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PDF FILES OF ALL LECTURES ARE POSTED ON THESE WEBSITES

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"
CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expéri-mentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

CONTENT OF THIS YEAR'S LECTURES

OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Non-linearity of Josephson tunnel junctions
3. Readout of qubits
4. Amplification of quantum signals and quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Defying the fine structure constant: Fluxonium qubit and the prospect of the observation of Bloch oscillations.

NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS
## LECTURE IV : READOUT OF SUPER-CONDUCTING ARTIFICIAL ATOMS

1. Readout resonators and circuits  
2. Principle of dispersive readout  
3. Calculation of cavity pull for transmon  
4. Escaping from the Purcell effect

## OUTLINE

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THE CONTINUOUS "TABLE" OF SUPERCONDUCTING ARTIFICIAL ATOMS

0 1
10
100
1000
10000
100000

"Cooper Pair Box" → 0
"Quantronium" → 1
"Transmon" → 10
"Fluxonium" →

$E_J / E_C$

"Flux qubit"

"Phase qubit"

analogous to atomic Z (?)

analogous to atomic A-Z (?)

09-IV-6a
THE QUBIT MEMORY READOUT PROBLEM

FIDELITY: \[ F = 1 - \varepsilon_0 - \varepsilon_1 \]

imperfect : "dark current” error
imperfect : “quantum efficiency” error
imperfect : decoherence of qubit

DISPERSSIVE READOUT STRATEGY


rf signal in \[ \omega_r \neq \omega_{01} \]

rf signal out

A) SHELTER QUBIT FROM ALL RADIATION EXCEPT READOUT RF @ \( \omega_r \)
B) AMPLIFY OUTGOING SIGNAL WITH LOWEST ADDED NOISE
C) SEND ENOUGH PHOTONS TO BEAT ADDED NOISE
UNLIKE NATURAL ATOMS, ARTIFICIAL SUPERCONDUCTING ATOMS INTERACT ALMOST TOO MUCH WITH ELECTROMAGNETIC RADIATION

SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY

\( \nu \sim 5-10\text{GHz} \)

\( w \sim 20\mu\text{m} \)

length \( L \sim 1\text{cm} \), but photon decay length \( \sim 10\text{km} \)!
SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY

place transmon here at a voltage antinode

resonator acts as a filter rejecting all frequencies which are not a multiple of $v_p/(2L)$

Walraff et al., 2004

OTHER TYPE OF COUPLING

or place transmon here at a voltage antinode of a 1-port resonator

Bertet et al., 2009
COUPLED MICROSTRIP RESONATOR

or place qubit at a voltage antinode of "\(\lambda/4\)" 1-port resonator

(Manucharyan et al., arXiv:0906.0831; 2009)

TRANSMON WITH DISPERSIVE READOUT

transmon

CHIP

\(\lambda/2\) filter

IN/OUT

circulator

amplifiers

mixer

microwave generator for readout

ADC
HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?

\[ h\omega = \frac{1}{2} L\langle i^2 \rangle + \frac{1}{2} C\langle V^2 \rangle \]

\[ \frac{1}{\sqrt{LC}} \approx \omega \]

\[ \sqrt{\frac{L}{C}} \approx Z_e \]
HOW MUCH CURRENT AND VOLTAGE FOR 1 MICROWAVE PHOTON?

\[ h\omega_c = \frac{1}{2} L \langle I^2 \rangle + \frac{1}{2} C \langle V^2 \rangle \]

\[ \frac{1}{\sqrt{LC}} \approx \omega_r \]
\[ \langle I^2 \rangle \approx h\omega_r^2 \frac{1}{Z_c} \]
\[ \langle V^2 \rangle \approx h\omega_r^2 Z_c \]

1 photon @ 10GHz: \( \sim 100nA \)

\[ \sqrt{\frac{L}{C}} \approx Z_c \]

\( \frac{h\omega}{\sqrt{LC}} \approx \Theta_r \)
OUTLINE

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PRINCIPLE OF DISPERSIVE READOUT

- microwave generator for readout frequency $f$
- circulator
- amplifiers
- mixer
- ADC
- transmon
- phase shift (deg) vs. $f$
How well can we discriminate the qubit states?

width determined by resonator Q

PRINCIPLE OF DISPERSIVE READOUT
**PRINCIPLE OF DISPERSIVE READOUT**

- Transmon
- Chip
- Circulator
- Amplifiers
- Mixer
- Microwave generator for readout frequency $f$
- Width determined by resonator $Q$
- Phase shift (deg)
- $|1\rangle - |0\rangle$
- "Cavity pull"

**READOUT PROTOCOL: EXCITATION OF QUBIT**

- Transmon
- Chip
- Circulator
- Amplifiers
- Mixer
- Microwave generator for qubit excitation
READOUT PROTOCOL: EXCITATION OF QUBIT

|0⟩ or |1⟩

READOUT PROTOCOL: SEND READOUT CW TONE

|0⟩ or |1⟩
READOUT PROTOCOL: SEND READOUT CW TONE

\[ \tau = \frac{1}{\kappa} = \frac{2q}{\omega} \]

number of incident signal modes: \( M_s = T_R / \tau \)
READOUT PROTOCOL: SEND READOUT CW TONE

transmon

cavity ring-down time \( \tau = \frac{1}{\kappa} = \frac{2Q}{\omega_0} \)

CHIP

IN/OUT

|0\rangle or |1\rangle

quadrature

signal mode

in-phase

number of ind\textsuperscript{ident} signal modes: \( M_\delta = T_\delta / \tau \)

READOUT PROTOCOL: SEND READOUT CW TONE

transmon

cavity ring-down time \( \tau = \frac{1}{\kappa} = \frac{2Q}{\omega_0} \)

CHIP

IN/OUT

|0\rangle or |1\rangle

quadrature

signal mode

in-phase

number of ind\textsuperscript{ident} signal modes: \( M_\delta = T_\delta / \tau \)
READOUT PROTOCOL: SEND READOUT CW TONE

transmon

cavity ring-down time \( \tau = \frac{1}{\kappa} = \frac{2Q}{\omega_c} \)

circulator

amplifiers

mixer

numeral of ind signal modes: \( M_s = T_s / \tau \)

\[ \begin{align*}
\text{in-phase} & \quad \langle 0 \rangle \\
\text{quadrature} & \quad \langle 1 \rangle 
\end{align*} \]

homodyne signal distributon

READOUT PROTOCOL: SEND READOUT CW TONE

transmon

cavity ring-down time \( \tau = \frac{1}{\kappa} = \frac{2Q}{\omega_c} \)

circulator

amplifiers

mixer

numeral of ind signal modes: \( M_s = T_s / \tau \)

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READOUT PROTOCOL: SEND READOUT CW TONE

transmon
cavity ring-down time \( \tau = \frac{1}{\kappa} = \frac{2\Omega}{\omega_c} \)
circulator
amplifiers
mixer
ADC

|0\rangle or |1\rangle

in-phase

quadrature

signal mode

number of ind\textsuperscript{dent} signal modes: \( M_s = T_R / \tau \)

cavity ring-down time

GAUSSIAN ANALYSIS OF READOUT FIDELITY

2\sigma

\text{prob. distr}^{\text{ion}}

\text{homodyne signal]\ X\ discrimination threshold}

\text{1} \quad \text{0}

\text{1} \quad \text{0}
GAUSSIAN ANALYSIS OF READOUT FIDELITY

\[ \exp \left[ \frac{-(X \pm S)^2}{2\sigma^2} \right] \]

prob. dist\textsuperscript{ion} $\rightarrow \sigma$

$1$ $0$

discrimination threshold

$2S$

homodyne signal $X$

\[ \frac{\rho}{\sqrt{2\pi}\sigma} \]

SIGNAL-TO-NOISE RATIO (AMPLITUDE):

\[ \rho = \frac{2S}{\sqrt{2\sigma}} \]
GAUSSIAN ANALYSIS OF READOUT FIDELITY

\[ \exp \left[ -\frac{(X \pm S)^2}{2\sigma^2} \right] \]

signal-to-noise ratio (amplitude):

\[ \rho = \frac{2S}{\sqrt{2\sigma}} \]

\[ F = \text{Erf} \left( \frac{S}{\sqrt{2\sigma}} \right) = \text{Erf} \left( \frac{\rho}{2} \right) \]

\[ \frac{S}{\sigma} = 1 \Rightarrow F = 0.683... \]

09-IV-16d

HERE, PHASE SHIFT IS SUPPOSED TO BE \(<\ 1\)
OUTLINE

1. Readout resonators and circuits
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ORIGIN OF CAVITY PULL FOR TRANSMON

\[
\begin{align*}
\omega_{r} & = \omega_{n} \\
\omega_{r} & = \omega_{n} \\
\omega_{r} & = \omega_{n} \\
\omega_{r} & = \omega_{n} \\
\end{align*}
\]
ORIGIN OF CAVITY PULL FOR TRANSMON

TRANSMON COUPLED TO A CAVITY

\[ \hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}} \]
TRANSMON COUPLED TO A CAVITY

\[ \hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}} \]

\[ \hat{H}_{\text{qubit}} = \hbar \omega_q \hat{c}^\dagger \hat{c} + \hbar \frac{\alpha}{2} \left( \hat{c}^\dagger \hat{c} \right)^2 \]

\[ \hbar \omega_q = \sqrt{8 E^\text{eff}_c} = \frac{\hbar}{\sqrt{L_c C_q}} \]

\[ \hbar \alpha = -E^\text{eff}_c = -\frac{e^2}{2C_q} \]

\[ \hat{H}_{\text{cavity}} = \hbar \omega_r \hat{a}^\dagger \hat{a} \]

\[ \omega_r = \frac{1}{\sqrt{L_r C_r}} \]

\[ \hat{H}_{\text{coupling}} = \hbar g \left( \hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger \right) \]

\[ g = \frac{C_e \sqrt{\omega_q \omega_r}}{2 \sqrt{C_q C_r}} \]

see last year's (08) lecture II slide 24
In the dispersive limit $\Delta \gg g$, 

$$\omega'_q = \omega_q + \frac{g^2}{\Delta}; \quad \omega'_r = \omega_r - \frac{g^2}{\Delta};$$

$$\Delta = \omega_q - \omega_r$$
COUPLED OSCILLATORS

frequency detuning parameter

mode frequencies

detuning $\Delta$

coupling constant $g$

$\omega_r^{bare}$

$\omega_q^{bare}$

dispersive limit

$\frac{g^2}{\Delta}$

$\frac{g}{\Delta}$
**COUPLED OSCILLATORS**

In the dispersive limit, the detuning \( \Delta \) is given by:

\[
\frac{\Delta}{\Delta} = \omega_q - \omega_r
\]

The coupling constant \( g \) is related to the detuning by:

\[
\frac{g^2}{\Delta} = \frac{\Delta}{\Delta}^2
\]

**TRANSMON + CAVITY, DISPERSIVE LIMIT**

The Hamiltonian is given by:

\[
\hat{H} = \omega_q \hat{c}^{\dagger} \hat{c} + \frac{1}{2} \alpha \left( \hat{c}^{\dagger} \hat{c} \right)^2 + \omega_r \hat{a}^{\dagger} \hat{a} + g \left( \hat{a}^{\dagger} \hat{c} + \hat{c}^{\dagger} \hat{a} \right)
\]

In the dispersive limit \( \Delta \gg g \), the Hamiltonian simplifies to:

\[
\hat{H}_{\text{lin}} = \omega_q' \hat{c}^{\dagger} \hat{c} + \omega_r' \hat{a}^{\dagger} \hat{a}
\]

\[
\frac{\hat{H}_{\text{lin}}}{\hbar} = \frac{\omega_q' n_q + \frac{1}{2} \alpha n_q^2 + \omega_r' n_r + \frac{g^2}{\Delta} n_q n_r}{\hbar}
\]

The bare mode frequencies are:

\[
\omega_q^\text{bare} = \omega_q - \Delta
\]

\[
\omega_r^\text{bare} = \omega_r - \Delta
\]

The coupling constant in the dispersive limit is:

\[
g = \sqrt{\frac{\Delta}{\Delta} \frac{\Delta}{\Delta}}
\]
TRANSMON + CAVITY, DISPERSIVE LIMIT

\[
\hat{H} = \omega_q \hat{c} \hat{c}^\dagger + \frac{1}{2} \alpha (\hat{c} \hat{c}^\dagger)^2 + \omega_q \hat{a} \hat{a}^\dagger + g (\hat{a} \hat{c}^\dagger + \hat{c} \hat{a}^\dagger)
\]

\[
\hat{H}_{\text{lin}} = \omega_q \hat{c} \hat{c}^\dagger + \omega_r \hat{a} \hat{a}^\dagger
\]

\[
n_q = \hat{c} \hat{c}^\dagger
\]

\[
n_r = \hat{a} \hat{a}^\dagger
\]

In the dispersive limit \( \Delta \gg g \), \( \omega_q' = \omega_q + \frac{g^2}{\Delta} \), \( \omega_r' = \omega_r - \frac{g^2}{\Delta} \).

\[
\hat{H}_{\text{eff}} = \omega_q' n_q + \frac{1}{2} \alpha n_q^2 + \omega_r' n_r + \left( \omega_r' + \frac{g^2}{\Delta^2} n_q \right) n_r
\]

\[
\Delta = \omega_q - \omega_r
\]

READOUT FIDELITY vs QUBIT LIFETIME

\[
\chi_1 - \chi_0 = \alpha \frac{g^2}{\Delta^2}
\]

cavity pull:

- non-linearity
- coupling
- detuning
READOUT FIDELITY vs QUBIT LIFETIME

\[ \chi_1 - \chi_0 = \frac{\alpha g^2}{\Delta^2} \]

cavity pull:

\[ \Gamma_1 = \kappa \frac{g^2}{\Delta^2} \]

relaxation via Purcell effect:

cavity damping rate

fidelity and relaxation rate tend to be antagonistic

\[ (\chi_1 - \chi_0) T_i \quad \text{independent of } g & \Delta \]
**OUTLINE**

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**THE MERIT OF COMBINED STRONG ANHARMONICITY AND SYMMETRY**

Example of 3-level atom:

- Qubit transition
- Readout transition
- Forbidden transition

Energy scales:

- $|e:1\rangle$
- $|g:0\rangle$
- $|f:2\rangle$

Availability of two energy scales is a useful resource.
**FLUXONIUM ATOM**

\[ E_J > E_L \gg E_C \]

- \[ \Phi = \frac{\phi_0}{2\pi} \]
- \[ \Phi_{ext} = 0.6 \Phi_0 \]
- \[ \Phi_{ext} = \frac{1}{2} \Phi_0 \]

**3-STATE DISPERSIVE READOUT SCHEME**

- \[ n_r = 1 \]
- \[ n_r = 0 \]
- \[ |0\rangle \]
- \[ |1\rangle \]
- \[ |2\rangle \]
3-STATE DISPERSIVE READOUT SCHEME

$n_r = 1$  
\[ |0\rangle \quad |1\rangle \quad |2\rangle \]

$n_r = 0$

09-IV-25a
3-STATE DISPERSIVE READOUT SCHEME

\[ n_r = 1 \quad n_r = 1 \quad n_r = 0 \quad n_r = 0 \]

\[ |0\rangle \quad |1\rangle \quad |2\rangle \]

AS MUCH CAVITY PULL \((\chi_1 - \chi_0)\) AS IN TRANSMON BUT INDUCED ("PURCELL") RELAXATION IS NOW HIGHER ORDER PROCESS

END OF PRESENTATION