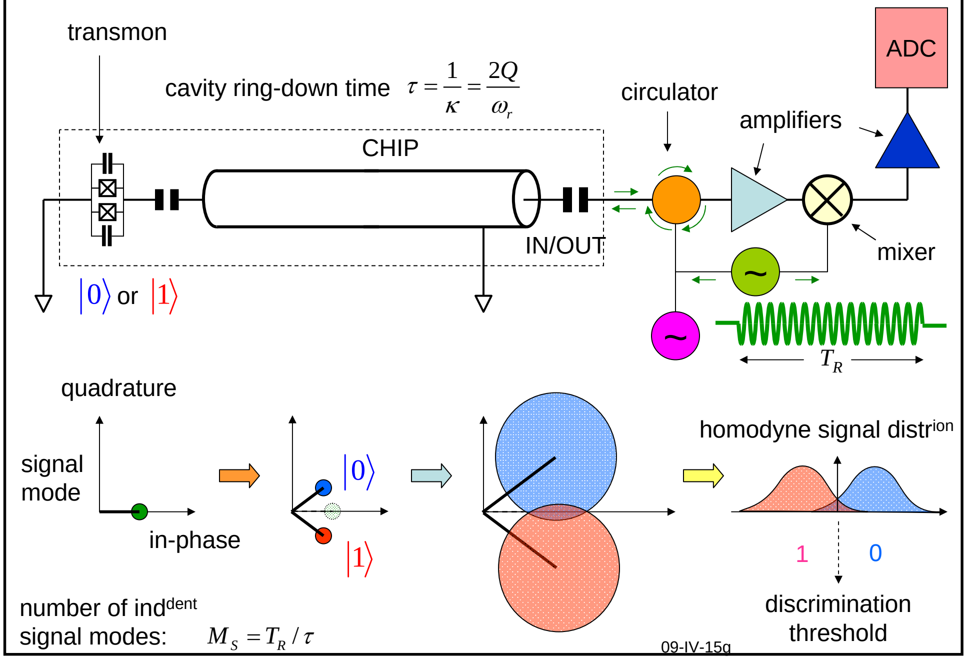
READOUT PROTOCOL: SEND READOUT CW TONE



READOUT PROTOCOL: SEND READOUT CW TONE

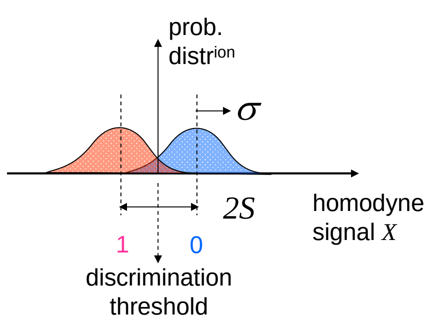


READOUT PROTOCOL: SEND READOUT CW TONE



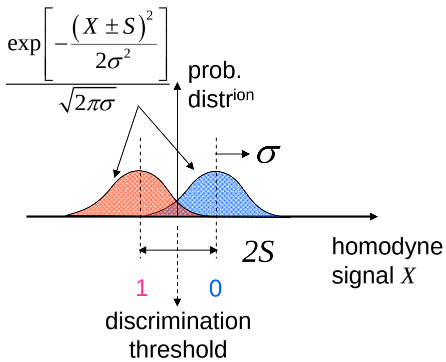
09-IV-15g

GAUSSIAN ANALYSIS OF READOUT FIDELITY



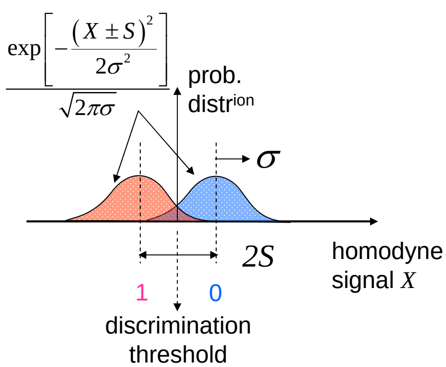
09-IV-16

GAUSSIAN ANALYSIS OF READOUT FIDELITY



09-IV-16a

GAUSSIAN ANALYSIS OF READOUT FIDELITY



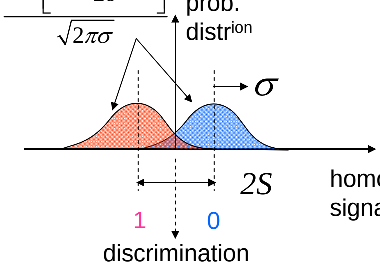
SIGNAL-TO-NOISE RATIO (AMPLITUDE):

$$\rho = \frac{2S}{\sqrt{2}\sigma}$$

09-IV-16b

GAUSSIAN ANALYSIS OF READOUT FIDELITY

$$\frac{\exp\left[-\frac{(X \pm S)^2}{2\sigma^2}\right]}{\sqrt{2\pi}\sigma}$$



SIGNAL-TO-NOISE RATIO (AMPLITUDE):

$$\rho = \frac{2S}{\sqrt{2}\sigma}$$

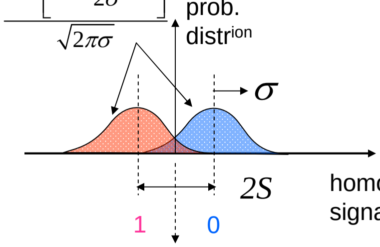
$$F = \mathrm{Erf}\left(\frac{S}{\sqrt{2}\sigma}\right) = \mathrm{Erf}(\rho/2)$$

$$\frac{S}{\sigma} = 1 \quad \Rightarrow \quad F = 0.683...$$

09-IV-16c

GAUSSIAN ANALYSIS OF READOUT FIDELITY

$$\frac{\exp\left[-\frac{(X \pm S)^2}{2\sigma^2}\right]}{\sqrt{2\pi}\sigma}$$



SIGNAL-TO-NOISE RATIO (AMPLITUDE):

$$\rho = \frac{2S}{\sqrt{2}\sigma}$$

cavity pull

ring-down time

\# of modes

$$\rho = \frac{(\chi_1 - \chi_0)\tau\sqrt{M_S}\sqrt{n_r}}{2\sqrt{n_A + 1/2}}$$

number of photons added by amplifier

number of readout photons

$$F = \mathrm{Erf}\left(\frac{S}{\sqrt{2}\sigma}\right) = \mathrm{Erf}(\rho/2)$$

$$\frac{S}{\sigma} = 1 \quad \Rightarrow \quad F = 0.683...$$

HERE, PHASE SHIFT IS SUPPOSED TO BE << 1

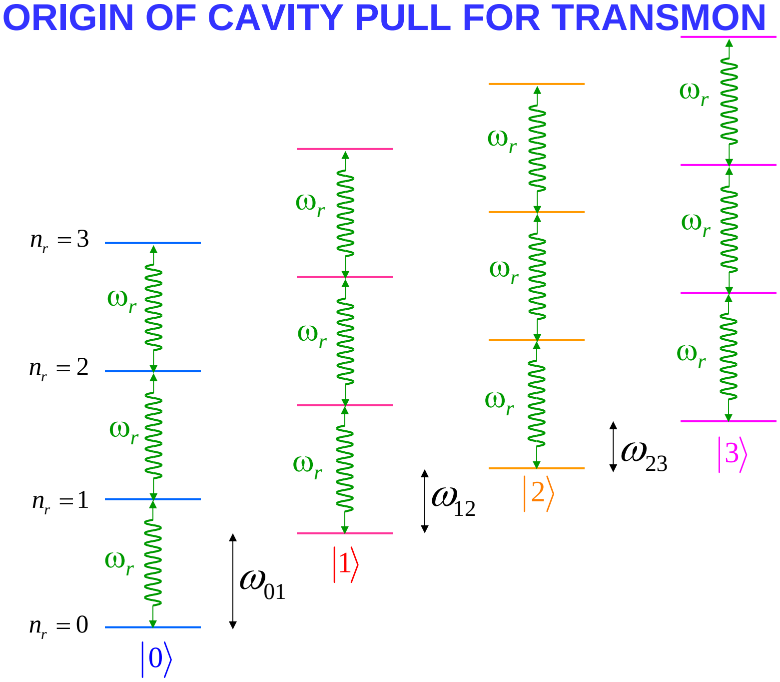
09-IV-16d

OUTLINE

1. Readout resonators and circuits
2. Principle of dispersive readout
3. Calculation of cavity pull for transmon
4. Escaping from the Purcell effect

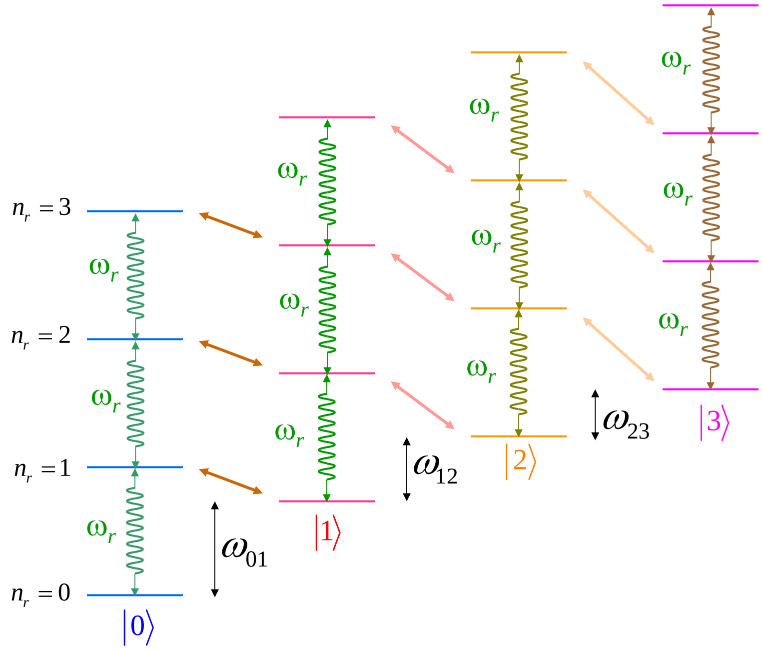
09-III-5c

ORIGIN OF CAVITY PULL FOR TRANSMON



09-IV-17

ORIGIN OF CAVITY PULL FOR TRANSMON



09-IV-17a

TRANSMON COUPLED TO A CAVITY

$$\hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}}$$

09-IV-18

TRANSMON COUPLED TO A CAVITY

$$\hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}}$$

$$\hat{H}_{\text{qubit}} = \hbar\omega_q \hat{c}^\dagger \hat{c} + \hbar \frac{\alpha}{2}\left(\hat{c}^\dagger \hat{c}\right)^2$$

$$\hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}}$$

$$\hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q}$$

09-IV-18a

TRANSMON COUPLED TO A CAVITY

$$\hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}}$$

$$\hat{H}_{\text{qubit}} = \hbar\omega_q \hat{c}^\dagger \hat{c} + \hbar \frac{\alpha}{2}\left(\hat{c}^\dagger \hat{c}\right)^2$$

$$\hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}}$$

$$\hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q}$$

$$\hat{H}_{\text{cavity}} = \hbar\omega_r \hat{a}^\dagger \hat{a}$$

$$\omega_r = \frac{1}{\sqrt{L_r C_r}}$$

$$\hat{H}_{\text{coupling}} = \hbar g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$g = \frac{C_c\sqrt{\omega_q\omega_r}}{2\sqrt{C_q C_r}}$$

see last year's (08) lecture II slide 24

09-IV-18b

TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{C}^\dagger \hat{C} + \omega_r' \hat{A}^\dagger \hat{A}$$

09-IV-19

TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

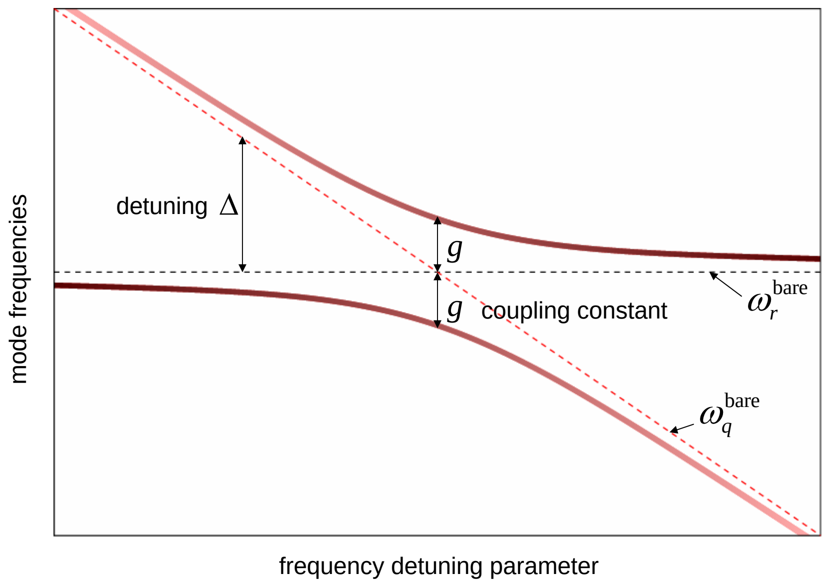
$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{C}^\dagger \hat{C} + \omega_r' \hat{A}^\dagger \hat{A}$$

In the dispersive limit $\Delta \gg g$ $\quad \omega_q' = \omega_q + \frac{g^2}{\Delta}; \;\; \omega_r' = \omega_r - \frac{g^2}{\Delta};$

$$\Delta = \omega_q - \omega_r$$

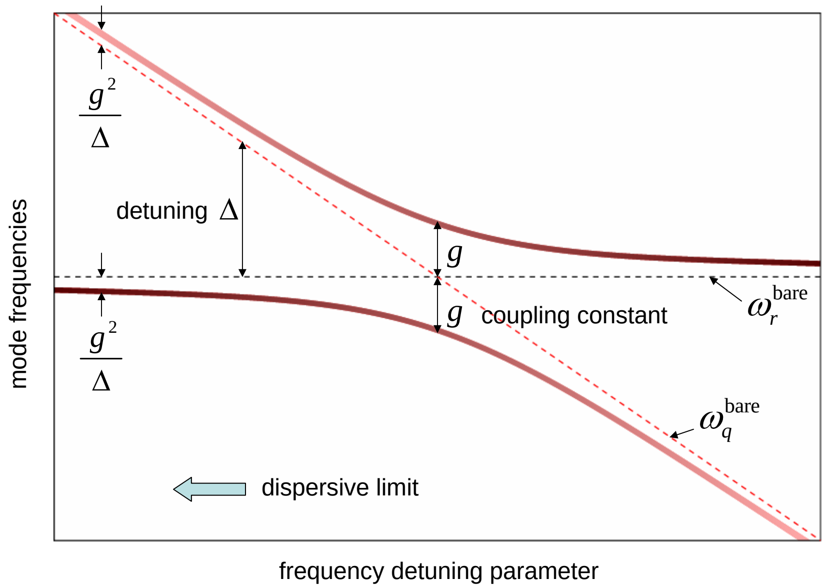
09-IV-19a

COUPLED OSCILLATORS



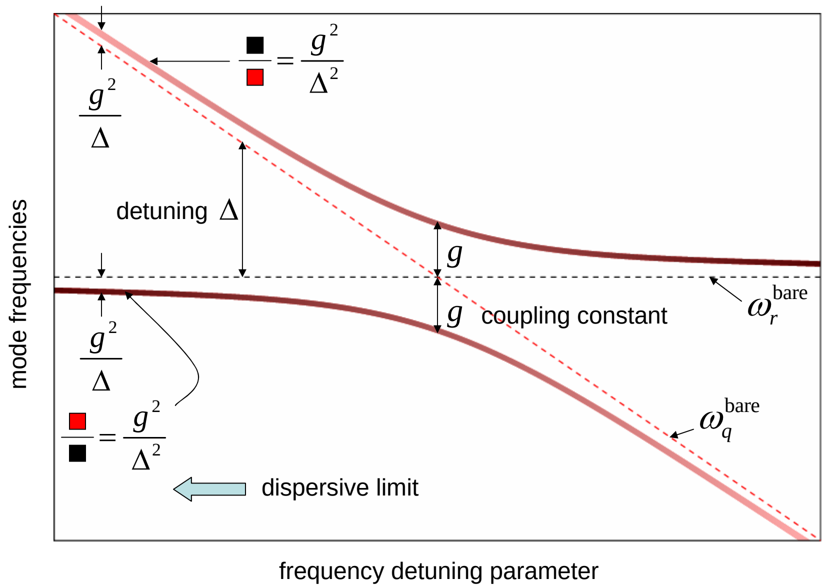
09-IV-20

COUPLED OSCILLATORS



09-IV-20a

COUPLED OSCILLATORS



09-IV-20b

TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{C}^\dagger \hat{C} + \omega_r' \hat{A}^\dagger \hat{A}$$

$$n_q = \hat{C}^\dagger \hat{C}$$

$$n_r = \hat{A}^\dagger \hat{A}$$

In the dispersive limit $\Delta \gg g$ $\quad \omega_q' = \omega_q + \frac{g^2}{\Delta}; \;\; \omega_r' = \omega_r - \frac{g^2}{\Delta};$

$$\Delta = \omega_q - \omega_r$$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega_q' n_q + \frac{1}{2}\alpha n_q^2 + \omega_r' n_r + \alpha \frac{g^2}{\Delta^2} n_q n_r$$

09-IV-21

TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{C}^\dagger \hat{C} + \omega_r' \hat{A}^\dagger \hat{A}$$

$$n_q = \hat{C}^\dagger \hat{C}$$

$$n_r = \hat{A}^\dagger \hat{A}$$

In the dispersive limit $\Delta \gg g$ $\quad \omega_q' = \omega_q + \frac{g^2}{\Delta}; \;\; \omega_r' = \omega_r - \frac{g^2}{\Delta};$

$$\Delta = \omega_q - \omega_r$$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega_q' n_q + \frac{1}{2}\alpha n_q^2 + \underbrace{\omega_r' n_r + \alpha \frac{g^2}{\Delta^2} n_q n_r}_{\left(\omega_r' + \alpha \frac{g^2}{\Delta^2} n_q\right) n_r = \left(\omega_r' + \chi_q\right) n_r}$$

09-IV-21a

READOUT FIDELITY vs QUBIT LIFETIME

non-linearity

coupling

cavity pull: $\quad \chi_1 - \chi_0 = \alpha \frac{g^2}{\Delta^2}$

detuning

09-IV-22

READOUT FIDELITY vs QUBIT LIFETIME

non-linearity

coupling

cavity pull: $$\chi_1 - \chi_0 = \alpha \frac{g^2}{\Delta^2}$$

detuning

relaxation via Purcell effect: $$\Gamma_1 = \kappa \frac{g^2}{\Delta^2}$$

cavity damping rate

09-IV-22a

READOUT FIDELITY vs QUBIT LIFETIME

non-linearity

coupling

cavity pull: $$\chi_1 - \chi_0 = \alpha \frac{g^2}{\Delta^2}$$

detuning

relaxation via Purcell effect: $$\Gamma_1 = \kappa \frac{g^2}{\Delta^2}$$

cavity damping rate

fidelity
and relaxation
rate tend
to be
antagonistic

$(\chi_1 - \chi_0)T_1$ independent of g & Δ

09-IV-22b

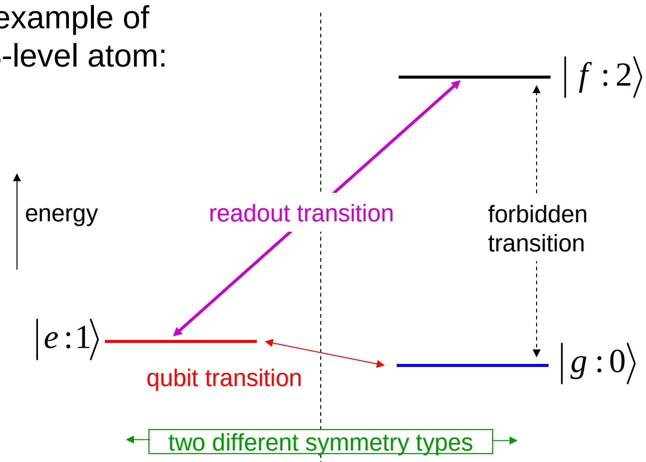
OUTLINE

1. Readout resonators and circuits
2. Principle of dispersive readout
3. Calculation of cavity pull for transmon
4. Escaping from the Purcell effect

09-III-5d

THE MERIT OF COMBINED STRONG ANHARMONICITY AND SYMMETRY

example of
3-level atom:

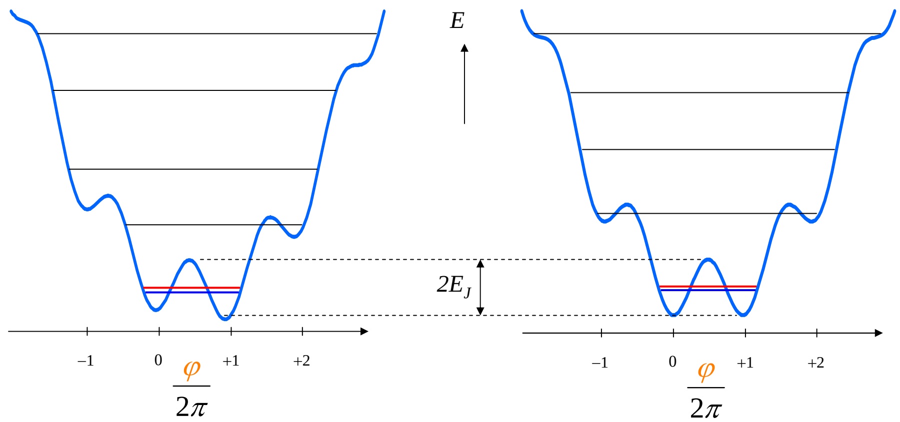


AVAILABILITY OF TWO ENERGY SCALES IS A USEFUL RESOURCE

09-IV-23

FLUXONIUM ATOM

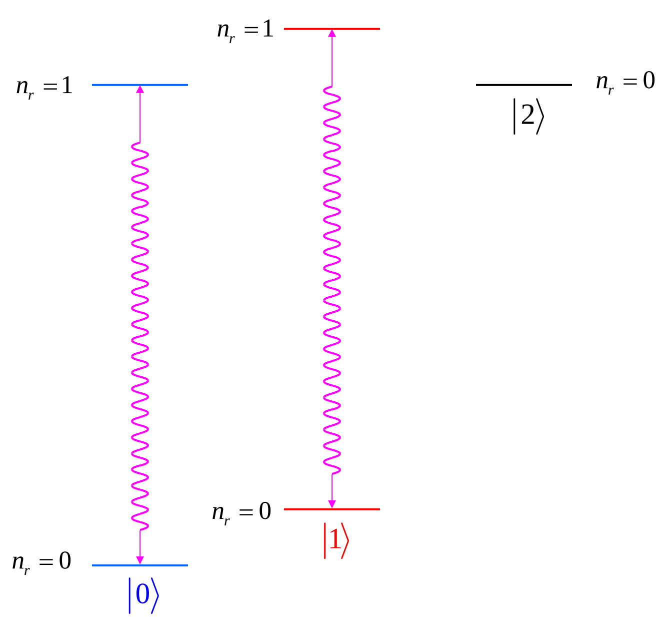
$$E_J > E_L \gg E_C$$



$\Phi_{\mathrm{ext}} = 0.6\,\Phi_0$ $\Phi_{\mathrm{ext}} = \frac{1}{2}\Phi_0$

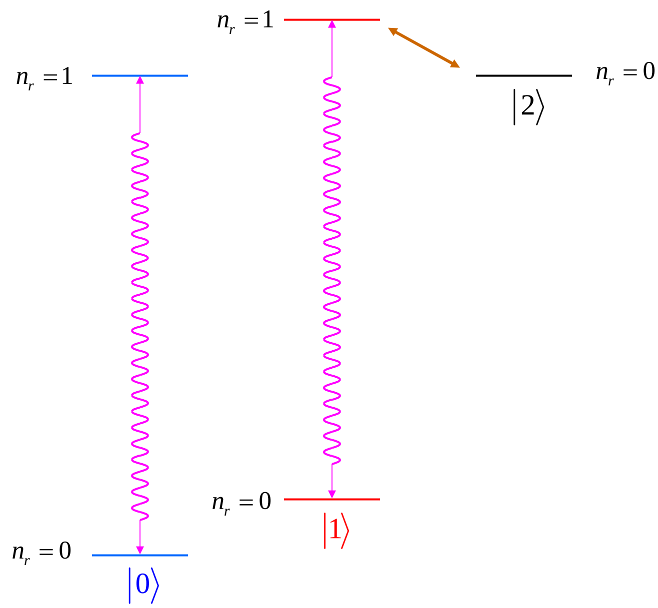
09-IV-24

3-STATE DISPERSIVE READOUT SCHEME



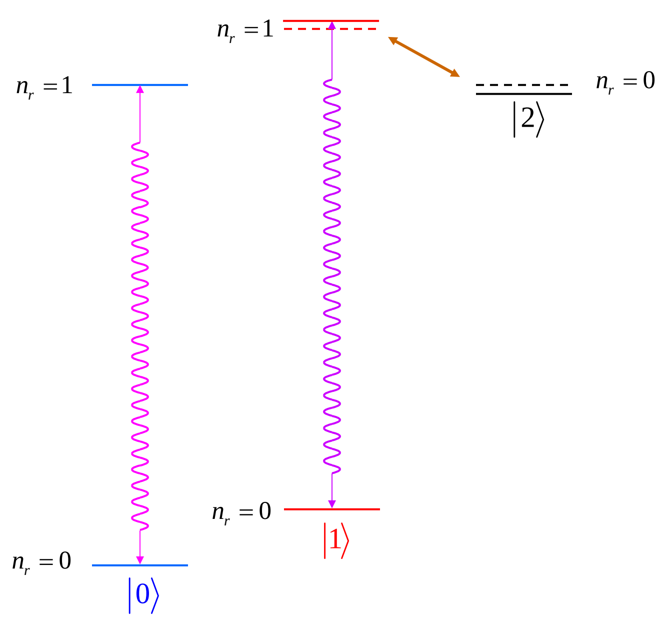
09-IV-25

3-STATE DISPERSIVE READOUT SCHEME



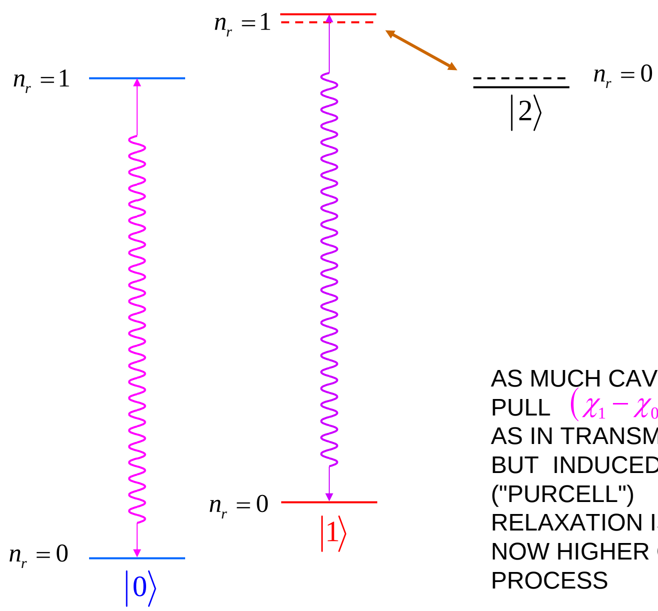
09-IV-25a

3-STATE DISPERSIVE READOUT SCHEME



09-IV-25b

3-STATE DISPERSIVE READOUT SCHEME



AS MUCH CAVITY
PULL $(\chi_1 - \chi_0)$
AS IN TRANSMON
BUT INDUCED
("PURCELL")
RELAXATION IS
NOW HIGHER ORDER
PROCESS

09-IV-25c

END OF PRESENTATION