PROGRAM OF THIS YEAR’S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model open, out-of-equilibrium, non-linear quantum systems?

Lecture III: Can we maintain the noise at the quantum limit while increasing gain, bandwidth and dynamic range?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can a continuous quantum measurement be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?
CALENDAR OF SEMINARS

May 10: Fabien Portier, SPEC-CEA Saclay
The Bright Side of Coulomb Blockade

May 17, 2011: Jan van Ruitenbeek (Leiden University, The Netherlands)
Quantum Transport in Single-molecule Systems

May 31, 2011: Irfan Siddiqi (UC Berkeley, USA)
Quantum Jumps of a Superconducting Artificial Atom

June 7, 2011: David DiVicenzo (IQI Aachen, Germany)
Quantum Error Correction and the Future of Solid State Qubits

June 14, 2011: Andrew Cleland (UC Santa Barbara, USA)
Images of Quantum Light

June 21, 2011: Benjamin Huard (LPA - ENS Paris)
Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

June 21, 2011 (3pm): Andrew Cleland (UC Santa Barbara, USA)
How to Be in Two Places at the Same Time ?

LECTURE VI : QUANTUM FEEDBACK CONTROL AND PERSISTENT RABI OSCILLATIONS

OUTLINE

1. Classical and quantum feedback; persistent Rabi oscillations
2. Stochastic differential equations for quantum trajectories
3. Fidelity of quantum feedback control
WHY FEEDBACK?

Excursions away from perturbation-less trajectory must be bounded!

Feedback establishes effective potential and damping, limiting excursions
PERSISTENT RABI OSCILLATIONS: MOTIVATIONS

- Simplest and non-trivial quantum feedback demonstration for qubits
- Metrological application: RF amplitude-to-frequency converter

Typical transmon parameters:

\[ \Omega_\phi / 2\pi = 2\text{MHz} \]
\[ T_1 = 2\pi \gamma_1^{-1} = 20\mu s \]
\[ T_2 = 2\pi \left( 2\gamma_1^{-1} + \gamma_\rho^{-1} \right) = 8\mu s \]

Courtesy of M. Mirrahimi

OPTIMAL FEEDBACK CONTROL

- SYSTEM \( Z(t) \)
- UNCONTROLLED DEGREES OF FREEDOM
- DETECTOR \( \zeta_r(t) \)
- CONTROLLER \( U_r(z,t) \)
- ESTIMATOR \( w_r(z,t) \)
- ENVIRONMENT
- TRANSLATOR

11-VI-7
QUANTUM FEEDBACK IS CURRENTLY USED TO MAINTAIN A FOCK STATE IN A RESONATOR [see S. HAROCHE’s 2010-2011 lectures and Dotsenko et al. Phys. Rev. A80, 013805 (2009)]

HERE WE ARE DISCUSSING QUANTUM FEEDBACK APPLIED TO THE PRESERVATION OF A QUBIT DYNAMICAL STATE
DISPERSSIVE CQED QUBIT MEASUREMENT

\[ H = \frac{\hbar}{2} \left( \omega R \sigma^+ \sigma^- + \chi \sigma^+ \sigma^- a a^+ + \omega \sigma^+ \sigma^- \right) \]

Phase shift of signal reflected off resonator

\[ \theta_s = 2 \tan^{-1} \frac{Z}{\kappa} \]

Width determined by resonator quality factor \( \frac{\omega_c}{\kappa} \)

Transmon cavity pull:

\[ \chi \equiv \frac{2 E_c g^2}{\Delta^2} \]

CONTINUOUS MEASUREMENT OF QUBIT

Qubit Bloch vector

Steady state inside outgoing pulse:

\[ A^m(t) \equiv \cos \left( \omega t + \theta_s(t) \right) \]

\[ Q(T_m) \equiv \int_0^{T_m} dt \sin (\omega t) A^m(t) \]
MEASUREMENT RECORD

Instantaneous growth rate of quadrature signal:

\[ \zeta_r(t) = \int d\tau \frac{dQ_r}{dt} f(t-\tau) \]

filter function

\[ Q_r(T_m) \equiv \int_0^{T_m} dt \zeta_r(t) \]

Steady-state relation

\[ dQ_r(t) = Z_r(t) dt + \frac{1}{\sqrt{\gamma_m}} dW(t) \]

\[ E[Z_r(t)] = \langle Z(t) \rangle = E\left[ \frac{dQ_r}{dt} \right] \]

Wiener increment:

\[ E\left[ dW(t) \right] = 0 \]

\[ \left[ dW(t) \right]^2 = dt \]

Idealized white noise

OUTLINE

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MASTER EQUATION OF THE QUBIT UNDER MEASUREMENT

\[
\dot{\rho} = \mathcal{L}\rho = -\frac{i}{\hbar} \left[ H_{\text{qubit}}, \rho \right] + \gamma_1 D(\sigma^-)\rho + \frac{1}{2} \left[ \gamma_\phi + \frac{\gamma_m}{2} \right] D(\sigma_z)\rho
\]

where \( D(A)\rho = A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A / 2 \)

- Markov
- RWA
- \( T=0 \)

relaxation rate: \( \gamma_\phi \)
dehphasering rate: \( \gamma_1 \)
measurement rate: \( \gamma_m = 4\chi^2 / (\chi^2 + \kappa^2) = \frac{d(SNR)}{dt} \)

Identical to the Bloch equations, but with one additional term:

\[
\begin{align*}
\dot{X} &= \Omega_R(t)Z(t) - \left( \frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2} \right)X \\
\dot{Y} &= -\left( \frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2} \right)Y \\
\dot{Z} &= -\Omega_R(t)X(t) - \gamma_1(Z - 1)
\end{align*}
\]

Rotating frame at \( \omega_{\text{eg}} \)

dispersive qubit readout with feedback

What should be the feedback law? How much delay is tolerable?
How pure is the state of the qubit with feedback?
EQUATIONS OF QUANTUM TRAJECTORIES

Introduce $\rho_r$, the qubit density matrix conditioned by the string of measurement results and represents all the information the observer has accumulated on the qubit.

It obeys:  
$$d\rho_r = \mathcal{L}\rho_r dt + \eta \gamma_m \mathcal{M}(\sigma_z) \rho_r (dQ_r - Z_r dt)$$

an update equation, where  
$$\mathcal{M}(A) \rho = \frac{1}{2} \left[ (A - \langle A \rangle) \rho + \rho (A^\dagger - \langle A^\dagger \rangle) \right]$$

This leads to the stochastic Bloch equations:

$$
\begin{align*}
  dX_r &= \Omega_r (t) Z_r - \left( \frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2} \right) X_r dt - \sqrt{\eta \gamma_m} X_r Z_r dW_r, \\
  dY_r &= -\left( \frac{\gamma_1}{2} + \gamma_\phi + \frac{\gamma_m}{2} \right) Y_r dt - \sqrt{\eta \gamma_m} Y_r Z_r dW_r, \\
  dZ_r &= -\Omega_r (t) X_r - \gamma_1 (Z_r - 1) + \sqrt{\eta \gamma_m} \left( 1 - |Z_r|^2 \right) dW_r.
\end{align*}
$$

The point $[X_r(t), Y_r(t), Z_r(t)]$ is in general inside the Bloch sphere. Its time evolution is the quantum trajectory of the system.

THE QUANTUM TRAJECTORIES EQUATIONS ANSWER THE FOLLOWING QUESTIONS QUANTITATIVELY:

IN QUANTUM REGIME,
HOW MANY BITS OF INFORMATION DOES THE MEASUREMENT ACQUIRE PER UNIT OF TIME?
WHAT IS THE CORRESPONDING BACK-ACTION ON THE QUBIT?

A SIMPLER MODEL CAN ANSWER THESE QUESTIONS SEMI-QUANTITATIVELY.
**SPIN MODEL OF CONTINUOUS MEASUREMENT**

Bloch sphere representation of qubit under measurement

\[ |\Psi_0\rangle = \alpha |+Z\rangle + \beta |-Z\rangle \]

\[ \alpha = \cos \frac{\theta}{2} e^{-i\phi/2} \]
\[ \beta = \sin \frac{\theta}{2} e^{i\phi/2} \]

\[ \langle \Psi_0 | Z | \Psi_0 \rangle = |\alpha|^2 - |\beta|^2 = Z_0 \]

An ancilla is prepared along \( X \) and moved towards qubit for partial measurement.
SPIN MODEL OF CONTINUOUS MEASUREMENT

Bloch sphere representation of qubit under measurement

Bloch sphere representation of ancilla qubit for partial measurement

\[ H_{\text{int}} = \frac{1}{2} \hbar \omega_c Z_i \cdot Z^i \]

ancilla and qubit interact during \( \tau \)

INTERACTION IS QND IF QUBIT 1 ONLY SEES A FIELD ALONG Z
ANCILLA AND QUBIT

SPIN MODEL OF CONTINUOUS MEASUREMENT

Interaction has produced:

\[ U = e^{i \varepsilon Z_1 Z'^1 / 2} \]

interaction strength:

\[ \varepsilon = \omega_c \tau \]

ancilla and qubit separate.

ancilla and qubit have become partially entangled (partial measurement).

Bloch vectors are insufficient!

full QND projective measurement if \( \varepsilon = \pi/2 \)

Next, ancilla is measured projectively along Y.

ONE GETS FOR INSTANCE, AFTER MEASUREMENT OF ANCILLA:

measurement record:

\[ \zeta_1 = +1 \]
ancilla is measured projectively along $Y$

$\zeta_1 = -1$

REPEAT $N$ TIMES, TOTAL TIME = $T_M$

This generates measurement record $(\zeta_1, \zeta_2, ..., \zeta_N)$

$k$-th ancilla is prepared along $X$ and moved towards qubit etc, etc, etc, etc...
SPIN MODEL OF CONTINUOUS MEASUREMENT

MAKE $N$ TEND TOWARD INFINITY WHILE $\varepsilon$ TENDS TOWARD ZERO

$$(\zeta_1, \zeta_2, \ldots, \zeta_N) \rightarrow \zeta(t)$$

UPDATE FORMULA

After $k$ partial measurement, qubit wavefunction is:

$$|\Psi_{k-1}\rangle = \alpha_{k-1}|+Z\rangle + \beta_{k-1}|-Z\rangle$$

$$\begin{cases}
\alpha_{k-1} = \cos \frac{\theta_{k-1}}{2} e^{i\phi} \\
\beta_{k-1} = \sin \frac{\theta_{k-1}}{2} e^{i\phi}
\end{cases}$$

$\phi$ is unchanged, qubit state is pure!

$$\langle Z \rangle_{k-1} = \cos \theta_{k-1} = Z_{k-1}$$

$$\sin \varepsilon = q \quad 0 < q \leq 1$$

$$\zeta_k = \pm 1 \quad p_k^\pm = \frac{1 \pm q \cdot Z_{k-1}}{2}$$

update qubit state conditioned by measurement result:

$$Z_k = \frac{q\zeta_k + Z_{k-1}}{1 + q\zeta_k Z_{k-1}}$$

stationary if

$$Z_{k-1} = \pm 1$$
HEISENBERG-LIKE RELATION

Auxiliary spins measure at a rate:
\[
\frac{d (SNR)}{dt} = q^2 \frac{N}{T_m} = \gamma_m
\]
Information on qubit goes out.... Brownian motion on circle due to back-action.

\[
\text{Average } X \text{ component of Bloch vector decreases like } q^2 N/2
\]

\[
\gamma_{m, ba} \gamma_{\phi} = \frac{1}{2}
\]
similar to \[
\Delta X_m \cdot \Delta P_{ba} = \frac{\hbar}{2}
\]

Measurement back-action dephasing rate:
\[
\gamma_{\phi, ba} = \frac{q^2 N}{2 T_m} = \frac{\gamma_m}{2}
\]

Measurement time

\[
\gamma_m \gamma_{\phi, ba} = \frac{1}{2}
\]

QUANTUM TRAJECTORY OF RABI OSCILLATIONS

\[
\Omega_R = 2 \gamma_m, \quad \eta = 1
\]
Filter time constant = 10^2

Measurement record:
\[
\zeta_r(t)
\]

Z component of Bloch vector conditioned by measurement:
\[
Z_r(t)
\]
(What a perfect observer would reconstruct)

Z component of Bloch vector averaged over all realizations of trajectories:
\[
Z(t)
\]

\[
\text{time (step units)}
\]

11-VI-17
QUANTUM JUMPS

For $\gamma_m \gg \Omega_R$, trajectories turn into "telegraphic" noise

$\zeta_r(t)$

$Q_r(t)$

$Z_r(t)$

What a perfect observer would reconstruct

Filter time constant = $5 \times 10^3$

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KOROTKOV’S PROPORTIONAL CONTROLLER

ESTIMATED DEPHASING OF RABI OSCILLATIONS:

\[ \delta\theta(t) = \tan^{-1} \left( \frac{X_r(t)}{Z_r(t)} \right) + \frac{\pi}{2} \left[ 1 - \text{sgn}(Z_r(t)) \right] - \Omega_R t \]

\[ -\pi < \delta\theta(t) < +\pi \]

FEEDBACK LAW:

\[ \Omega_R(t) = \bar{\Omega}_R t - G_{FB} \bar{\epsilon}_R \delta\theta(t) \]

FIDELITY OF PROPORTIONAL CONTROL

Simulations for transmon qubit

\[ \eta = \frac{1}{2} \quad G_{FB} = \frac{1}{2} \quad \bar{n}_r = 0.13 \quad \frac{\kappa}{2\pi} = 8\text{MHz} \quad \frac{\chi}{2\pi} = 1.2\text{MHz} \]

Courtesy of M. Mirrahimi
IMPROVEMENT OF FEEDBACK SCHEME USING STRONGER, PULSED MEASUREMENT

- Partial projection on the poles $Z = \pm 1$ (Quantum Zeno effect)
- Correction by a $\pi$-pulse around $Y$-axis if $Z = -1$ is detected (similar to adding a day on a leap year).

FIDELITY OF PULSED CONTROL

$$\bar{n}_r = 2 \quad \frac{\kappa}{2\pi} = 20\text{MHz} \quad \frac{\chi}{2\pi} = 4\text{MHz} \quad \text{delay of 100ns}$$
CONCLUSIONS

A quantum system driven out-of-equilibrium and in contact with several reservoirs can be seen from a scattering point of view emphasizing the notion of information channels.

Increasing the bit rate of monitored information channels over that of un-monitored ones is a necessary condition for increasing fidelity of quantum feedback control.

The key bi-directionality property of information channels manifests itself in powerful relations linking dissipation/amplification and noise, on one hand, as well as measurement precision/speed and the corresponding inevitable back-action, on the other hand.

END OF 2011 COURSE ON QUANTUM AMPLIFICATION AND FEEDBACK

NEXT YEAR: NANOMECHANICAL RESONATORS IN QUANTUM REGIME

ACKNOWLEDGEMENTS