

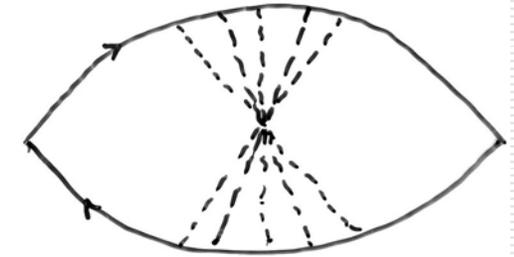
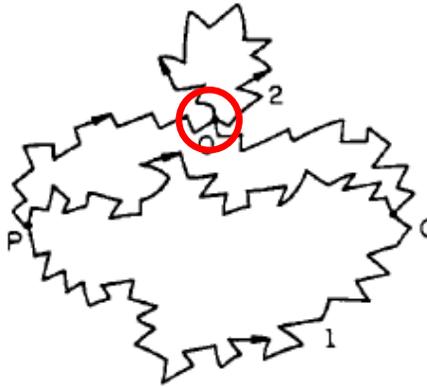
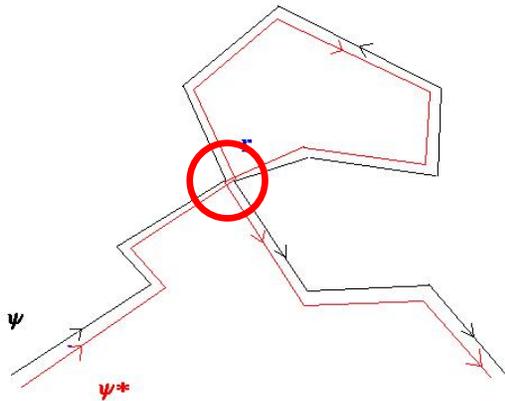
Interférences quantiques dans les systèmes désordonnés

G. Montambaux (LPS, Orsay)

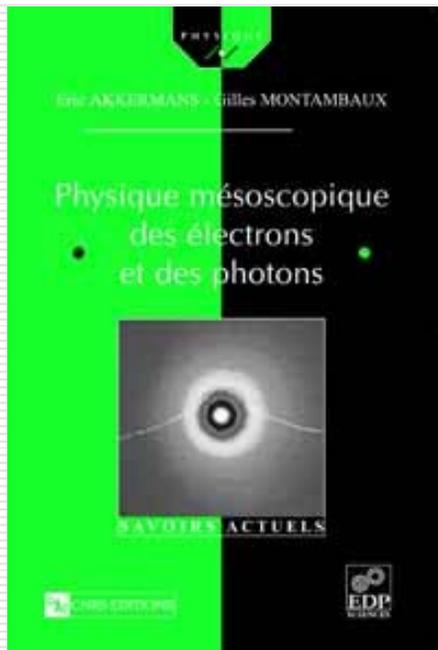
Effets de cohérence de phase dans la propagation des ondes

Electrons
Lumière

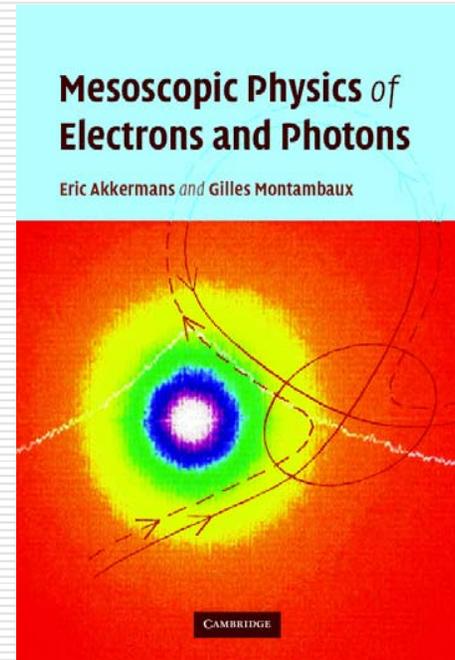
Localisation faible
Rétrodiffusion cohérente



- 1) Croisement quantique
- 2) conservation de l'énergie
- 3) formulation dans l'espace réel



EDP Sciences, 2004



Cambridge University Press, 2007

Collaboration: Eric Akkermans, Technion, Haifa

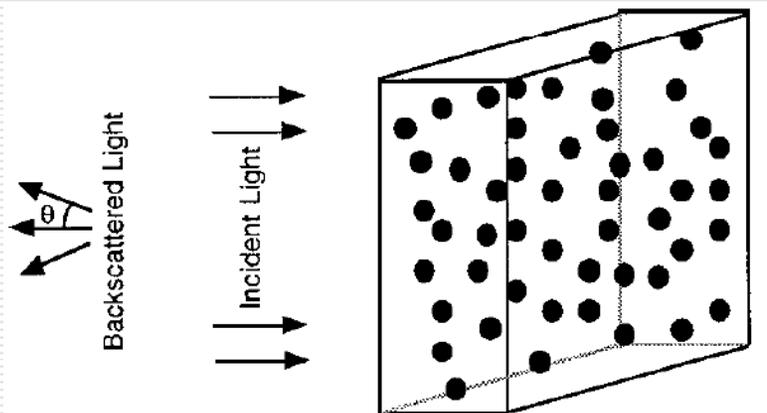
Conservation of energy in coherent backscattering of light

S. FIEBIG¹, C. M. AEGERTER¹, W. BÜHRER¹, M. STÖRZER¹, E. AKKERMANS², G. MONTAMBAUX³
and G. MARET¹

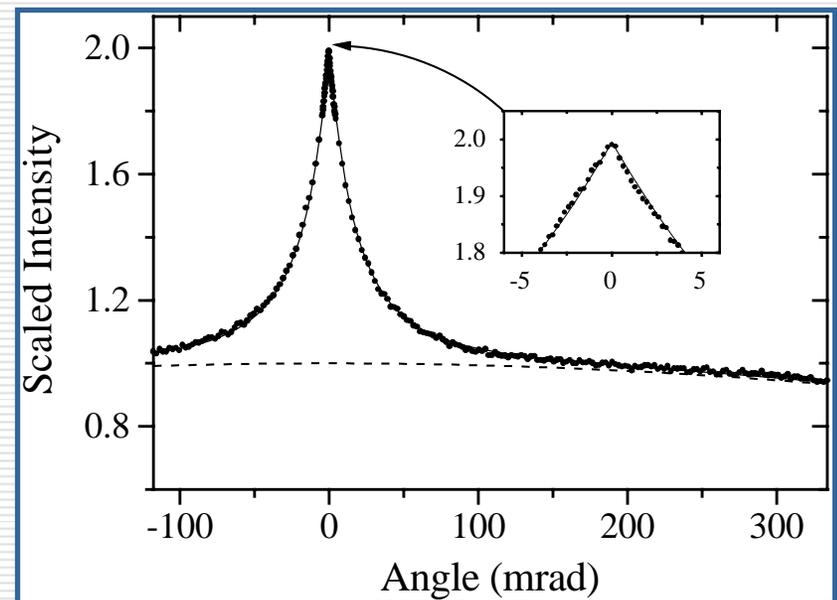
¹ *Fachbereich Physik, University of Konstanz - Box M621, 78457 Konstanz, Germany*

² *Department of Physics, Technion Israel Institute of Technology - 32000 Haifa, Israel*

³ *Laboratoire de Physique des Solides, CNRS UMR 8502, Université Paris-Sud - 91405 Orsay, France*



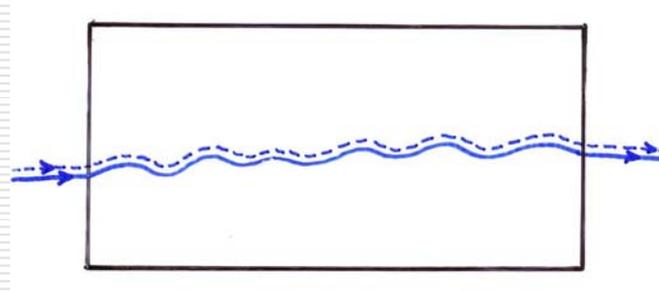
albedo



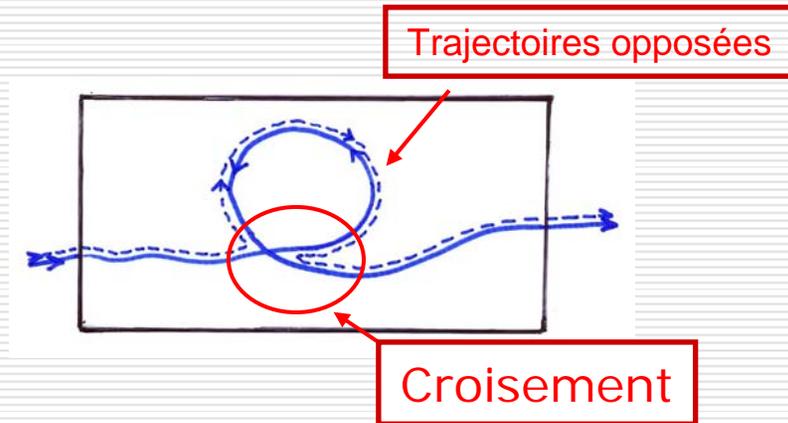
Localisation faible : correction au transport classique

Conductance classique

$$G_{cl} = g \frac{e^2}{h}$$



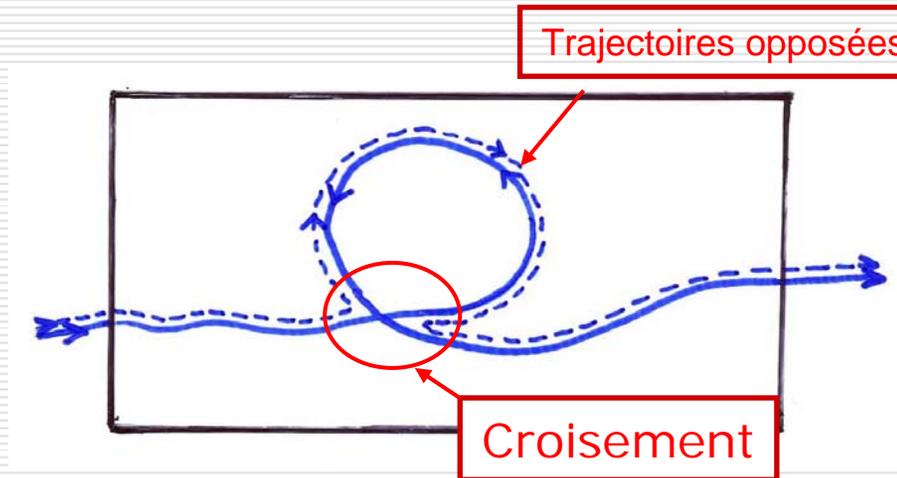
Correction quantique d'ordre 1/g



$$\frac{\Delta G}{G_{cl}} \sim - \frac{\lambda_F^{d-1} v_F}{Vol} \int_{\tau_e}^{\infty} P(t) e^{-t/\tau_\phi} dt$$

Coherence de phase

Les phases des deux trajectoires opposées sont identiques



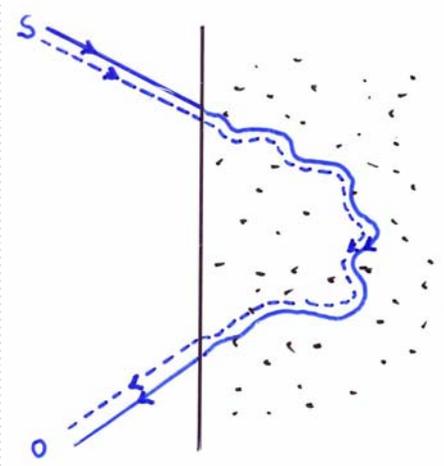
$$\varphi_j = \frac{1}{\hbar} \int_j \vec{p} \cdot d\vec{l}$$

$$\vec{p} \rightarrow -\vec{p}$$

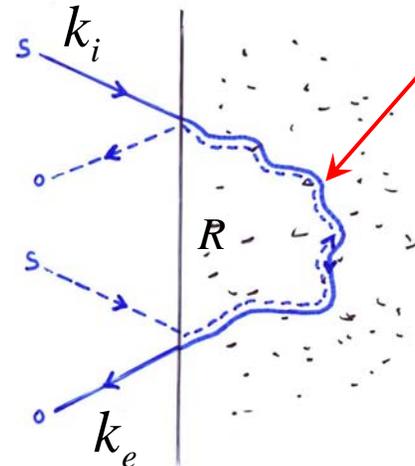
$$\varphi_{j_T} = \frac{1}{\hbar} \int_j (-\vec{p}) \cdot (-d\vec{l})$$

$$d\vec{l} \rightarrow -d\vec{l}$$

Rétrodiffusion cohérente en optique: Albedo



Diffusion classique

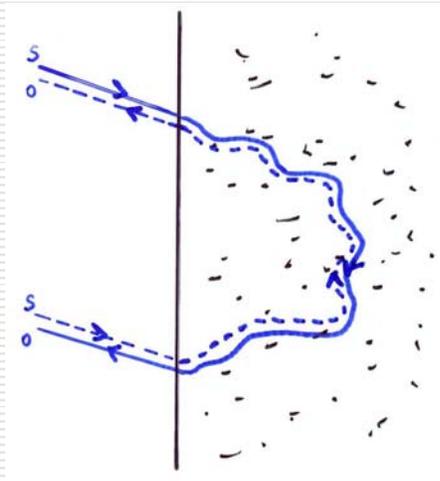


Contribution cohérente

Trajectoires opposées

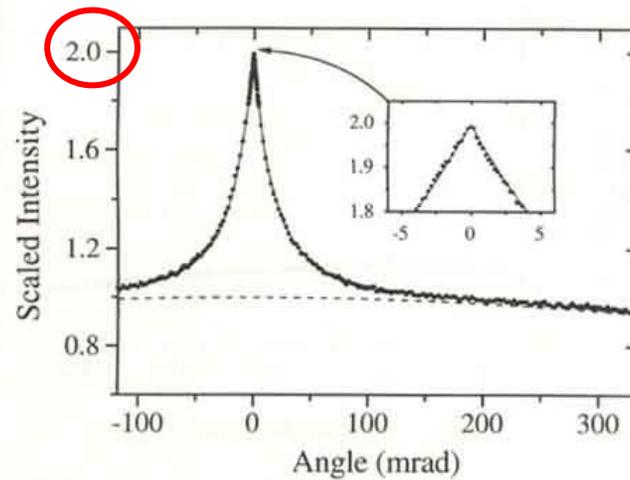
déphasage

$$e^{i(\vec{k}_i + \vec{k}_e) \cdot \vec{R}}$$

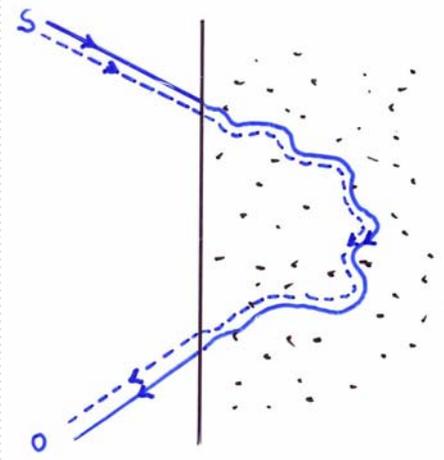


Contribution cohérente si $k_e = -k_i$

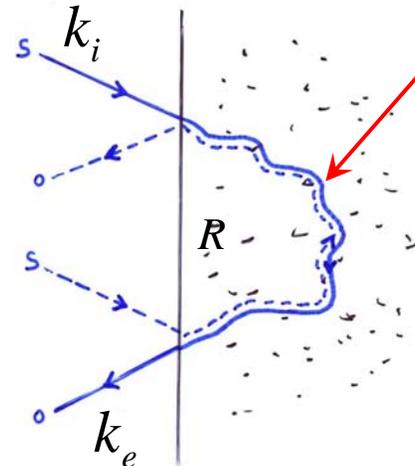
D. Wiersma et al.



Rétrodiffusion cohérente en optique: Albedo



Diffusion classique

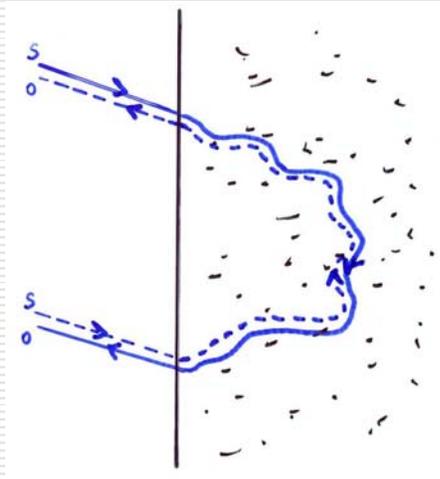


Contribution cohérente

Trajectoires opposées

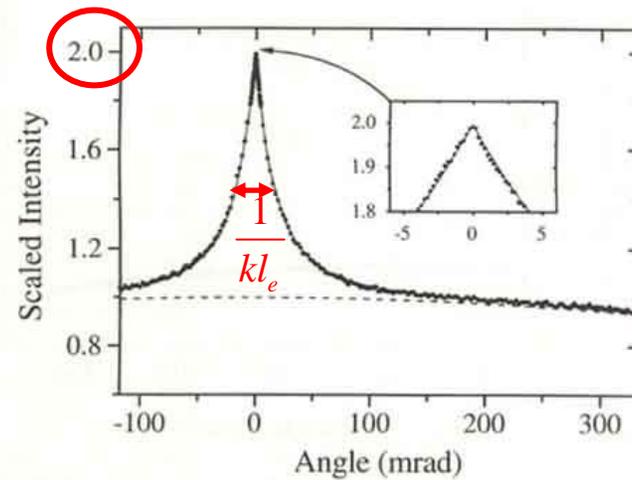
déphasage

$$\left\langle e^{i(\vec{k}_l + \vec{k}_e) \cdot \vec{R}} \right\rangle_{\vec{R}}$$



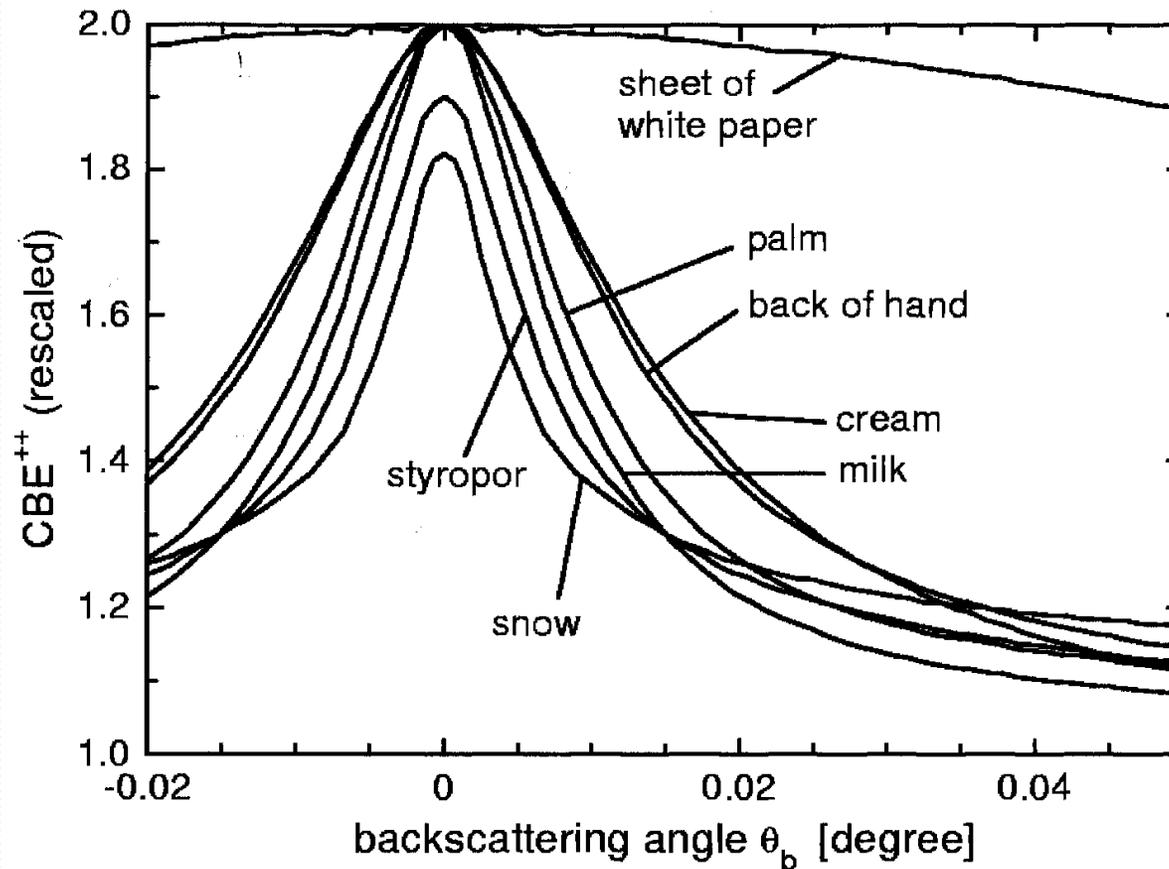
Contribution cohérente si $k_e = -k_i$

D. Wiersma et al.



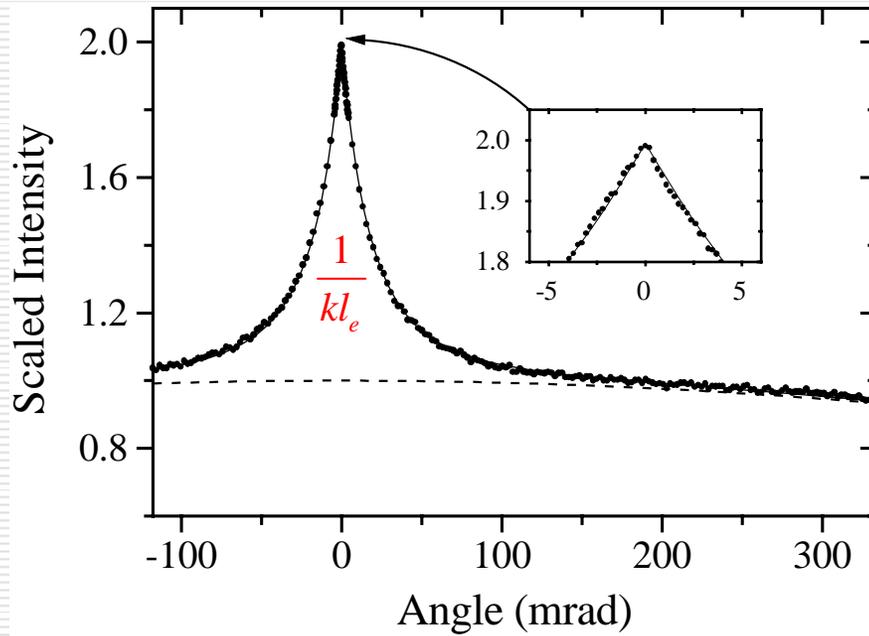
« Cône de rétrodiffusion cohérente »

Ça se voit partout...



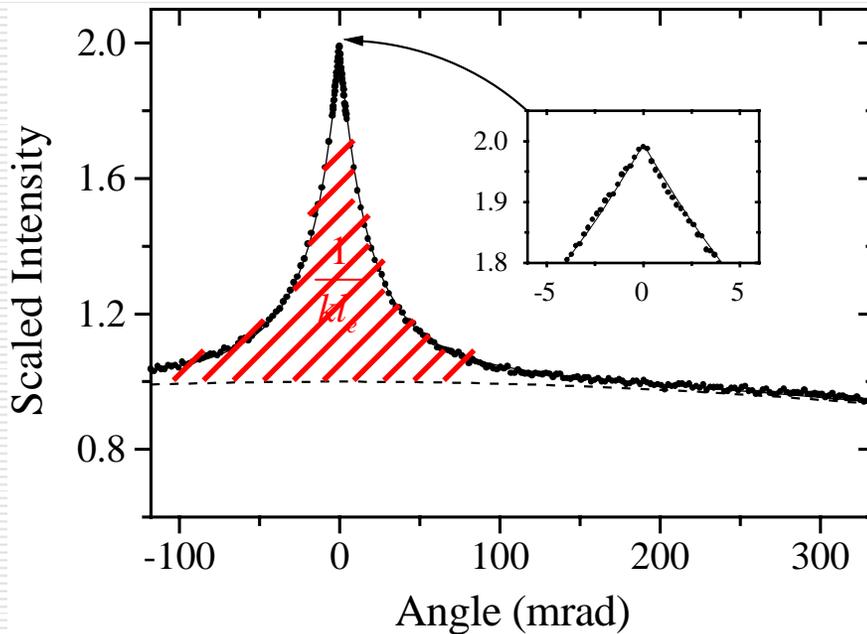
Autres ondes, acoustiques, etc...
aussi en astrophysique (anneaux de saturne)

Une question...



Wolf, Maret, 1985
Van Albada, Lagendijk, 1985
Wiersma et al.

Une question...



Wolf, Maret, 1985
Van Albada, Lagendijk, 1985
Wiersma et al.

La contribution intégrée du pic est d'ordre

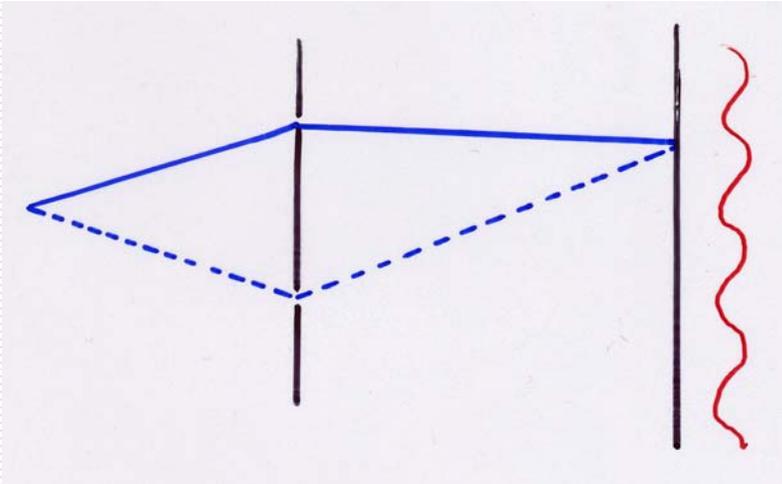
$$\int_0^{1/kl_e} (1 - kl_e \theta) \sin \theta d\theta \sim \frac{1}{(kl_e)^2}$$

Normalisation

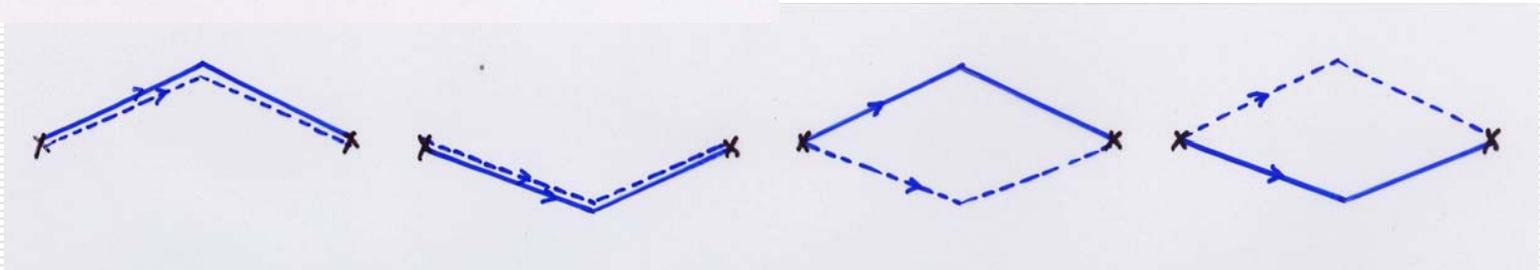
d'où vient cette énergie supplémentaire ?

$$\int I_{\text{int}}(\theta) \sin \theta d\theta = 0$$

$$I = \left| \sum_j A_j \right|^2 = \sum_j |A_j|^2 + \sum_{j \neq j'} A_j A_{j'}^*$$

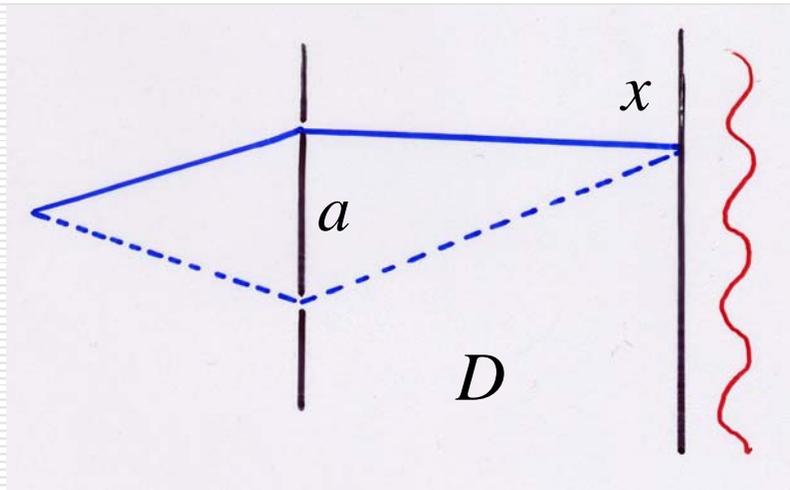


$$\int I_{int} = 0$$



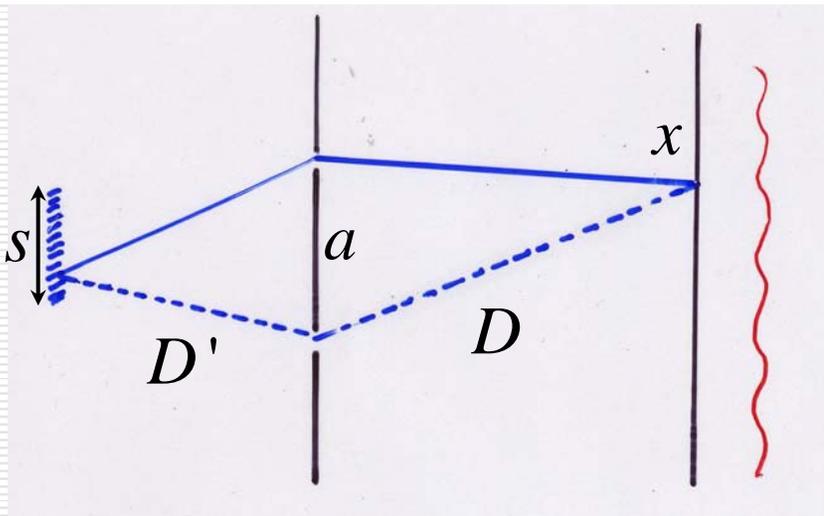
$$I = |A_1|^2 + |A_2|^2 + A_1 A_2^* + A_2 A_1^*$$

Rappels simples ...



$$I = I_{cl} \left(1 + \cos \frac{kax}{D} \right)$$

$$I = I_{cl} + I_{int} \quad \int I_{int} = 0$$



$$I = I_{cl} \left(1 + \frac{\sin \frac{kas}{D'}}{\frac{kas}{D'}} \cos \frac{kax}{D} \right)$$

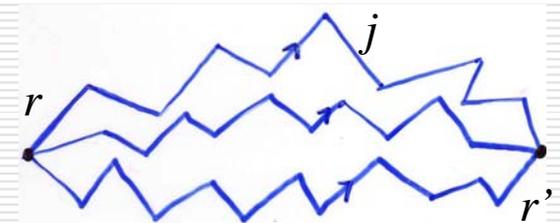
Conductance, Intensité et transmission

La conductance est reliée à la probabilité de faire passer des électrons d'un bord à l'autre de l'échantillon (Landauer)

On introduit $P(r, r', t)$, probabilité de trouver une particule en r' si elle a été injectée en r

Amplitude quantique $G(r, r') = \sum_j A_j(r, r')$

$$A_j(r, r') = |A_j(r, r')| e^{i\varphi_j(r, r')}$$



La probabilité est le module carré de l'amplitude

$$P(r, r') = \left| \sum_j A_j(r, r') \right|^2 \quad I(r, r')$$

Conductance, Intensité et transmission (2)

Deux types de contribution à la probabilité

Moyenne sur le désordre

$$P(r, r') = \sum_j |A_j(r, r')|^2 + \sum_{j \neq j'} A_j(r, r') A_{j'}^*(r, r')$$

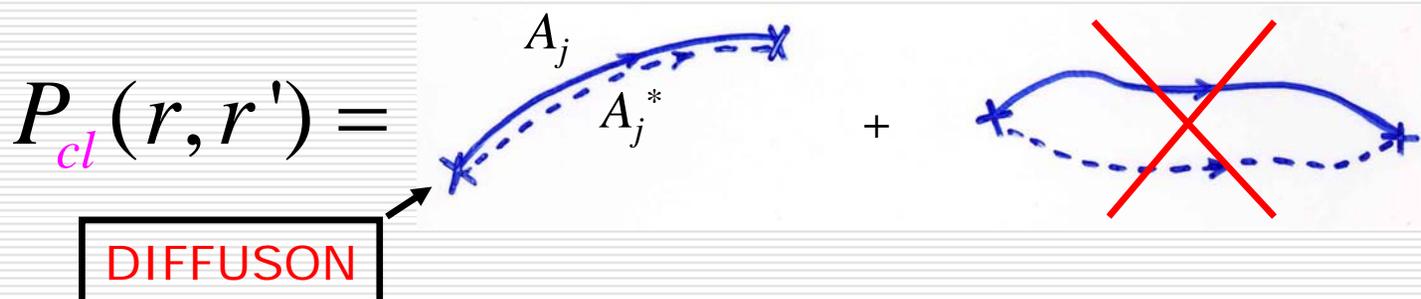
Terme classique

Terme d'interférence

Effets quantiques

Transport classique : seules les trajectoires appariées A_j, A_j contribuent
Si trajectoires différentes, les amplitudes A_j et $A_{j'}$ sont différentes

- les phases non corrélées
- le terme d'interférence disparaît



$$P(r, r') = \sum_j \overbrace{\left| A_j(r, r') \right|^2} + \sum_{j \neq j'} \overbrace{A_j(r, r') A_{j'}^*(r, r')}$$

$$P(r, r') = P_{cl}(r, r') + P_{int}(r, r')$$

La probabilité totale doit être normalisée

$$\int P(r, r', t) dr' = 1 \quad \xrightarrow{TF} \quad P(q=0, \omega) = \frac{i}{\omega}$$

Or, la probabilité classique est déjà normalisée

$$P_{cl}(q, \omega) = \frac{1}{-i\omega + Dq^2} \quad P(q=0, \omega) = \frac{i}{\omega}$$

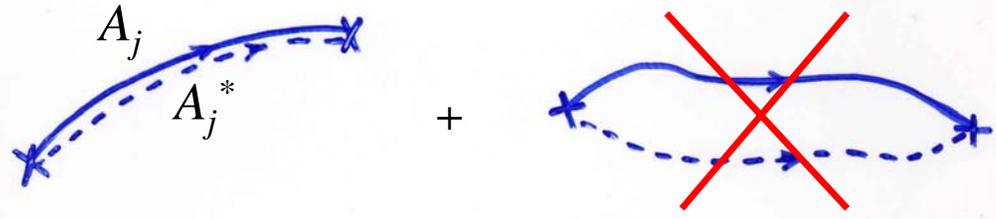


$$P_{int}(q=0, \omega) = 0$$

$$\int P_{int}(r, r', t) dr' = 0$$

Effets quantiques

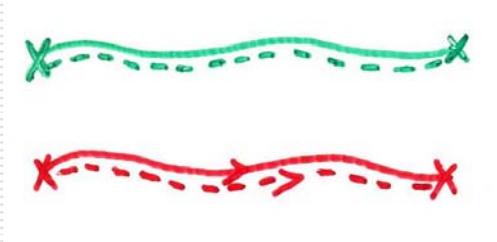
$$P(r, r') =$$



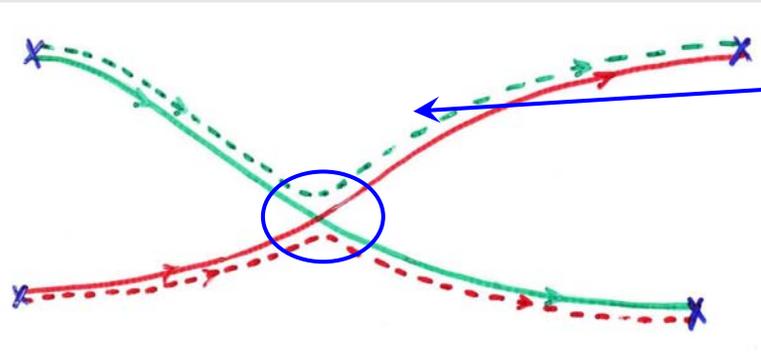
Q: Comment apparaissent les effets quantiques ?

R: Quand les trajectoires se croisent

Exemple : 2 particules de r_1, r_2 vers r_1', r_2'



Classiquement : produit des probabilités (intensités)



Croisement quantique

Échange d'amplitudes

correction

Croisement quantique

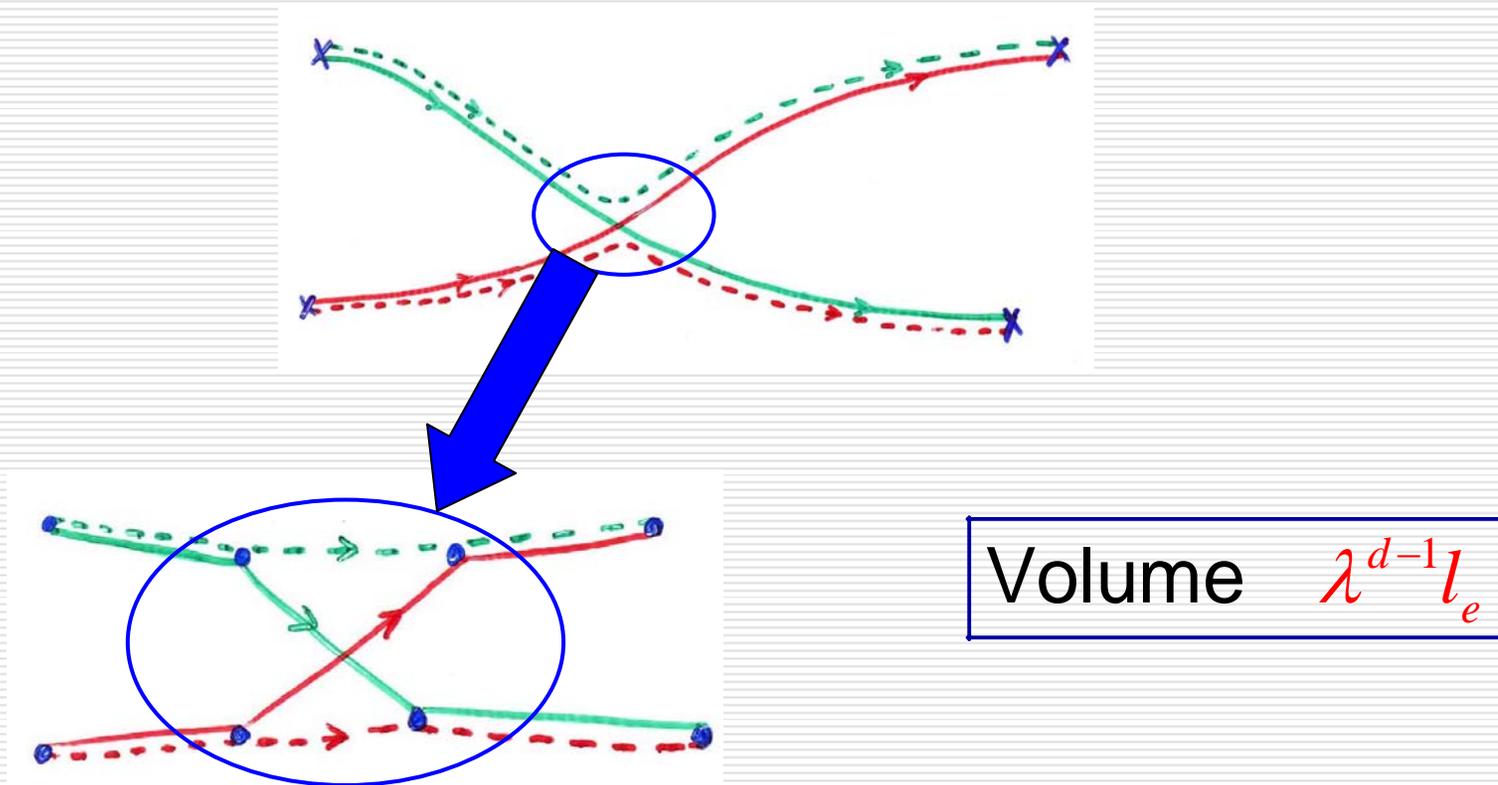


Image simple : le **diffuson** $P(r, r', t)$ est un objet de longueur vt et de section efficace λ^{d-1}

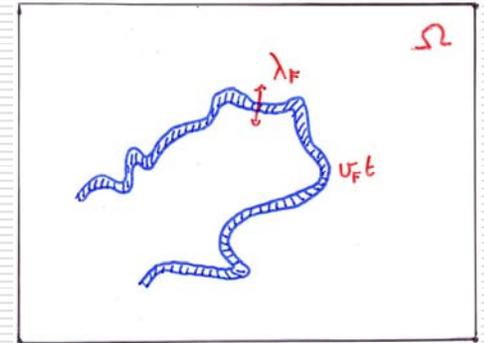
- * Les effets quantiques correspondent aux croisements
- * Importance des effets quantiques => évaluer la probabilité de croisement

Evaluer la probabilité de croisement quantique

Image simple : le diffuson $P(r, r', t)$ est un objet de longueur $v t$ et de section efficace λ^{d-1}

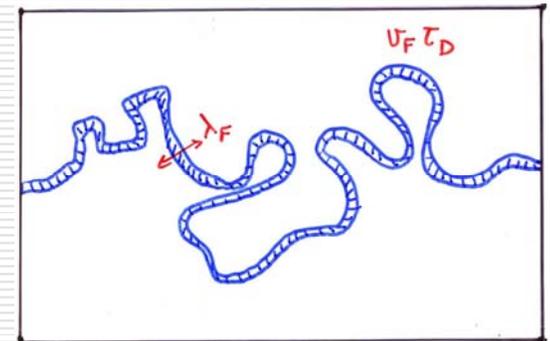
Probabilité de croisement pendant un temps t , dans un volume $V=L^d$

$$p_{\times}(t) = \frac{\lambda^{d-1} v t}{L^d}$$



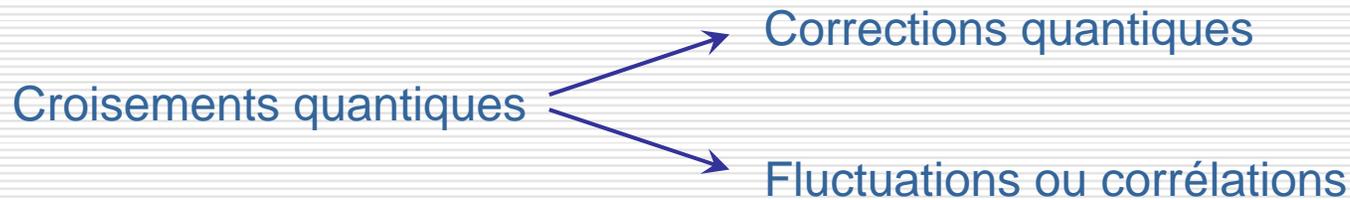
Phénomènes de transport : l'onde passe dans l'échantillon de taille L , un temps $\tau_D = L^2/D$
La probabilité de croisements quantiques **qui affectent les propriétés de transport** est donc

$$p_{\times}(\tau_D) = \frac{\lambda^{d-1} v \tau_D}{L^d} = \frac{\lambda^{d-1}}{l_e L^{d-2}} \sim \frac{1}{g}$$



Effets cohérents et croisements quantiques

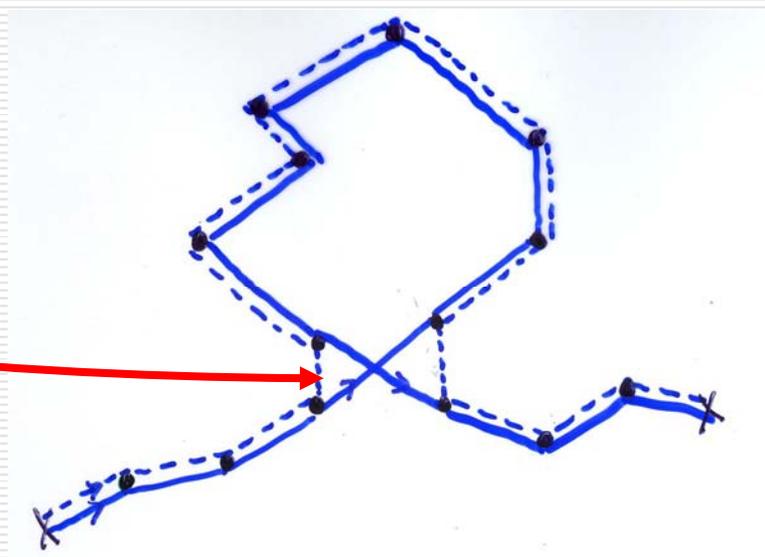
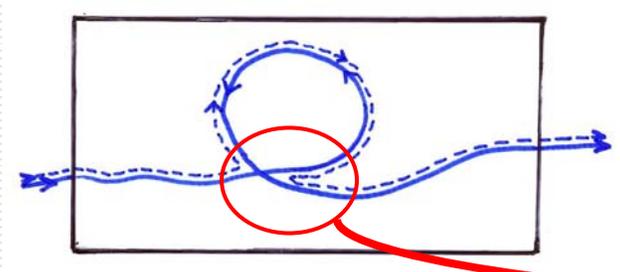
La probabilité de croisement quantique est $1/g$

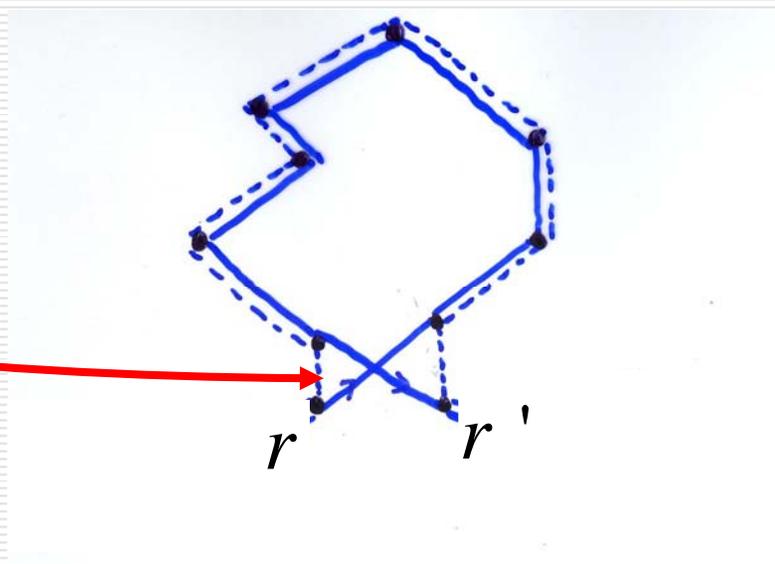
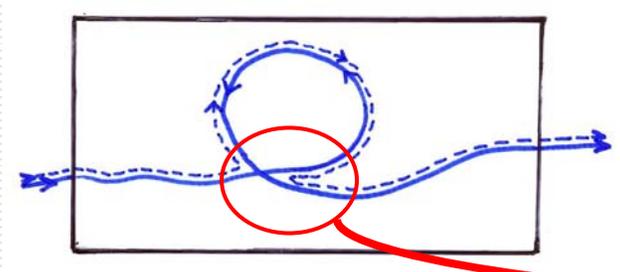


Transport classique $G = g \frac{e^2}{h}$

Les effets quantiques sont d'ordre $G \times \frac{1}{g} \sim \boxed{\frac{e^2}{h}}$ si cohérence de phase

Dans un bon métal ($g \gg 1$), les effets quantiques sont petits





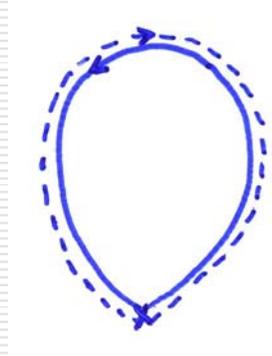
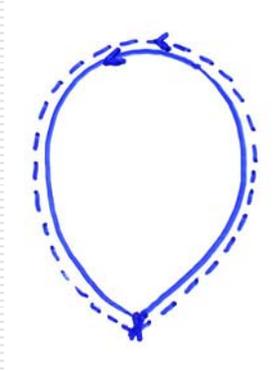
Cohérence de phase

$$P(r, r') = \sum_j |A_j(r, r')|^2 + \sum_{j \neq j'} A_j(r, r') A_{j'}^*(r, r')$$

Probabilité **classique** de retour à l'origine

$$r = r'$$

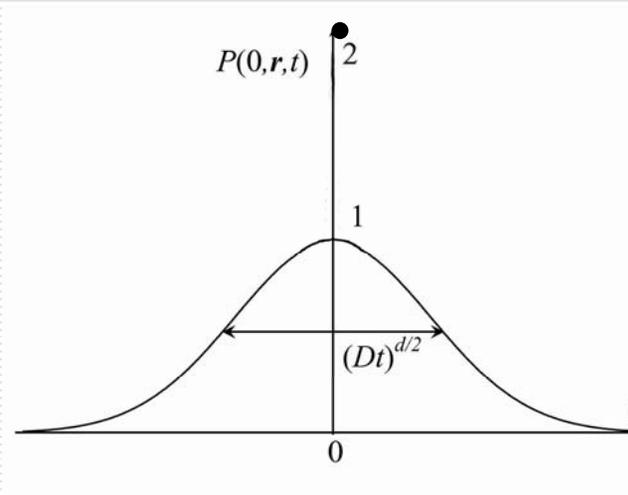
Diffuson



Terme d'**interférence**

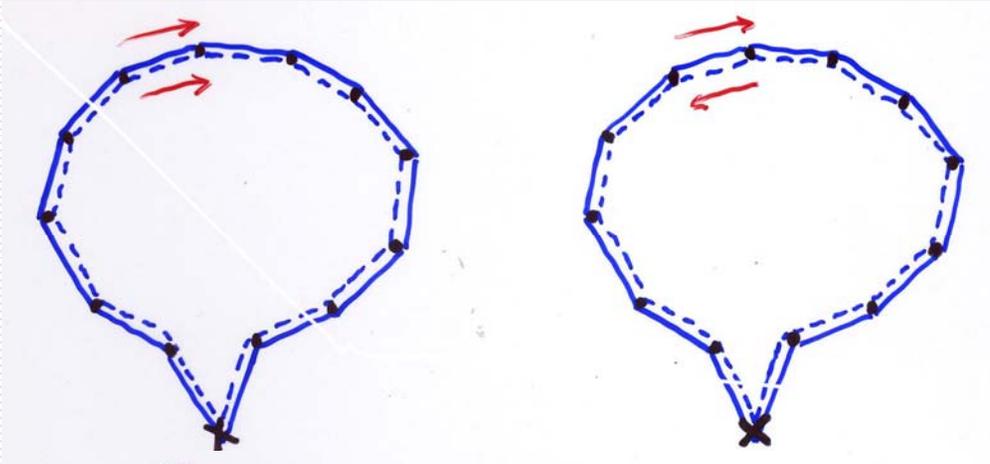
Cooperon

Si invariance par renversement du sens du temps

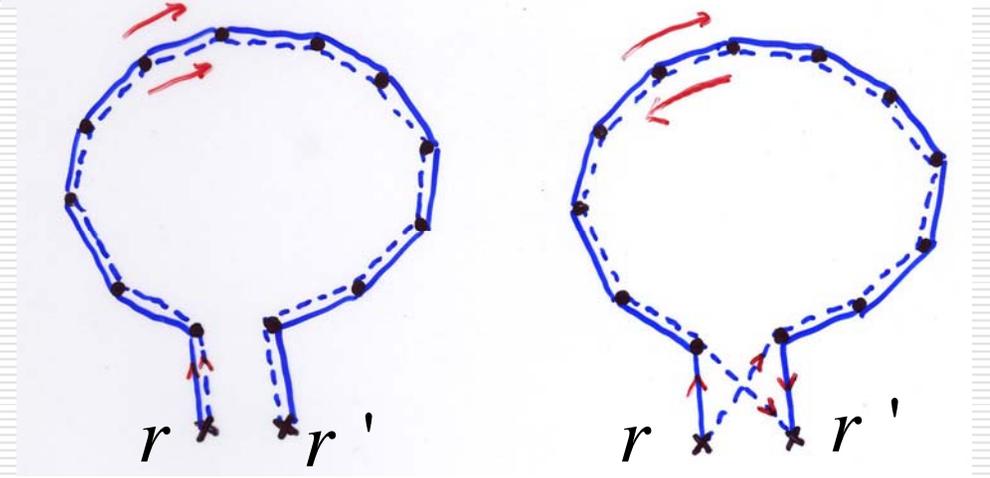


$$P(0, r, t) = \frac{e^{-r^2/4Dt}}{(4\pi Dt)^{d/2}}$$

$$r = r'$$

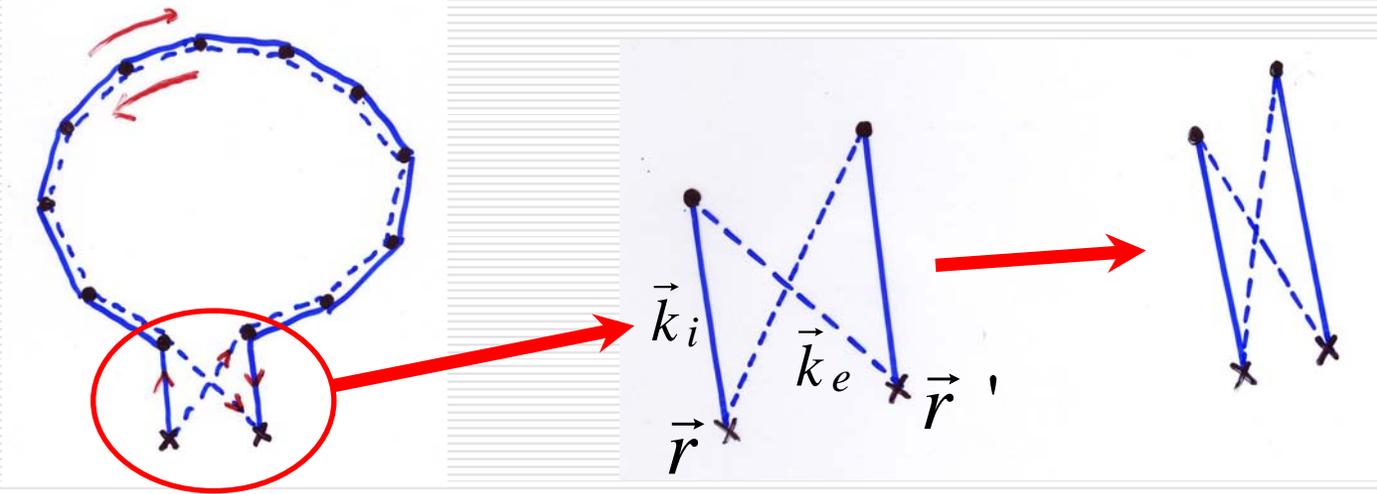


$$r \neq r'$$



Croisement quantique

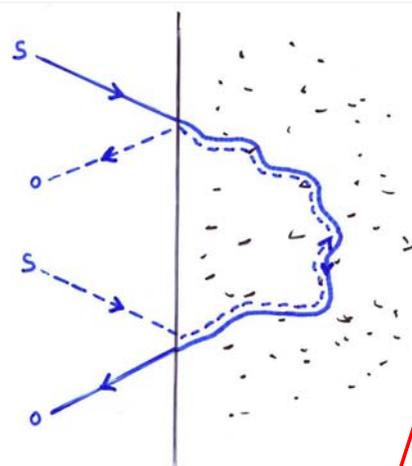
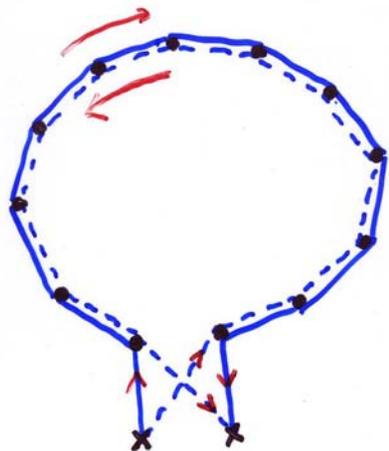
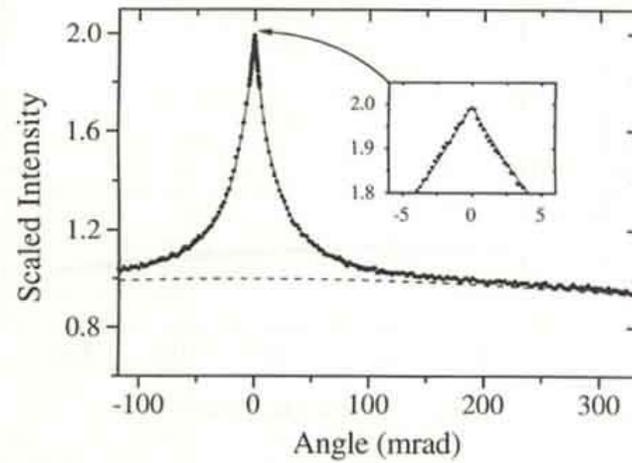
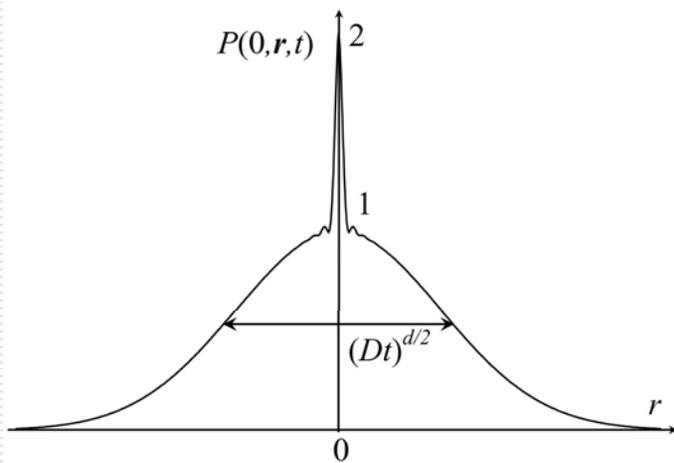
Déphasage



$$e^{i(\vec{k}_i + \vec{k}_e) \cdot (\vec{r} - \vec{r}')}$$

$$\left\langle e^{i(\vec{k}_i + \vec{k}_e) \cdot \vec{R}} \right\rangle_{\vec{k}_i, \vec{k}_e}^2 = \left(\frac{\sin k_F R}{k_F R} \right)^2 e^{-R/l_e}$$

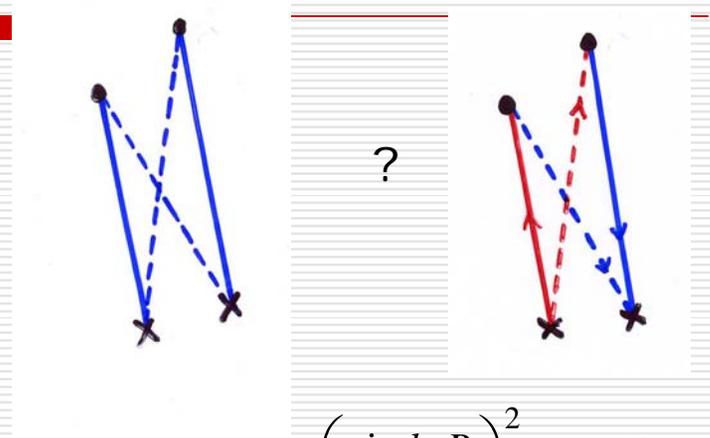
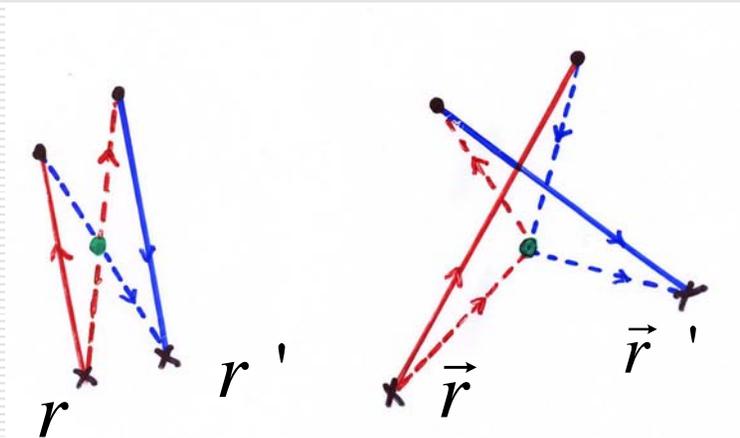
$$\vec{R} = \vec{r} - \vec{r}'$$



$$\left\langle e^{i(\vec{k}_i + \vec{k}_e) \cdot \vec{R}} \right\rangle_{\vec{k}_i, \vec{k}_e}$$

$$\left\langle e^{i(\vec{k}_i + \vec{k}_e) \cdot \vec{R}} \right\rangle_{\vec{R}}$$

Existe-t-il d'autre contribution que



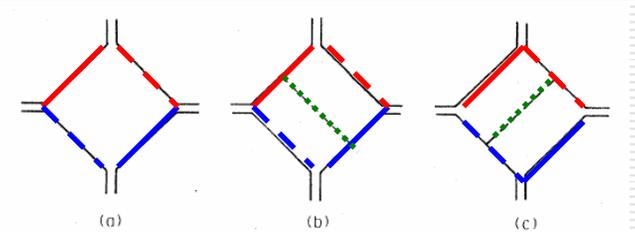
$$\left(\frac{\sin k_F R}{k_F R} \right)^2 e^{-R/l_e}$$

$$\sim - \frac{1}{Rl_e} e^{-R/l_e}$$

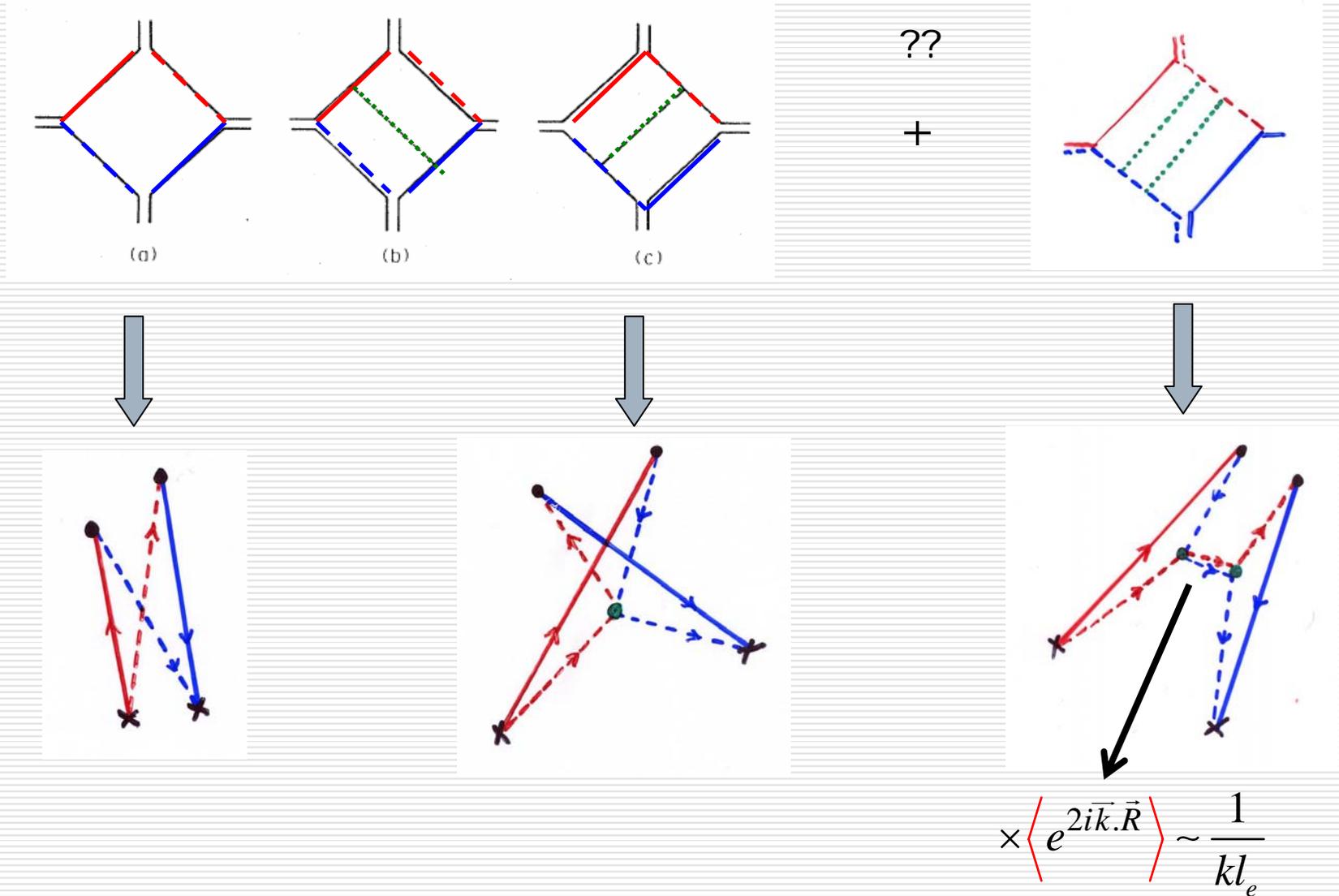
Cette contribution est **négative**,
de longue portée
 et **compense la contribution courte portée**

Oui, avec une impureté supplémentaire :

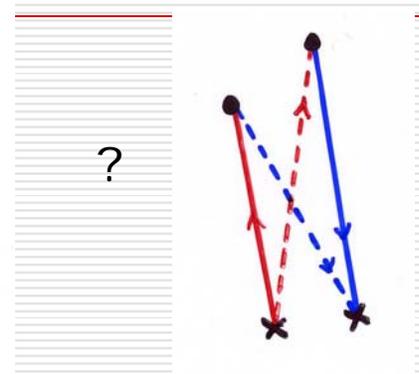
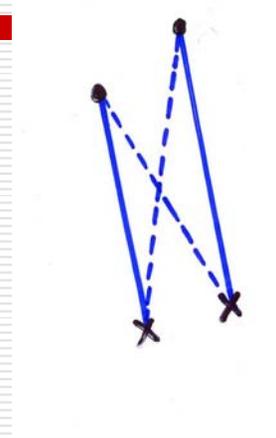
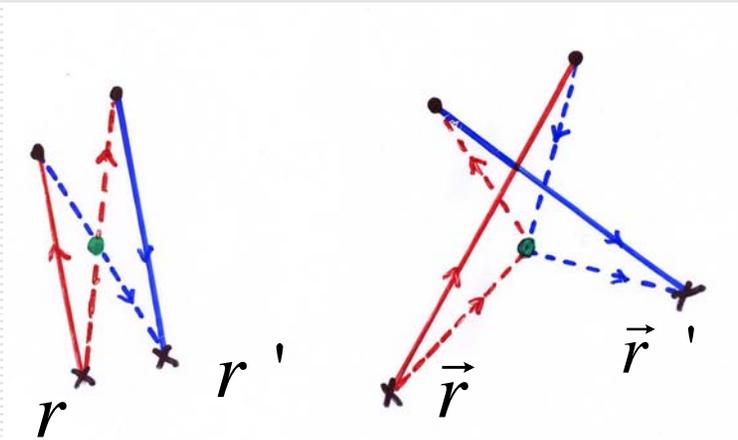
« Boîte de Hikami »
PRB 24, 2671 (1981)



Plus d'impuretés ???



Existe-t-il d'autre contribution que



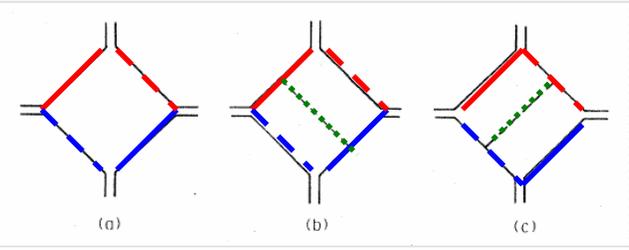
$$\left(\frac{\sin k_F R}{k_F R} \right)^2 e^{-R/l_e}$$

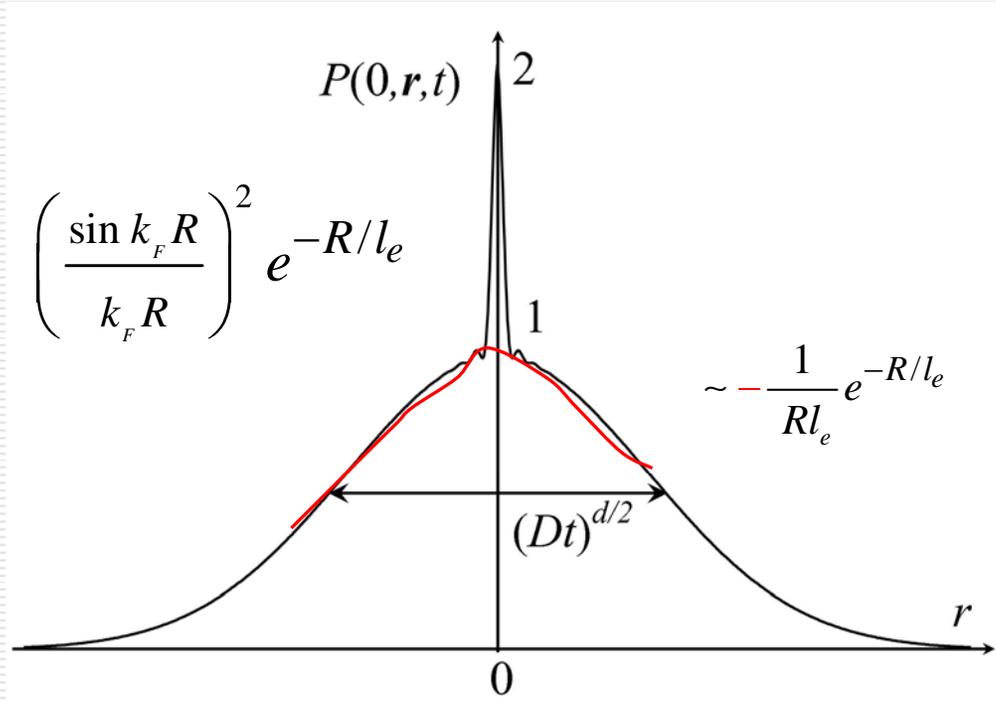
$$\sim - \frac{1}{Rl_e} e^{-R/l_e}$$

Cette contribution est **négative**,
de longue portée
 et **compense la contribution courte portée**

Oui, avec une impureté supplémentaire,
 et pas plus.

« Boîte de Hikami »



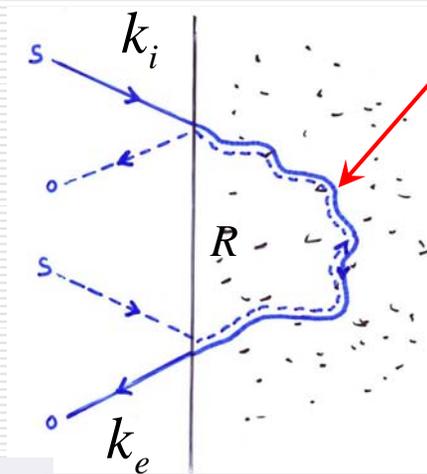


La probabilité est bien normalisée

Rétrodiffusion cohérente

Cherchons la contribution qui compense le cone de rétrodiffusion

Contribution cohérente

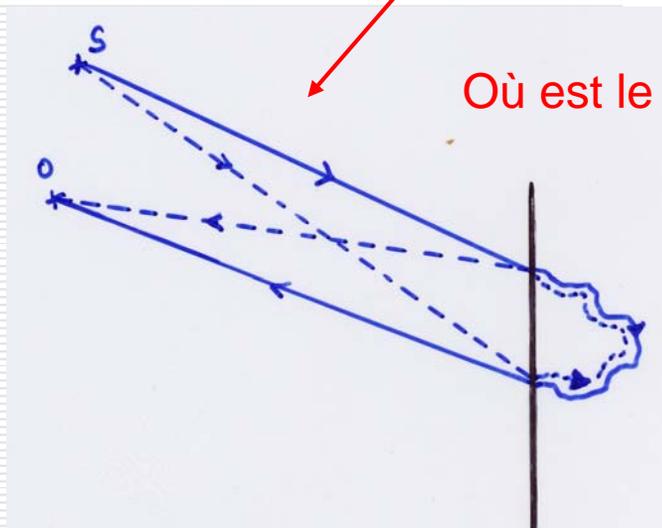


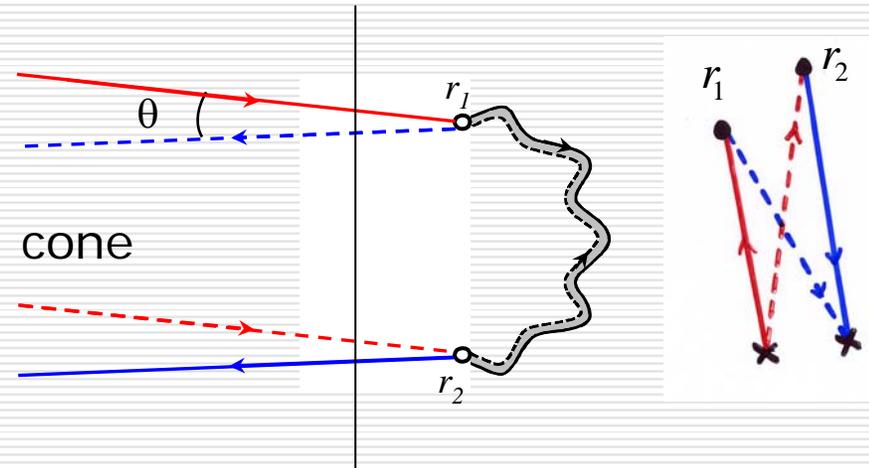
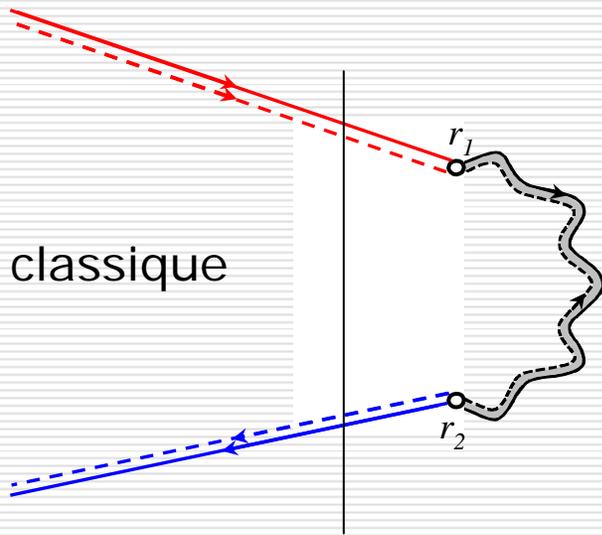
Trajectoires renversées

déphasage

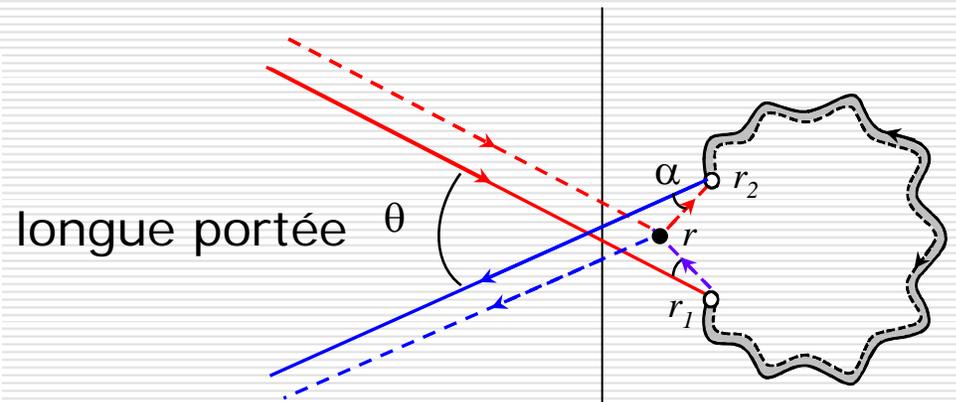
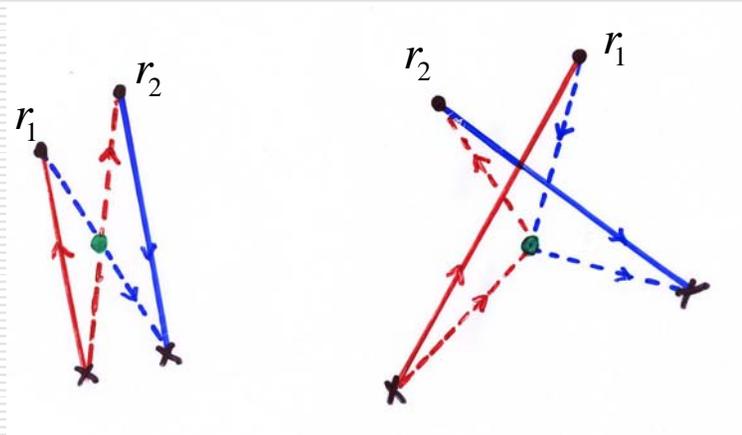
$$e^{i(\vec{k}_i + \vec{k}_e) \cdot \vec{R}}$$

Où est le croisement quantique ?

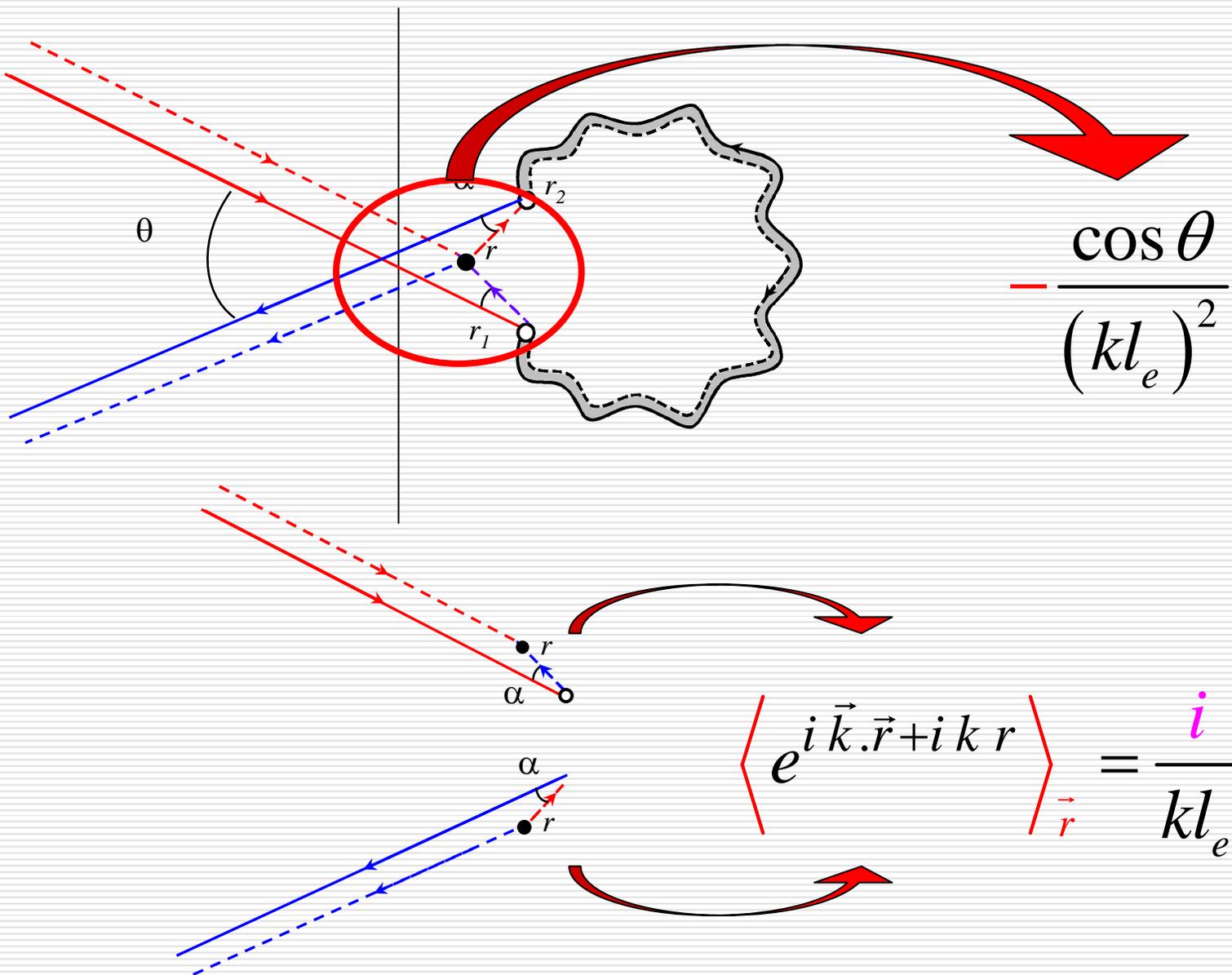


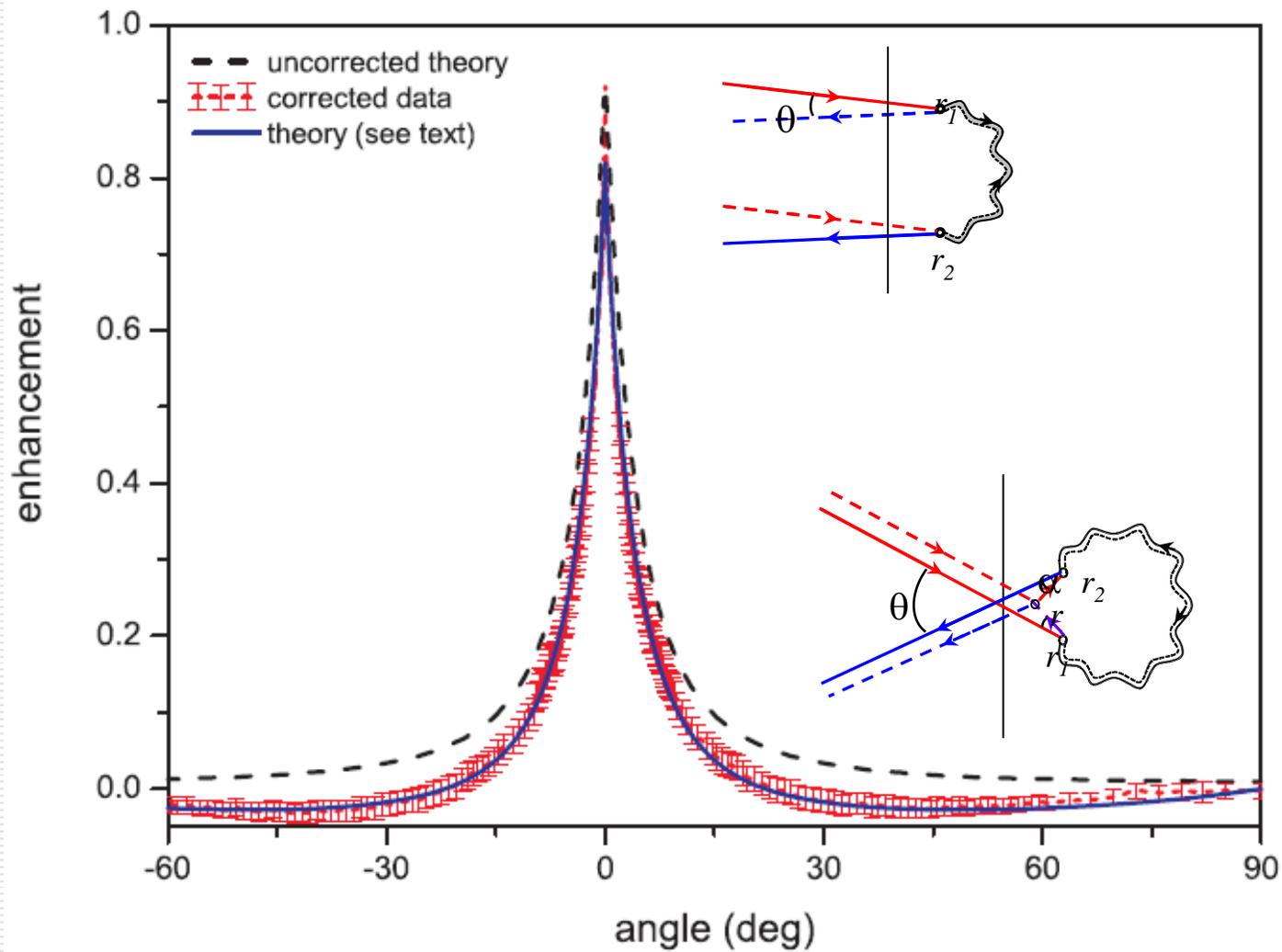


Ouverture angulaire $\theta_m \sim 1/kl_e$

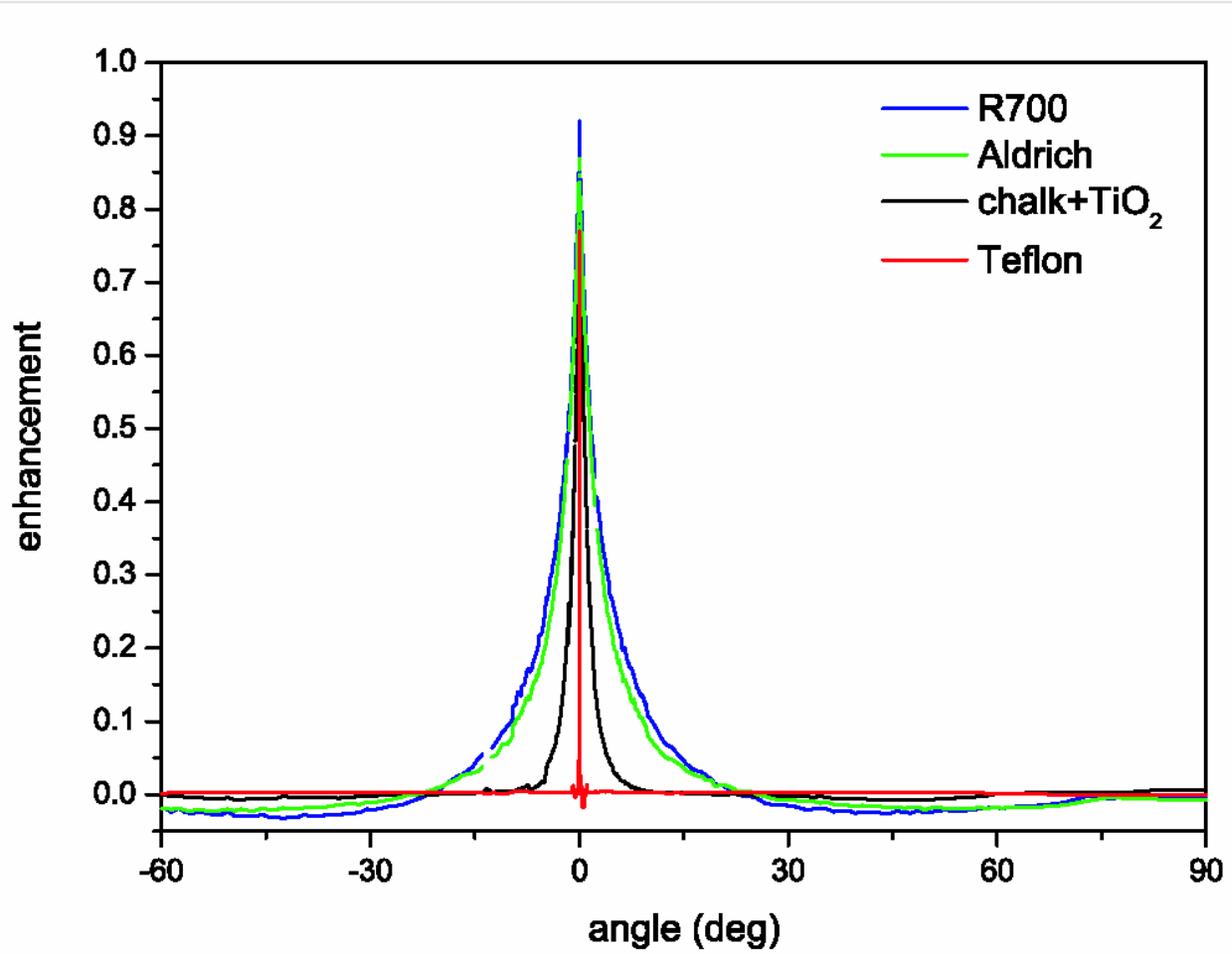


Ouverture angulaire $\theta_m \sim \pi/2$





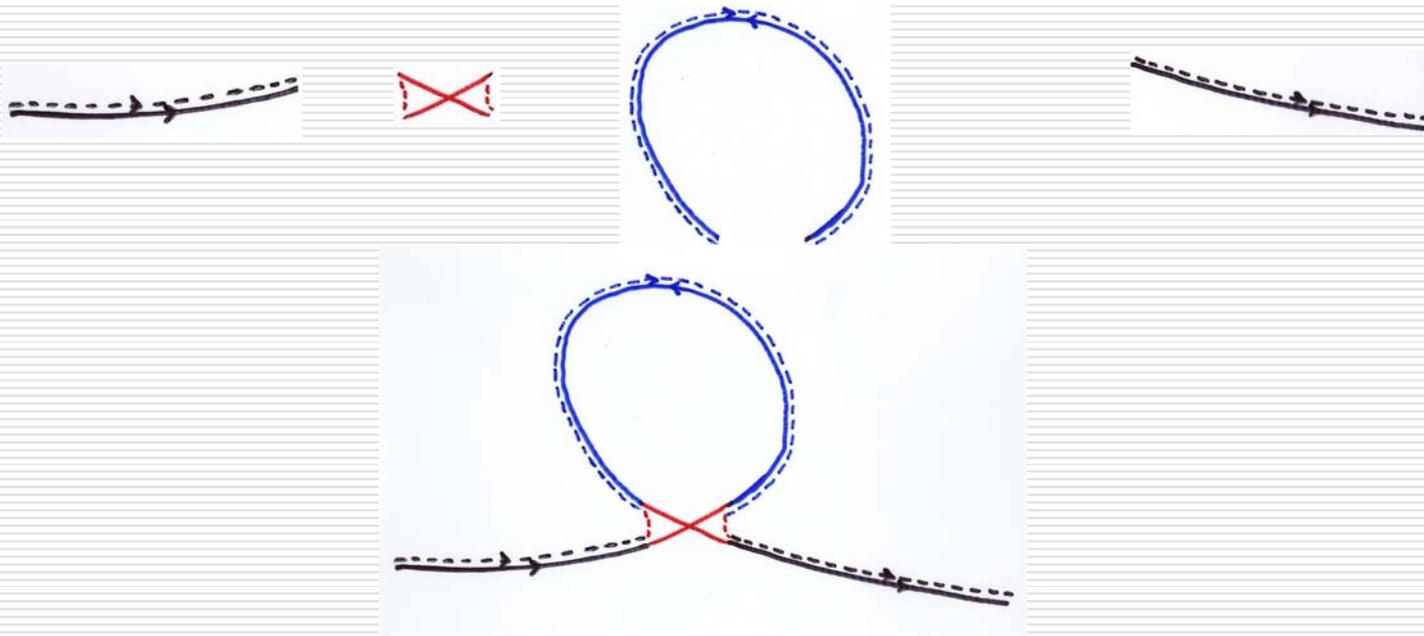
$$\int P_{\text{int}}(\theta) \sin \theta d\theta = 0$$



Conclusion

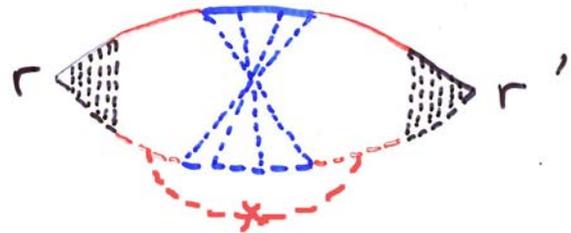
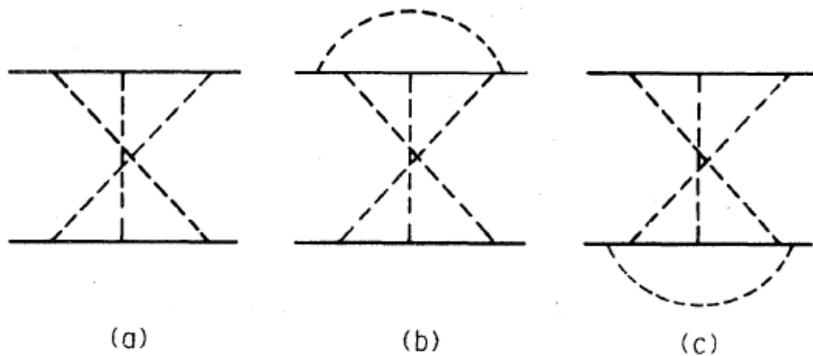
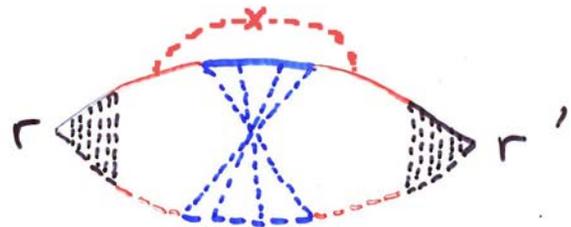
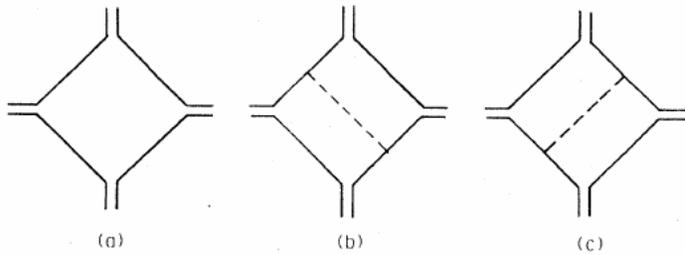
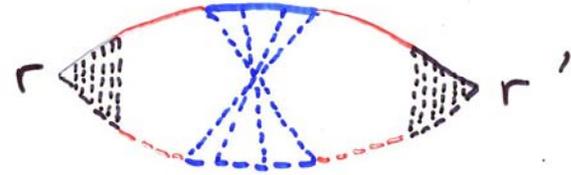
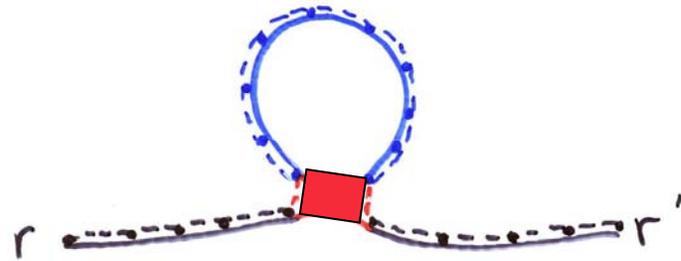
Transport quantique d'électrons ou de lumière dans les systèmes diffusifs

« Meccano » Diffusion classique (diffuson ou cooperon)
Croisements quantiques



Formulation simple des propriétés de cohérence de phase

et les diagrammes... ?

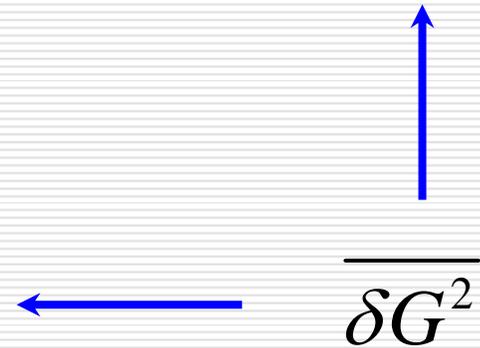
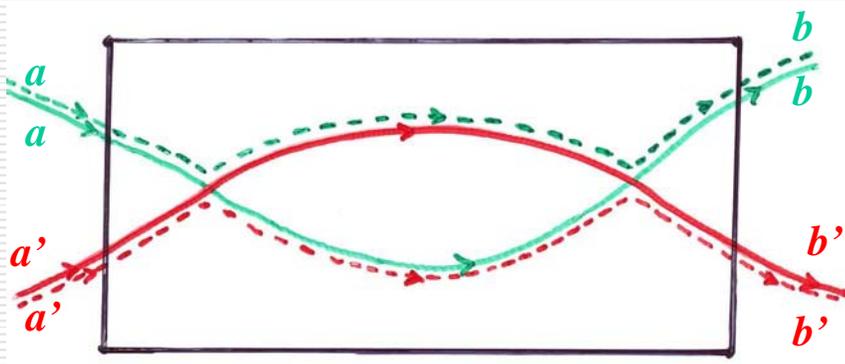


Fluctuations universelles de conductance

$$G = \frac{e^2}{h} T = \frac{e^2}{h} \sum_{a,b} T_{ab}$$

$$\overline{G^2} = \frac{e^4}{h^2} \left(\sum_{a,b,a',b'} \overline{T_{ab} T_{a'b'}} + \sum_{a,b,a',b'} \overline{\delta T_{ab} \delta T_{a'b'}} \right)$$

a, b : canaux d'entrée et de sortie



Fluctuations de conductance = 2 conductances et 2 croisements

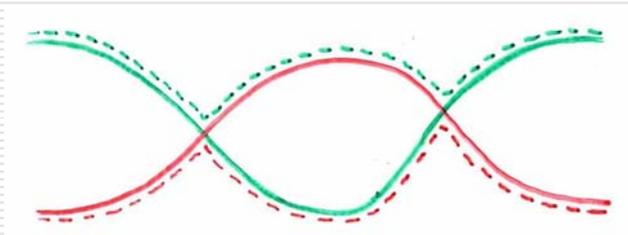
$$\overline{\delta G^2} = \left(\frac{e^2}{h} \right)^2 g^2 \times \frac{1}{g^2} \Rightarrow \left(\frac{e^2}{h} \right)^2 \quad \text{Universel}$$

Fluctuations gaussiennes ?

$$\overline{G} =$$



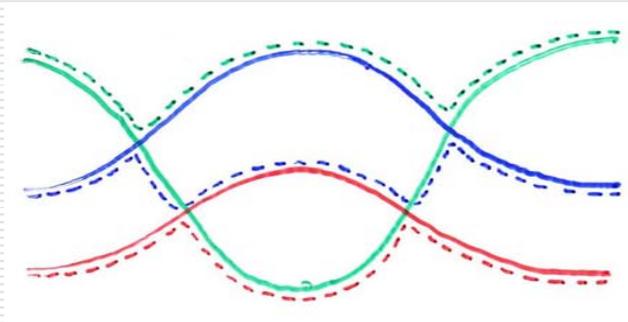
$$\overline{\delta G^2} =$$



2 conductances et 2 croisements

$$g^2 \times \frac{1}{g^2} \sim O(1)$$

$$\overline{\delta G^3} =$$



3 conductances et 4 croisements

$$g^3 \times \frac{1}{g^4} \sim O\left(\frac{1}{g}\right)$$

$$\overline{\delta G^n} =$$

n conductances et $(2n-2)$ croisements

$$\overline{\delta G^n} \sim g^n \times \frac{1}{g^{2n-2}} \sim O\left(\frac{1}{g^{n-2}}\right)$$

Fluctuations gaussiennes dans la limite $g \rightarrow \infty$