LECTURE I : INTRODUCTION TO AMPLIFICATION AND FEEDBACK OF ENGINEERED QUANTUM SYSTEMS

CONTENTS
1. Measurements, noise and amplification
2. Caves' theorem, link with detection in quantum optics
3. Are amplifiers and photomultipliers equivalent?
4. Principle of Josephson parametric amplifiers
5. Using quantum amplifiers in mesoscopic physics
6. Measurement based-feedback
**ELECTRICAL TRANSPORT MEASUREMENT**

Probe signal

\[ I_p(t) = I_p \cos(\omega T_m) w(t) \]

Response signal

\[ V_r(t) = V_r \cos(\omega t + \phi) w(t) \]

Processing

\[ V_r \cos(\omega t + \phi) = \text{Re}[Z_{12}(\omega) e^{i\omega t} I_p] \]

Measurement value:

\[ M = \int_{-\infty}^{\infty} w(t) V_r(t) e^{i\omega t} dt = V_r [\omega] \]

\[ \equiv Z_{12}(\omega) I_p \int_{-\infty}^{\infty} w(t)^2 dt \]

\[ M = M_1 + iM_2 \] in-phase and quadrature components

Variations of \( M \) are acquired as a function of external drive and bias fields, and thermodynamic parameters like temperature.

**A FEW EXAMPLES FROM MESOSCOPIC PHYSICS**

probe → SAMPLE → response

- Tunnel junction
- Atomic point contact
- 2D-gas point contact
- Nanomagnetic system
- Qubit
- \( \mu \)w LC resonator
- Nano-mechanical resonator
**Noise Reduces the Information Extracted from Measurement**

\[
V_2(t) = V_r(t) + V_N(t)
\]

\[
M = V_r[\omega]_w + V_N[\omega]_w
\]

3 origins for noise: a) probe, b) sample, c) processing.

**Signal Energy**:
\[
E_s = \frac{1}{R} \int_{-\infty}^{\infty} V_r(t) \, dt \approx \frac{2}{R} |V_r[\omega]_w|^2
\]

**Noise Energy**:
\[
E_N = \frac{2}{R} |V_N[\omega]_w|^2
\]

**Signal-to-Noise Ratio**:
\[
\text{SNR} = \frac{E_s}{E_N}
\]

**Shannon Information**:
\[
I = \ln_2 \left(1 + \frac{E_s}{E_N}\right) \quad \text{(strict validity requires noise be Gaussian)}
\]

**The Noise of One Physicist...**

... *may be the signal of another*

**Noise Spectral Density**:
\[
S_{V_N}[\omega] = \lim_{T_{_w} \to \infty} \frac{\langle |V_N[\omega]_w|^2 \rangle}{\int_{-\infty}^{\infty} w(t)^2 \, dt}
\]

\[
\langle V_N[\omega] V_N[\omega_2] \rangle = 2\pi \delta(\omega + \omega_2) S_{V_N}[\omega]
\]

**Stationarity**:
\[
V_N[\omega] = \int_{-\infty}^{\infty} V_N(t)e^{i\omega t} \, dt
\]

**Wiener-Kinchin Theorem**:
\[
S_{V_N}[\omega] = \int_{-\infty}^{\infty} e^{i\omega \tau} \langle V_N(t+\tau)V_N(t) \rangle \, d\tau
\]

**In Thermal Equilibrium at** \( T \):
\[
S_{V_N}[\omega] = \hbar \omega \left(1 + \coth \frac{\hbar \omega}{2k_b T} \right) \text{Re} \left[ Z_{22}(\omega) \right]
\]

**Fluctuation-Dissipation Theorem**:
\[
\hbar \omega \ll k_b T \Rightarrow S_{V_N}[\omega] = 2k_b T \text{Re} \left[ Z_{22}(\omega) \right] \quad \text{Johnson-Nyquist}
\]

\[
\hbar \omega \gg k_b T \Rightarrow S_{V_N}[\omega] = \hbar \omega + \hbar \omega
\]

Out-of-equilibrium, noise can reveal useful information on steady-state properties.
Noise in measurement has 3 origins: 

a) lack of control of probe

b) sample itself

c) processing → cause removed by amplification

Idealized representation of op-amp type amplifier:

AMPLIFIERS REMOVE NOISE IN PROCESSING STAGE BUT ADD NOISE OF THEIR OWN

2 added noises, characterized by 3 spectral densities:

\[ S_N[\omega], S_{VV}[\omega], S_{IV}[\omega] \]

Total noise energy:

\[ E_N^2 = \frac{2}{R} \left( |Z|^2 S_N[\omega] + S_{VV}[\omega] + 2 \text{Re}[Z(\omega)S_{IV}[\omega]] \right) \]

NOISE TEMPERATURE

\[ T_N^R = \frac{E_N^R}{4k_B} \]  

: equivalent temperature of load giving the same output noise

\[ k_B T_N^{R,X} = \frac{(R^2 + X^2)}{2} S_N + \frac{1}{2R} S_{VV} + \text{Re}[S_{IV}] - \frac{X}{R} \text{Im}[S_{IV}] \]

Optimize \( X \):

\[ k_B T_N^R = \frac{R}{2} S_N + \frac{1}{2R} S_{VV} - \frac{[\text{Im}[S_{IV}]]^2}{2RS_N} + \text{Re}[S_{IV}] \quad X^w = \frac{\text{Im}[S_{iv}]}{S_N} \]
NOISE TEMPERATURE

\[ T_N^R = \frac{E_N^R}{4k_B} \]  : equivalent temperature of load giving the same output noise
temperature of input load

noise power at output of amplifier

quantum regime

\[ \frac{T_N^R}{T^R} = \frac{\hbar \omega}{k_B} \]

Results are easily generalized to finite input impedance amplifiers

OPTIMUM NOISE TEMPERATURE
OF AMPLIFIER

\[ \log T_N^R \]

\[ \log T_{N_{\text{opt}}} \]

\[ R_{\text{opt}} = \sqrt{\frac{S_{11}}{S_{22}} - \frac{\text{Im}[S_{12}]}{(S_{22})^2}} = R_N \]

noise resistance of amplifier

\[ T_{N_{\text{opt}}} \] is intrinsic to the amplifying process itself

variations of noise with source impedance

Results are easily generalized to finite input impedance amplifiers
**THE QUANTUM LIMIT OF AMPLIFICATION**

Optimal noise energy:  \[ k_B T_N^{\text{opt}} = \sqrt{S_{VV} S_{NN} - [\text{Im} S_{NV}]^2} - \text{Re} S_{NV} \]

**QUANTUM LIMIT**

\[ k_B T_N^{\text{opt}} \geq \frac{\hbar \omega}{2} \]

(Caves, 1982)

The minimum noise energy is equivalent to half-a-photon at the signal frequency.

Relevant in RF mesoscopic measurements: \( f = 10 \text{GHz} \) implies \( T_N > 250 \text{mK} \gg T_{\text{bath}} \)

Can be understood as a generalization of the Fluctuation-Dissipation Theorem to active systems (Clerk et al., 2010)

**CAVEAT:** IT IS POSSIBLE TO PERFORM OTHER KINDS OF AMPLIFICATION WITHOUT NOISE

**DIFFERENT DESCRIPTIONS OF A SIGNAL**

**CURRENT**

**VOLTAGE**

**WAVE AMPLITUDE**

\[ A^\pm (t) = \frac{V(t)}{2 \sqrt{Z_c}} \pm \frac{Z_c I(t)}{2} \]

\[ A^+(t + \frac{d}{v}) = A^+(t) \]

\[ |A^+|^2 - |A^-|^2 = P^+ - P^- = P : \text{total energy flux through a section of line} \]
SPATIAL MODE DESCRIPTION:
SIGNAL PORTS

notation: port index vs signal amplitude symbol

REPRESENTATIONS OF A LINEAR, PHASE-PRESERVING AMPLIFIER

OP-AMP

\[
\begin{bmatrix}
V_b \\
I_a
\end{bmatrix} = \begin{bmatrix}
g_V & z_{output} \\ y_{input} & g_I
\end{bmatrix} \begin{bmatrix}
V_a \\
I_b
\end{bmatrix} + \begin{bmatrix}
g_v V_N \\
I_N
\end{bmatrix}
\]

ADMITTANCE

\[
\begin{bmatrix}
I_a \\
I_b
\end{bmatrix} = \begin{bmatrix}
y_{aa} & y_{ab} \\ y_{ba} & y_{bb}
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b
\end{bmatrix} + \begin{bmatrix}
I_{aN} \\
I_{bN}
\end{bmatrix}
\]

SCATTERING

\[
\begin{bmatrix}
A_{out}^a \\
B_{out}^b
\end{bmatrix} = \begin{bmatrix}
r_{aa} & t_{ab} \\ t_{ba} & r_{bb}
\end{bmatrix} \begin{bmatrix}
A_{in}^a \\
B_{in}^b
\end{bmatrix} + \begin{bmatrix}
A_{N}^{out} \\
B_{N}^{out}
\end{bmatrix}
\]

Noise:

\[
\begin{align*}
S_{aa}\{\omega\} & \\
S_{bb}\{\omega\} & \\
S_{ab}\{\omega\}
\end{align*}
\]

\[
|t_{ba}|^2 = G(\omega) : \text{Power gain}
\]
MICROWAVE AMPLIFIER CHARACTERISTICS

REPRESENTATION OF MATCHED 2-PORT AMPLIFIER

- Forward power gain \( G = \frac{P_{b_{\text{out}}}}{P_{a_{\text{in}}}} \)
- Added noise temperature \( T_{\text{add}} = \frac{S_{bb}}{Gk_B} = T_N \)
- Backaction noise temperature \( T_{\text{back}} = \frac{S_{a_b}}{Gk_B} \)
- Correlation temperature \( T_{\text{corr}} = \frac{|S_{ab}|}{k_B} \)
- Directionality \( D = \frac{G}{g} \)
- Signal bandwidth \( BW \)
- Tuning bandwidth
- Dynamic range

Commercial HEMT amplifiers: \( T_N \sim 5K, BW \sim 4\text{GHz@10GHz} \)

CLASSIFICATION OF AMPLIFIERS

"PHASE-INSENSITIVE" AMPLIFIERS:

Phase preserving: \[ B_{\text{out}}^\text{out} = \sqrt{G} A_{\text{in}}^\text{in} + B_N^\text{out} \]

Phase conjugating: \[ B_{\text{out}}^\text{out} = \sqrt{G} \left( A_{\text{in}}^\text{in} \right)^* + B_N^\text{out} \]

Gain is independent of signal phase

"PHASE-SENSITIVE" AMPLIFIERS:

\[ B_{\text{out}}^\text{out} = \sqrt{H} A_{\text{in}}^\text{in} + \sqrt{K} \left( A_{\text{in}}^\text{in} \right)^* + B_N^\text{out} \]

\[ = \sqrt{G} A_{\parallel}^\text{in} + i\sqrt{G} A_{\perp}^\text{in} + B_N^\text{out} \]

Gain depends on signal phase
GEOMETRIC REPRESENTATION OF SIGNAL MODE

\[ \text{Im} \left[ a_\mu \right] = a_\mu^\perp \]

\[ \text{Re} \left[ a_\mu \right] = a_\mu^\parallel \]

\( \mu \) : mode index (spatial and temporal)

\( N \) = signal mode energy in photon number

\( \theta \) = signal mode phase

FRESNEL VECTOR \[ \rightarrow \] FRESNEL "LOLLYPOP"

Classical \( A_\mu \rightarrow \) Quantum \( a_\mu \)

Thermal equilibrium \( \sigma_N = \sqrt{\frac{1}{2} \coth \frac{h \omega}{2 k_B T}} \)

QUANTUM LIMITED AMPLIFICATION WITH A LINEAR, PHASE-PRESERVING AMPLIFIER

STANDARD QUANTUM LIMIT: AMPLIFIER ADDS ONLY ANOTHER \( \frac{1}{2} \) PHOTON OF NOISE!
MINIMUM REQUIRED BY HEISENBERG PRINCIPLE FOR A MEASUREMENT OF BOTH QUADRATURES

\( A_{\text{HMTs}} \approx 20 \quad A_{\text{JPA}} < 1 \)

Shimoda, Takahasi and Townes, J. Phys. Soc. Jpn. 12, 686 (1957); Haus and Mullen, Phys. Rev. 128, 2407 (1962);
AMPLIFICATION AT THE QUANTUM LIMIT WITH A LINEAR, PHASE-SENSITIVE AMPLIFIER

\[ a^\perp \quad \sqrt{N} \quad \frac{1}{\sqrt{2}} \quad a^\parallel \]

\[ b^\perp \quad \sqrt{N}' \quad \sqrt{N} \quad b^\parallel \]

de-amplification \[ G_\perp \]

amplification \[ G_\parallel \]

\[ A_{\perp}^\text{min} A_{\parallel}^\text{min} = \frac{1}{16} \left(1 - G_\parallel^{-1/2} G_\perp^{-1/2}\right)^2 \]

QUANTUM LIMIT: NO ADDED NOISE (!) + SQUEEZING OF QUANTUM FLUCTUATIONS

(Caves, 1982)

RELATIONSHIP BETWEEN PHASE-PRESERVING AND PHASE-SENSITIVE AMPLIFICATION

\[ \phi \quad \phi + \pi/2 \]

phase sensitive amplifiers with two # phase references

UNVOIDABLE ADDED NOISE OF PHASE-PRESERVING AMPLIFIER CAN BE UNDERSTOOD AS AMPLIFIED QUANTUM NOISE OF HIDDEN PORT OF BEAM SPLITTER. HOWEVER A MORE PRECISE MODEL OF NOISE IS NEEDED TO OPTIMIZE THE AMPLIFIER
LINK WITH QUANTUM OPTICS

DIRECT DETECTION

atom(s)

photon counting

HOMODYNE DETECTION

intensity msmt

DIRECT DETECTION

HOMODYNE DETECTION

PARAMETRIC AMPLIFIER RESOURCE:
JUNCTION NON-LINEAR INDUCTANCE

\[ L_J = \frac{h^2}{(2e)^2 E_J} \]
\[ \delta L_J = L_J \left[ \left( 1 - \frac{I^2}{I_0^2} \right)^{-1} - 1 \right] \]
\[ = \frac{1}{2} L_J \left( \frac{I^2}{I_0^2} + O \left( \frac{I^4}{I_0^4} \right) \right) \]
MECHANICAL NANORESONATOR MEASURED WITH A JOSEPHSON PARAMP


FORCE SENSITIVITY MEASUREMENT

OBSERVATION OF MOTION IMPRECISION OF NANOMECHANICAL RESONATOR BELOW THE STANDARD QUANTUM LIMIT


READOUT OF A SUPERCONDUCTING QUBIT WITH A JOSEPHSON PARAMP

OBSERVATION OF QUANTUM JUMPS
IN A CONDENSED MATTER SYSTEM


SENSITIVITY AND QND:
1-PHOTON READOUT OF FLUXONIUM QUBIT

M. Hatridge et al. (in preparation)

Reflected Microwave Phase (°)

\[ f_{\text{cavity}} = 8.094 \text{ GHz} \]
\[ f_{\text{fluxonium}} = 3.672 \text{ GHz} \]
SENSITIVITY AND QND: 1-PHOTON READOUT OF FLUXONIUM QUBIT

M. Hatridge et al. (in preparation)

CAN WE PREVENT THE RABI OSCILLATIONS TO DECAY BY FEEDING BACK THE RESULT OF OUR MEASUREMENT IN THE DRIVE?

M. Hatridge et al. (in preparation)

- phase preserving, linear amplification
- equivalent to 20x increase in averages
- no impact on fluxonium $T_1$
Questions examined in this course:
What conditions need the feedback to satisfy for convergence?
How pure is the state of the qubit with feedback?
PROGRAM OF THIS YEAR’S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model out-of-equilibrium non-linear quantum systems?

Lecture III: How do we optimize the parametric amplifier characteristics while maintaining its noise at the quantum limit?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can continuous quantum measurements be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?

Please note that there will be no lecture on May 24

SELECTED BIBLIOGRAPHY

Books and series of lectures
Braginsky, V. B., and F. Y. Khalili, "Quantum Measurements" (Cambridge University Press, Cambridge, 1992)
Clarke, J. and Braginsky, A. I., eds., "The SQUID Handbook" (Wiley-VCH, Weinheim, Germany, 2006)
Haroche, S., Lectures at College de France, 2011
Haroche, S. and Raimond, J-M., "Exploring the Quantum" (Oxford University Press, 2006)
Nielsen, M. and Chuang, I., "Quantum Information and Quantum Computation" (Cambridge, 2001)
Walls, D.F., and Milburn, G.J. "Quantum Optics" (Springer, Berlin, 2008)
Wiseman, H.M. and Milburn, G.J., "Quantum Measurement and Control" (Cambridge, 2011)

Articles
### CALENDAR OF SEMINARS

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END OF LECTURE