Continuous Measurement of a Driven Quantum Electrical Circuit

QUANTUM ELECTRONICS GROUP

SPEC CEA-Saclay

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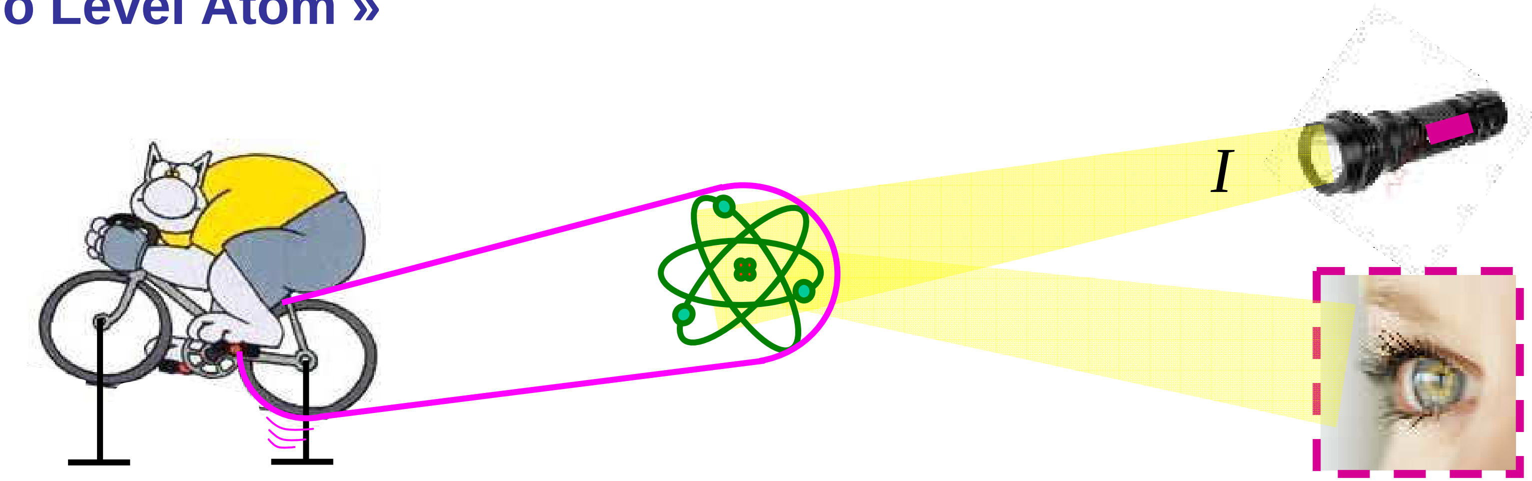
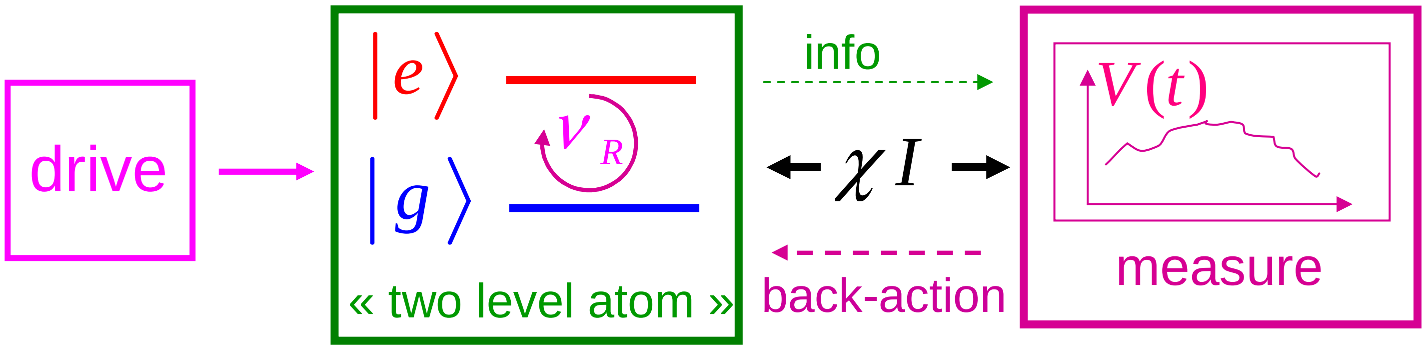
CDF, May 2008

Introduction: Continuous Measurement of a Driven «Two Level Atom »



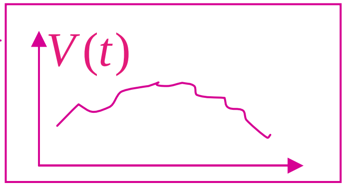


$$\hat{H} = -\frac{1}{2}h\nu_R\hat{\sigma}_X + h\chi\hat{v}\hat{\sigma}_Z$$

Influence of $\chi I / \nu_R$?

Introduction: influence of measurement strength χI

Quantum trajectory?

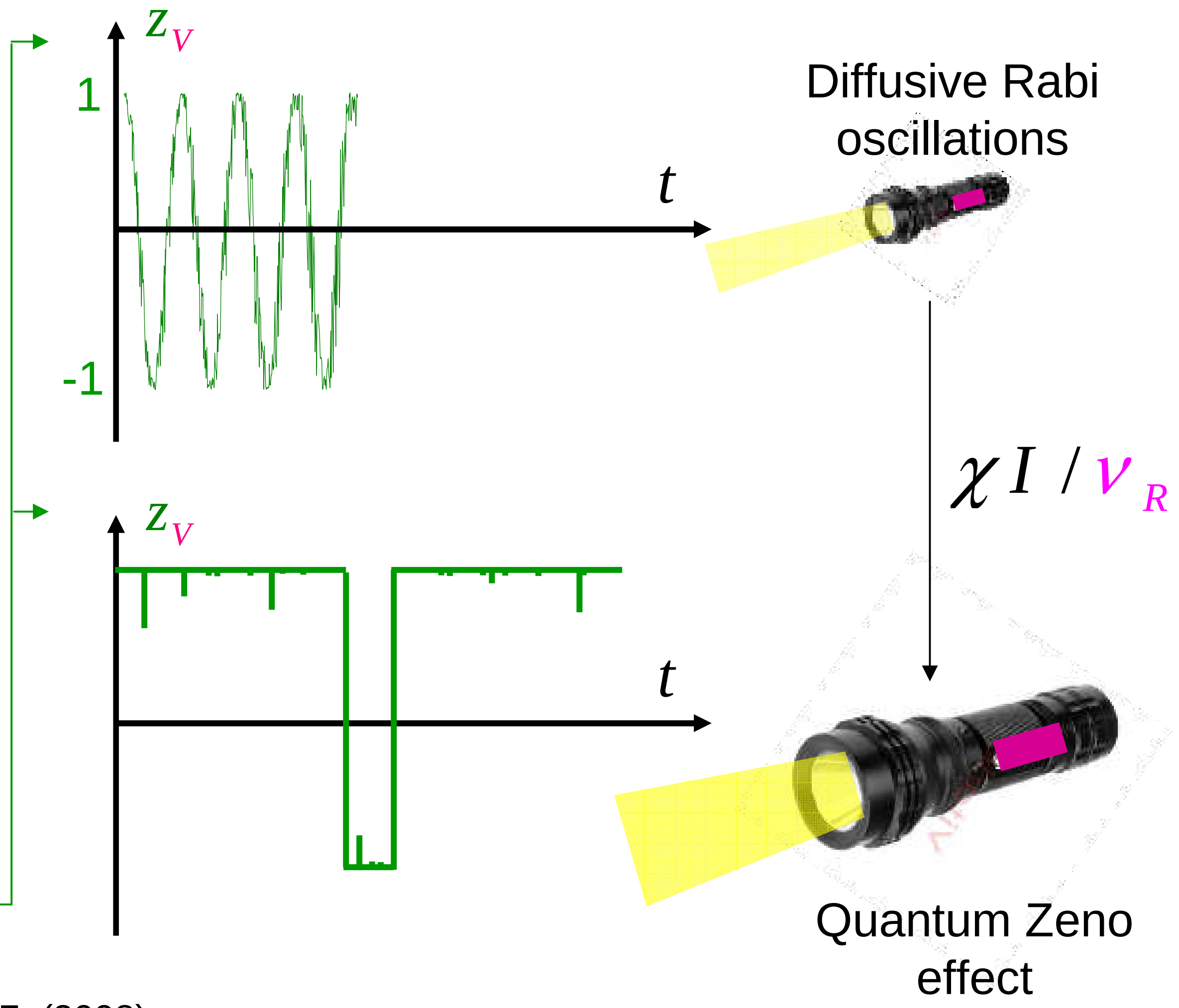


measure

density matrix
conditional to V(t):

$$\rho_V = \begin{pmatrix} \rho_{gg,V} & \rho_{ge,V} \\ \rho_{eg,V} & \rho_{ee,V} \end{pmatrix}$$

$$z_V = \rho_{gg,V} - \rho_{ee,V}$$

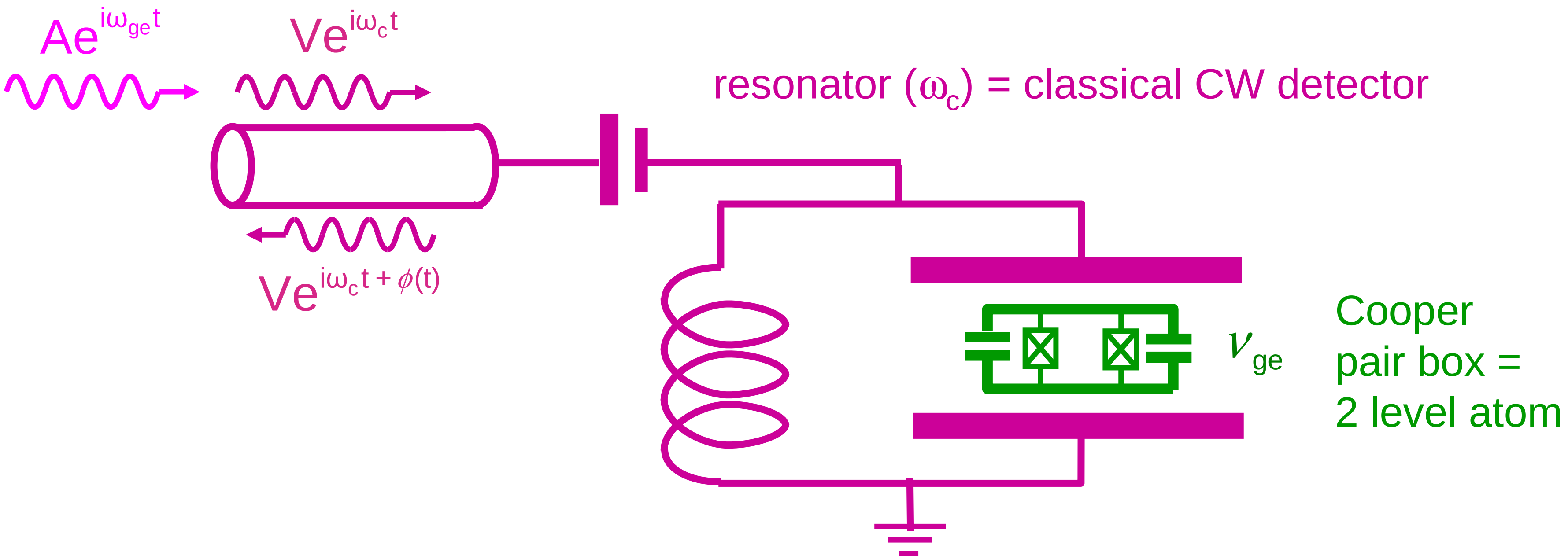


See J. Gambetta et al., Phys. Rev. A 77, (2008)

Can we address this problem with an electrical circuit ?

The transmon circuit

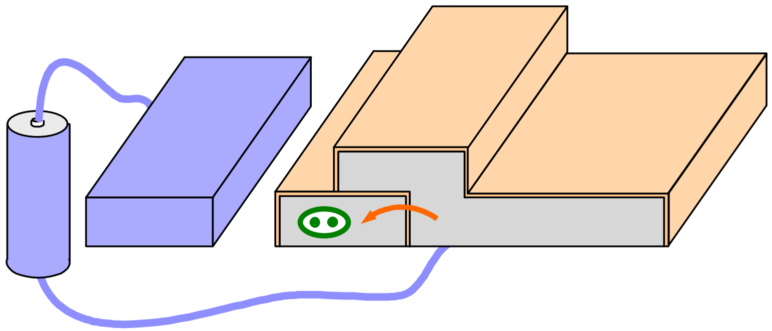
A. Blais *et al*., Phys. Rev. A **69**, 062320 (2004)
A. Walraff *et al*., Nature **431**, 162 (2004)
J. Koch *et al*., Phys. Rev. A **76**, 042319 (2007)



1) How does this system work?
2) Experimental implementation and results

The Cooper pair box (CPB)

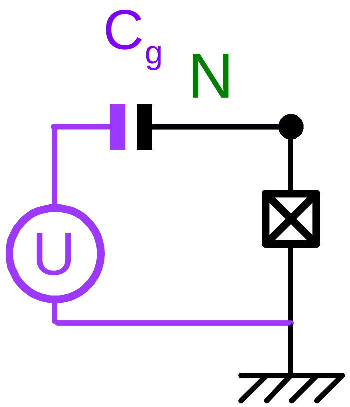
1 knob : $N_g = C_g U/2e$



1 d° of freedom $\left[\hat{\theta}, \hat{N}\right] = i$

2 energies:

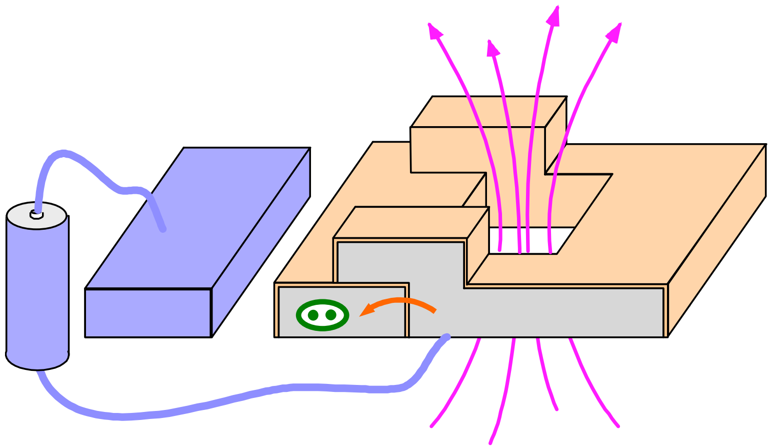
$$E_C = \frac{(2e)^2}{2C_{island}} \qquad E_J = \frac{h\Delta}{8e^2 R_t}$$



$$\hat{H} \approx E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\theta}$$

kinetic anharmonic potential

The split Cooper pair box (CPB)



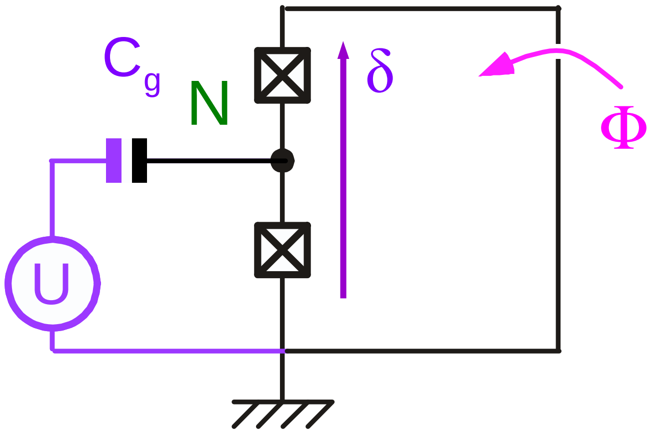
2 knobs : $N_g = C_g U/2e$

$$\delta = \Phi/\varphi_0$$

1 d°of freedom $\left[\hat{\theta},\hat{N}\right] = i$

2 energies:

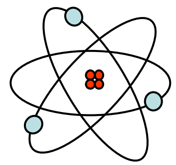
$$E_C = \frac{(2e)^2}{2C_{island}} \qquad E_J = \frac{h\Delta}{8e^2R_t}$$



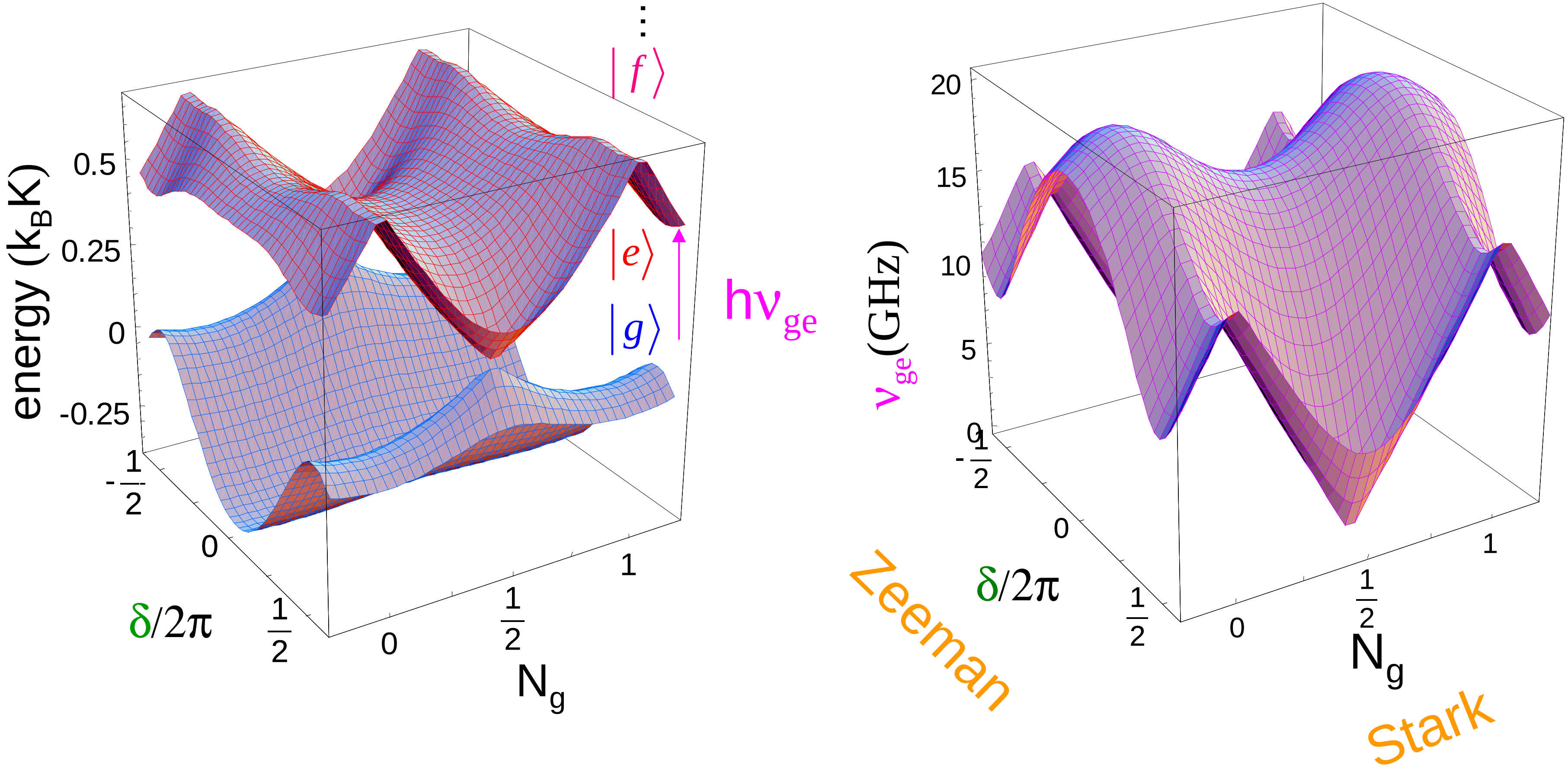
$$\hat{H} \approx E_C(\hat{N} - N_g)^2 - E_J \cos\frac{\delta}{2}\cos\hat{\theta}$$

tunable

The split CPB energy spectrum

artificial 

$E_J = 0.86\ k_B K \quad \sim \quad E_C = 0.68\ k_B K$

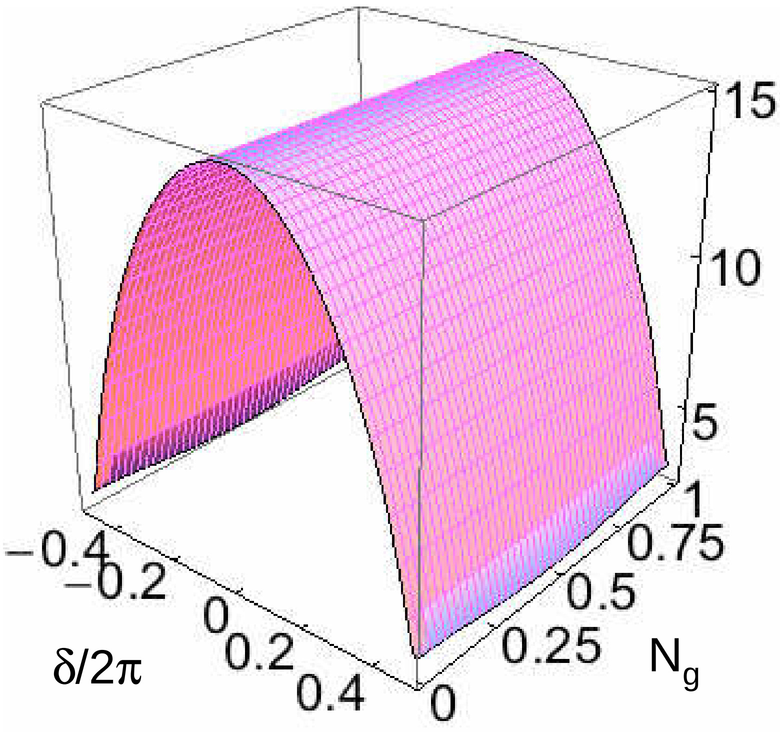


$$\hat{H} \approx E_C(\hat{N} - N_g)^2 - E_J \cos\frac{\delta}{2}\cos\hat{\theta}$$

(cool down below 0.1 K)

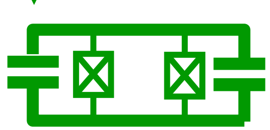
$$k_B T \ll h\nu_{ge}$$

The low Ec regime of the Cooper pair box $E_C << E_J$



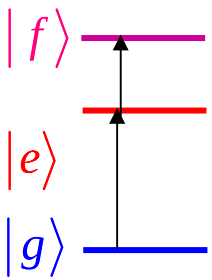
$E_J=3k_BK$, $E_c=0.1k_BK$

Extra C



$\nu_{ge}(\Phi)$

Charge noise insensitivity…



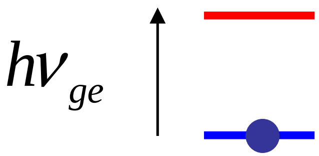
… but anharmonicity is low

$$|\nu_{fe} - \nu_{ge}| \sim \frac{E_c/4}{h} << \nu_{ge}$$

typically: 0.3GHz *6 GHz*

A two Level atom or fictitious spin

$$\psi(t) = \cos\frac{\theta}{2}|g\rangle + \sin\frac{\theta}{2} e^{\imath\varphi}|e\rangle$$

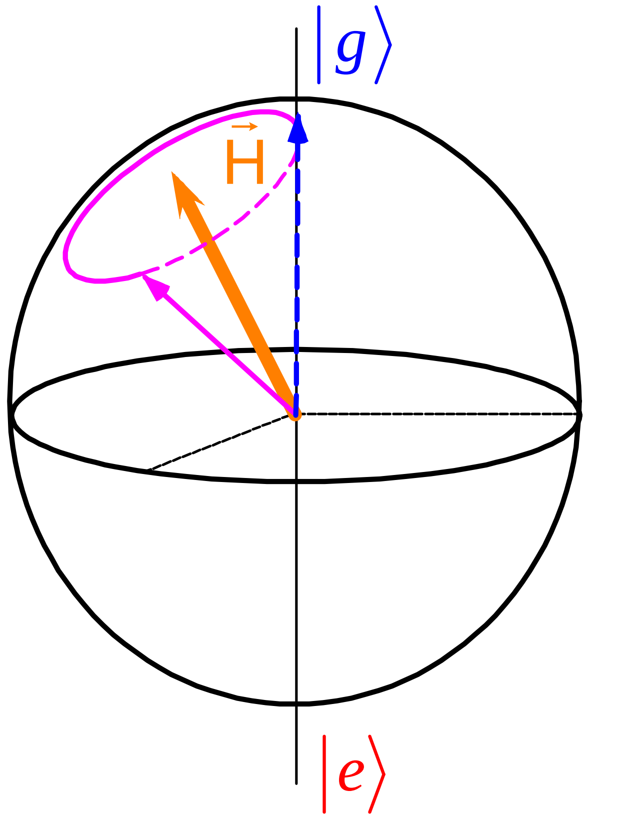


$$\hat{H} = -\frac{1}{2}\vec{H}(N_g, \delta).\vec{\hat{\sigma}}$$

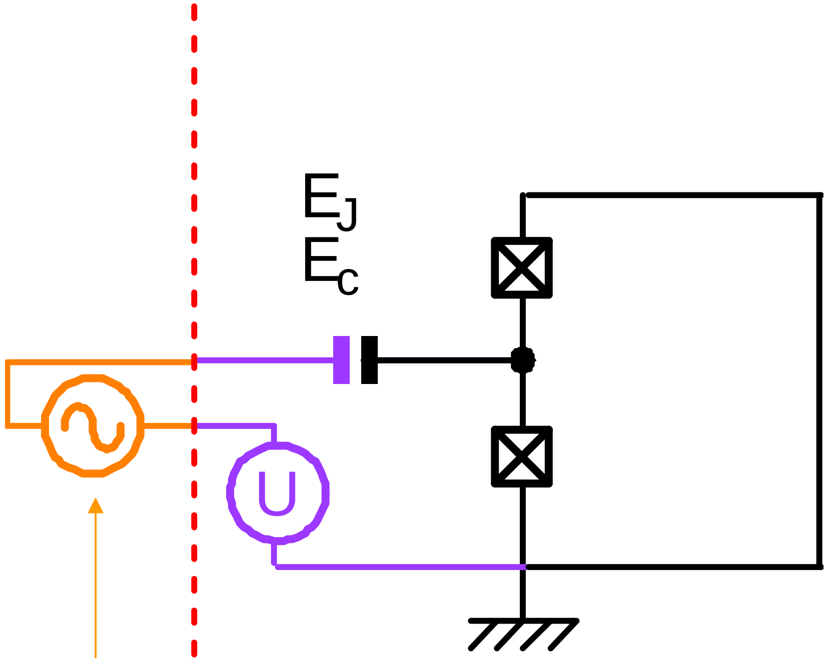
fictitious spin

in a fictitious magnetic field

$$\left\|\vec{H}\right\| = h\nu_{ge}$$



State manipulation of the CPB: 1) microwave drive at $\nu_{\mu w} \sim \nu_{ge}$

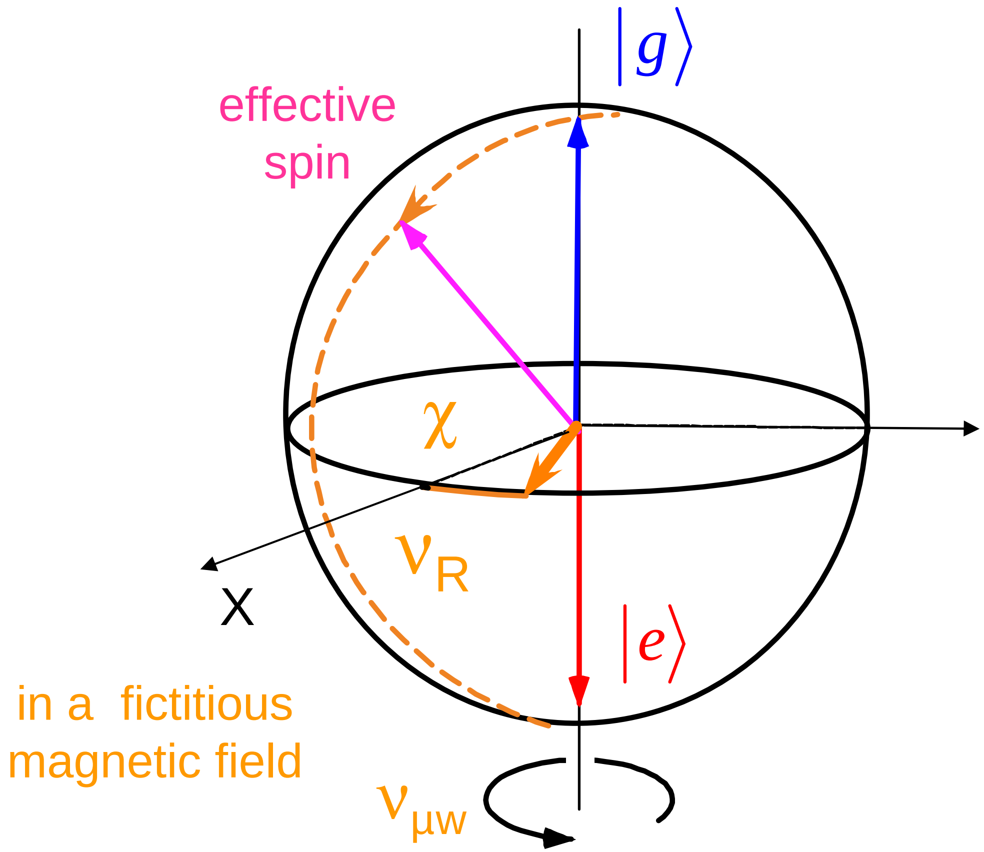


$$\Delta N_g \cos\left(2\pi\nu_{\mu w} t + \chi\right)$$

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J\cos\hat{\theta}$$

$$\left|\tilde{\psi}\right\rangle = e^{i\pi\nu_{\mu w} t\hat{\sigma}_Z}\left|\psi\right\rangle$$

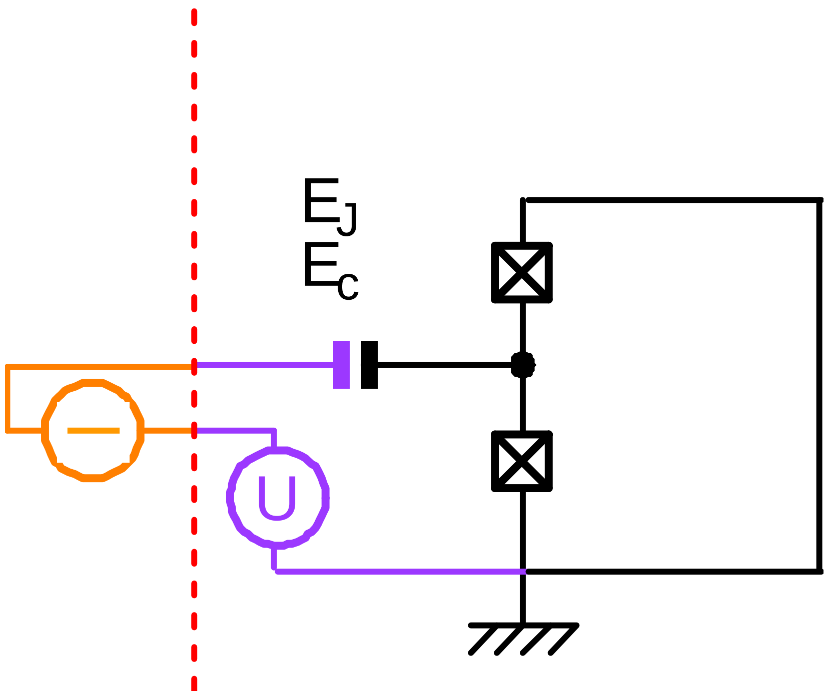
$$\tilde{\hat{H}} \approx -\frac{1}{2}h\Delta\nu\,\hat{\sigma}_Z - \frac{1}{2}h\nu_R\hat{\sigma}_X$$

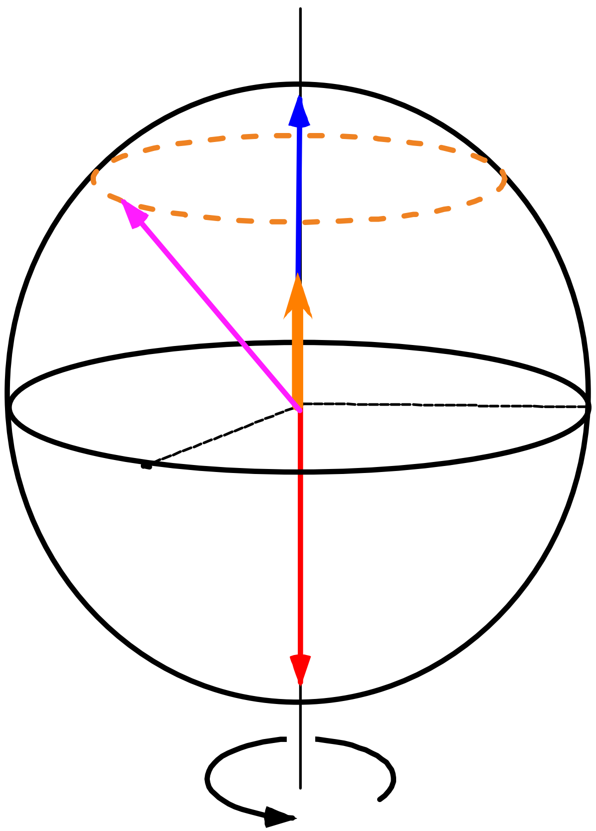


$$h\nu_R = 2E_C\left|\langle g|\hat{N}|e\rangle\right|\Delta N_g$$

$$\Delta\nu = \nu_{ge} - \nu_{\mu w}$$

State manipulation of the CPB: 2) Z rotations from detuning





$$\left|\tilde{\psi}\right\rangle = e^{i\pi\nu_{\mu w} t \hat{\sigma}_Z} \left|\psi\right\rangle$$

$$\tilde{\hat{H}} \approx -\frac{1}{2} h \Delta\nu \, \hat{\sigma}_Z - \frac{1}{2} h \nu_R \hat{\sigma}_X$$

$$\Delta\nu = \nu_{ge} - \nu_{\mu w}$$

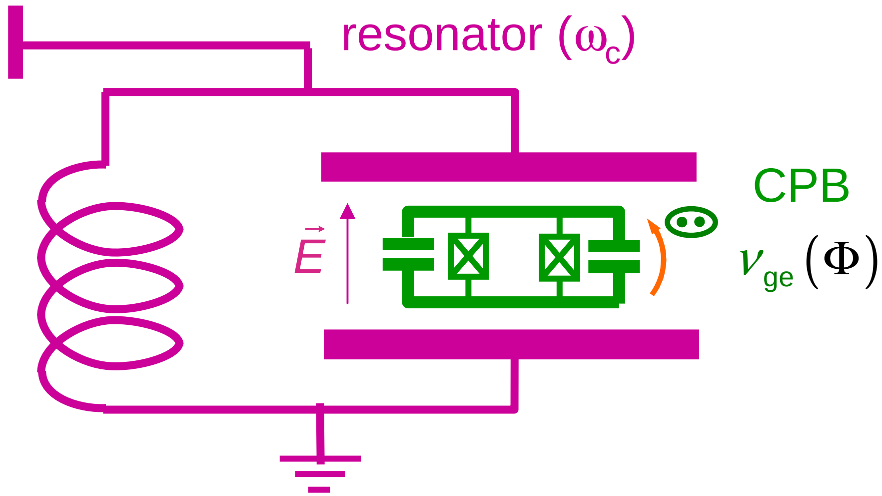
The coupled CPB-resonator system

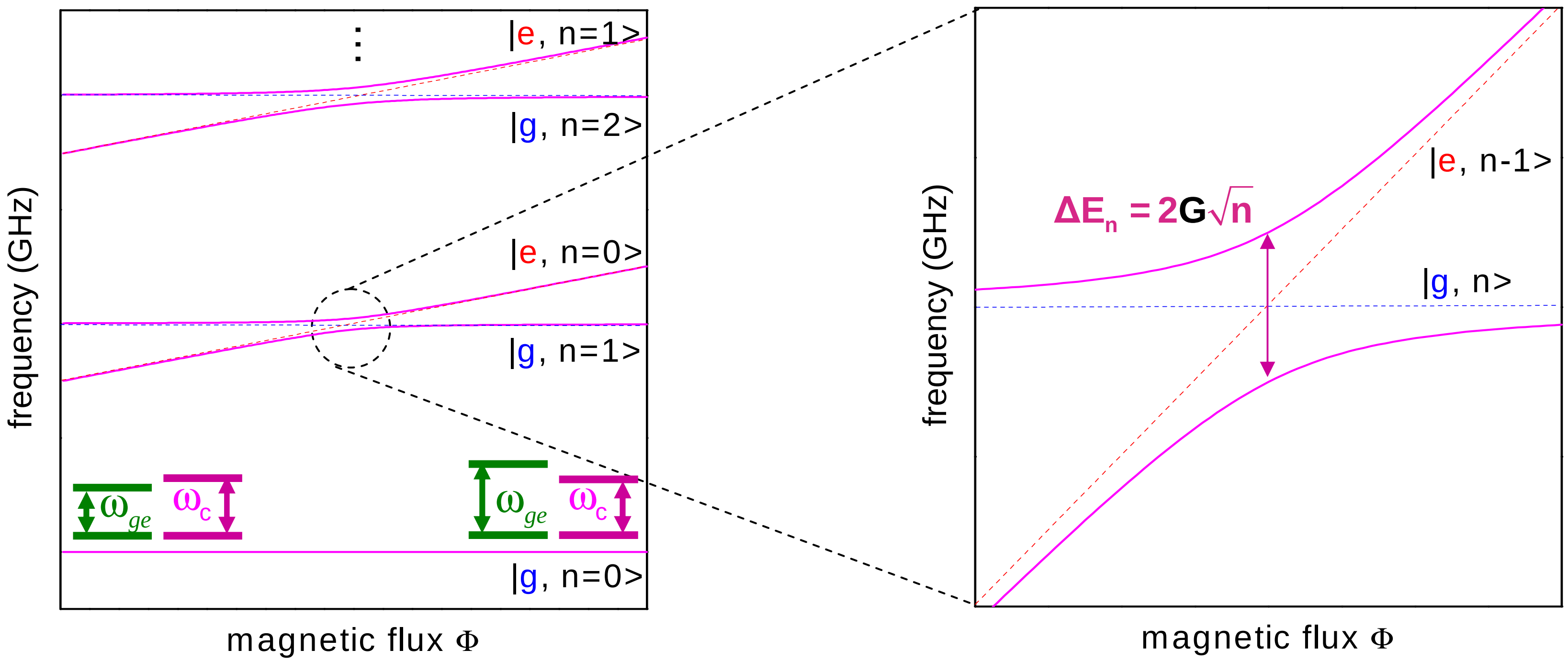
A. Blais *et al.*, Phys. Rev. A **69** (2004)

Jaynes-Cummings Hamiltonian
(Cavity Quantum Electrodynamics) :

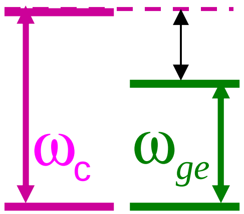
$$\hat{H} = -\frac{\hbar\omega_{ge}}{2}\hat{\sigma}_z + \hbar\omega_c\hat{a}^+\hat{a} + \hbar G(\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^+)$$

$$G \propto C_g$$



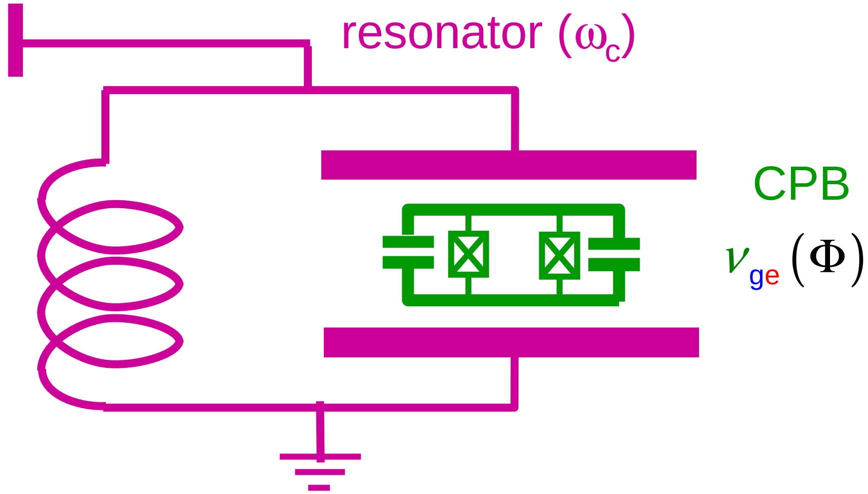


Jaynes-Cummings in the dispersive regime



$$|\Delta| = |\omega_{ge} - \omega_c| >> G$$

$$n << n_{crit} = \frac{\Delta^2}{4G^2}$$



$$\hat{H} = -\frac{\hbar\omega_{ge}^0}{2}\hat{\sigma}_z + \hbar\omega_c^0\hat{a}^+\hat{a} + \hbar G(\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^+)$$

$$\hat{H}_{eff} = -\frac{\hbar}{2}(\omega_{ge} + \chi)\hat{\sigma}_z + \hbar(\omega_c - \chi\hat{\sigma}_z)\hat{a}^+\hat{a}$$

$$\chi = G^2/\Delta$$

$$\omega_c = \omega_c^0 - \chi\sigma_z$$

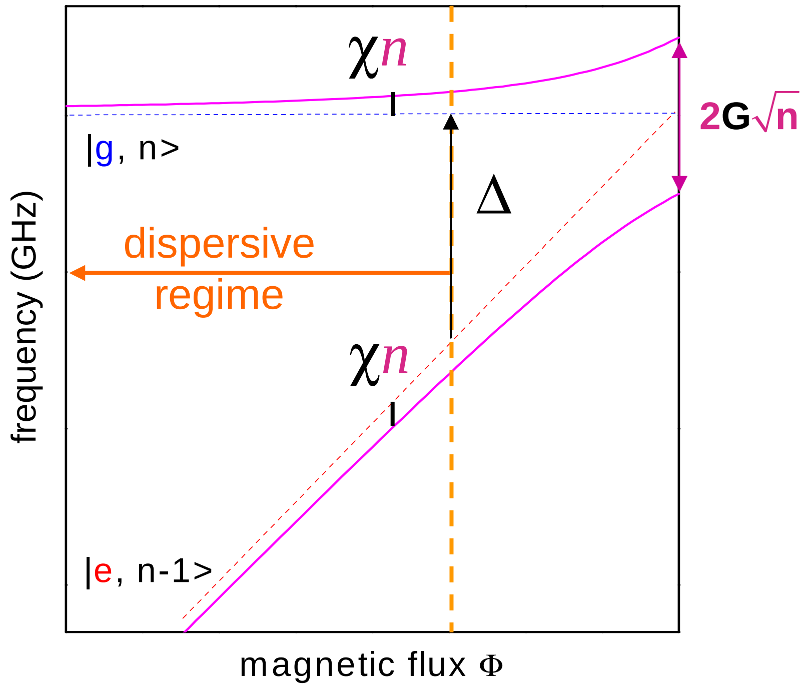
cavity pull

$$\omega_{ge} = \omega_{ge}^0 + (n + 1/2)\chi$$

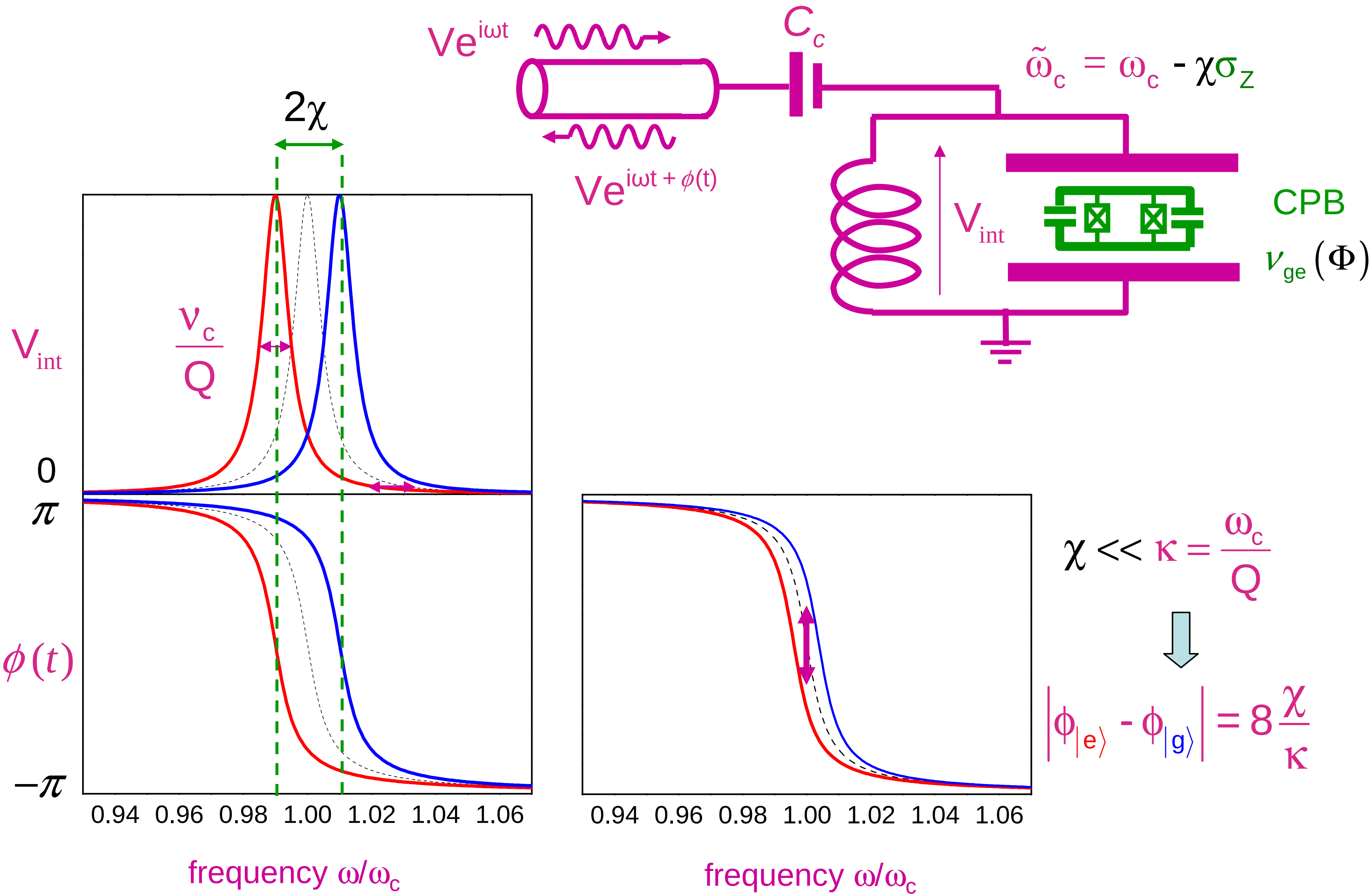
AC Stark shift

Lamb shift

measurement

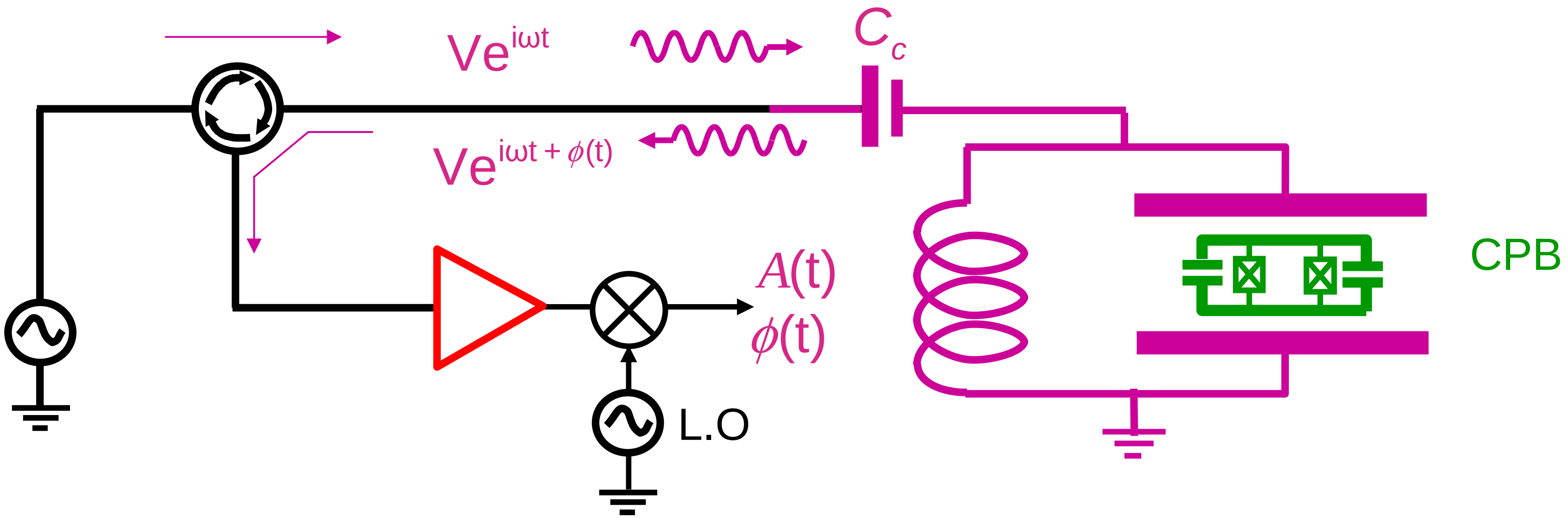


Use cavity pull to measure the CPB



Measure of $\phi(t)$?

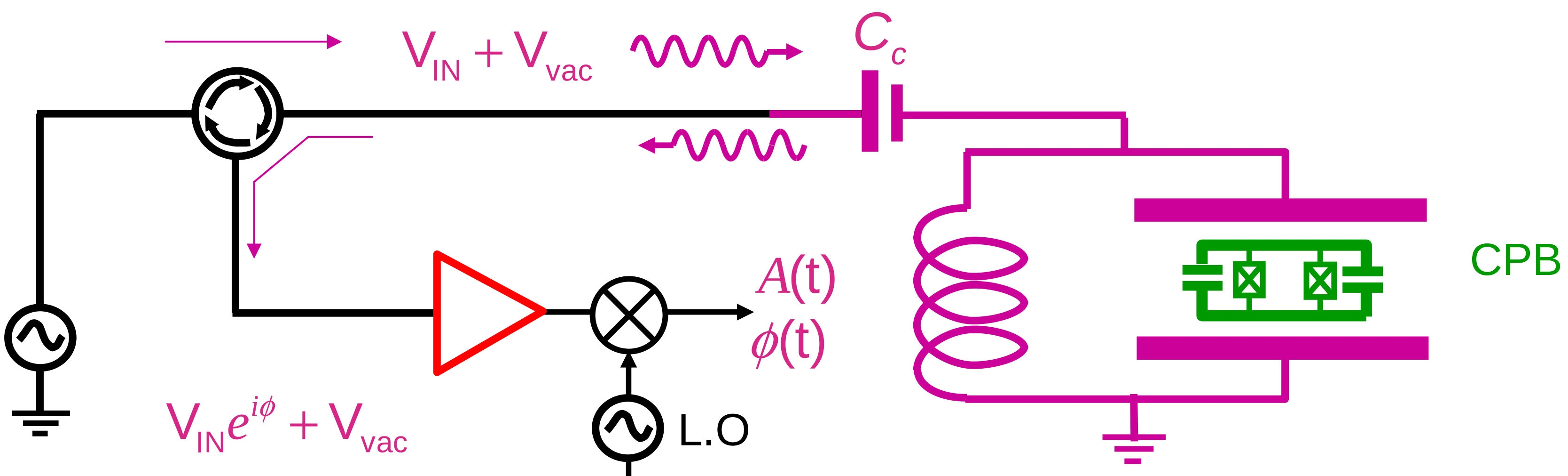
Homodyne detection of the reflected signal



Measurement process?

Sensitivity?

Back-action during the measurement process: ideal measurement time



$$\omega_{ge} = \omega_{ge}^0 + [n(t) + 1/2]\chi$$

1) ac Stark shift $\omega_{ge} = \omega_{ge}^0 + [\bar{n} + 1/2]\chi$

2) usefull dephasing $a|g\rangle + b|e\rangle \rightarrow a|g\rangle + be^{i\varphi}|e\rangle$

$$\varphi(t) = \bar{\omega}_{ge} t + 2\chi \int_0^t \delta n(t') dt'$$

Time T_{meas} to discriminate $|g\rangle$ and $|e\rangle$

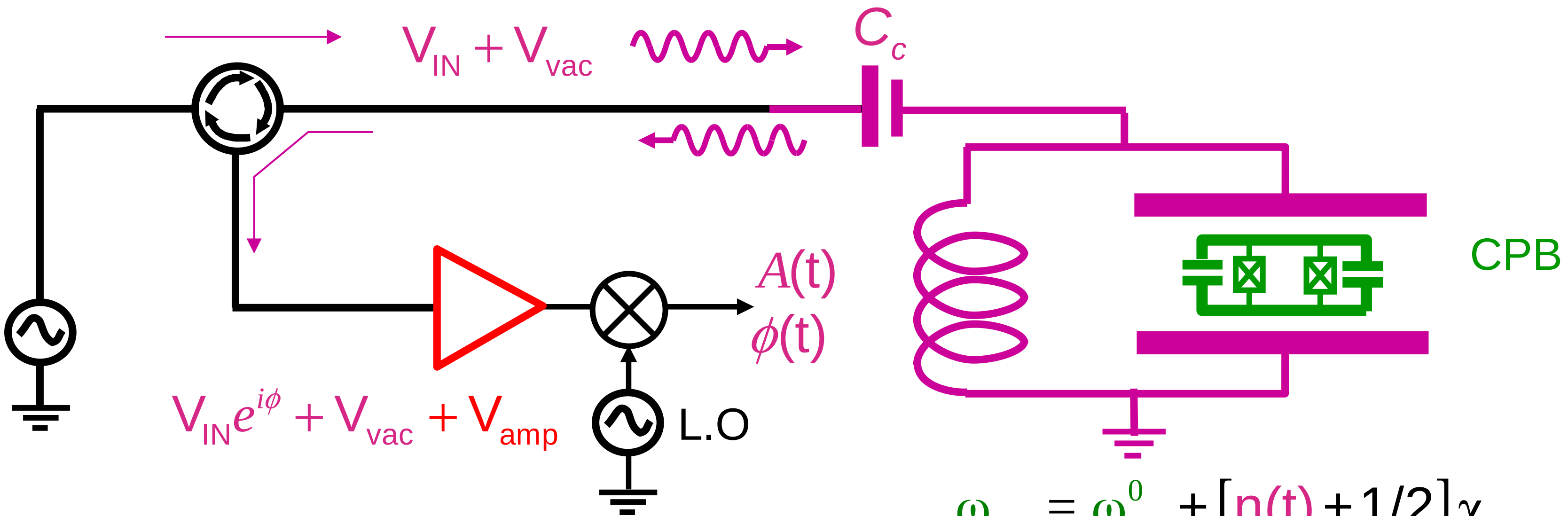
$$\Rightarrow 2T_{meas} = T_{\varphi, phot} = \frac{\kappa}{8\bar{n}\chi^2}$$

Projection:

$$|g\rangle \rightarrow p_g = |a^2|$$

$$|e\rangle \rightarrow p_e = |b^2|$$

Actual measurement and dephasing times



$$\omega_{ge} = \omega_{ge}^{0} + [n(t) + 1/2]\chi$$

1) noise of input signal $V_{vac}(t)$

2) noise of amplifier $S_{Vamp}(\omega) = Z_0 k T_N$

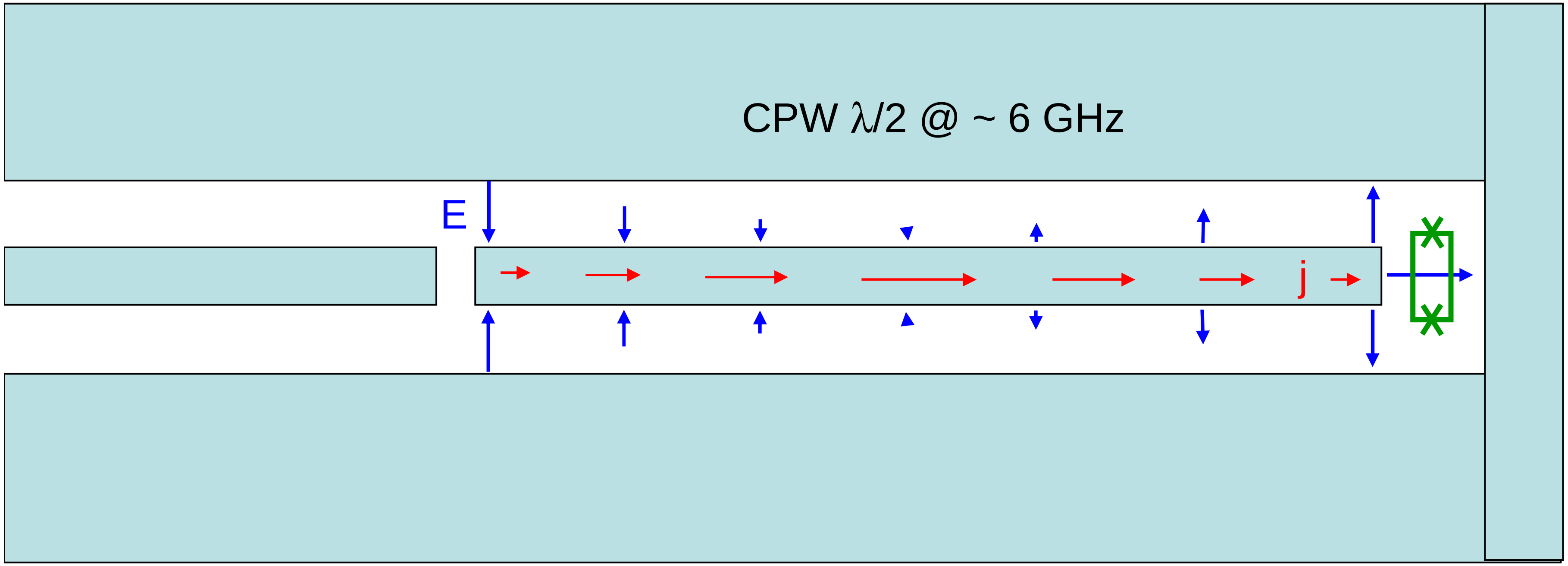
$$T_{meas} = \frac{\kappa}{16\eta\bar{n}\chi^2} \qquad \eta = \frac{1}{1 + \frac{kT_N}{\hbar\omega_c/2}} \text{ :measurement efficiency}$$

single shot measurement($T_{meas} < T_1$) : impossible $\qquad T_N \approx 2.5K$

Other decoherence channels for the CPB

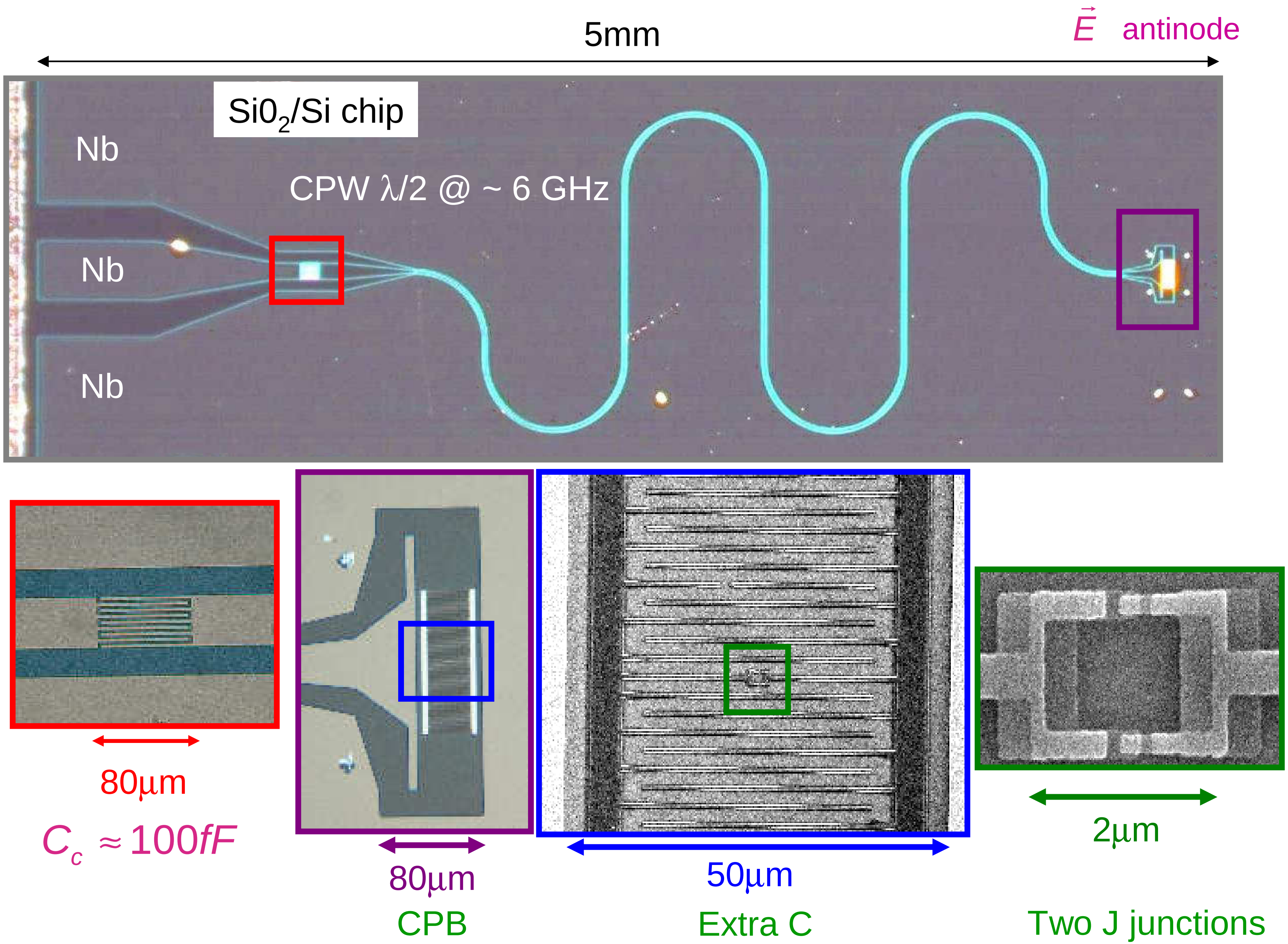
$$\chi \ll \kappa \quad \Longrightarrow \quad T_2^{-1} = (2T_1)^{-1} + T_{\varphi,0}^{-1} + T_{\varphi,\mathrm{phot}}^{-1} \qquad T_{\varphi,\mathrm{phot}} = \frac{\kappa}{8\bar{n}\chi^2}$$

Implementation



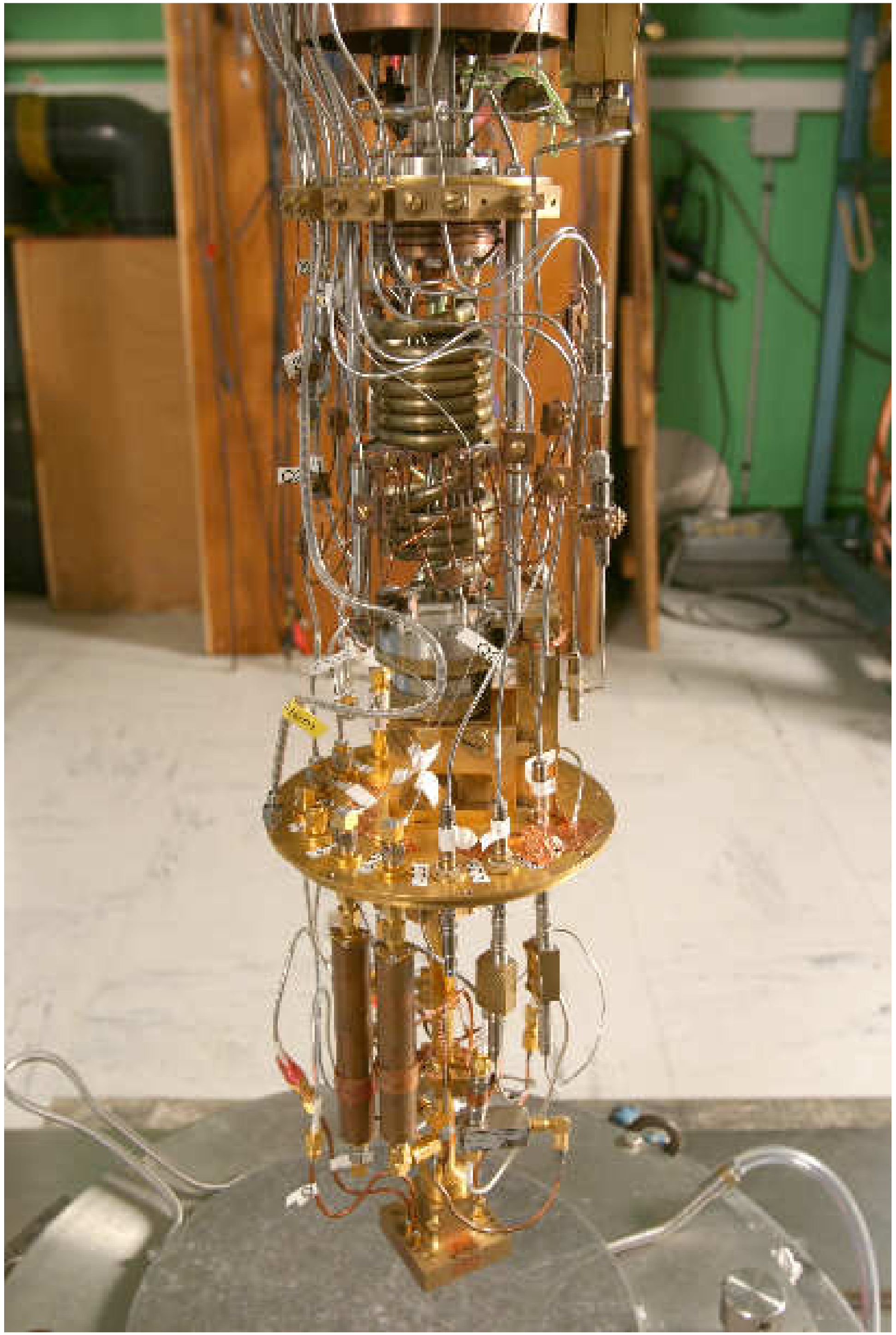
distributed resonator

Implementation

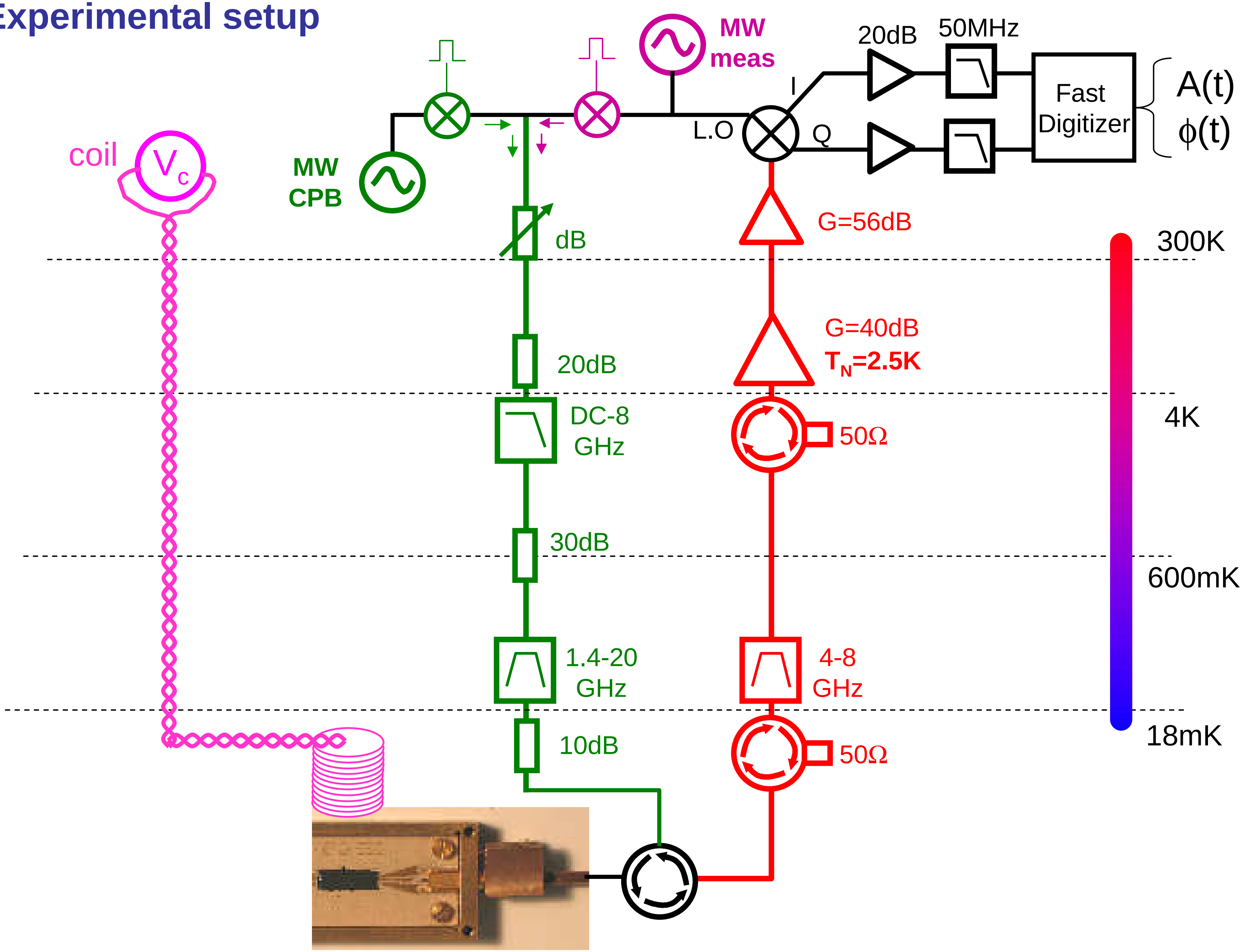


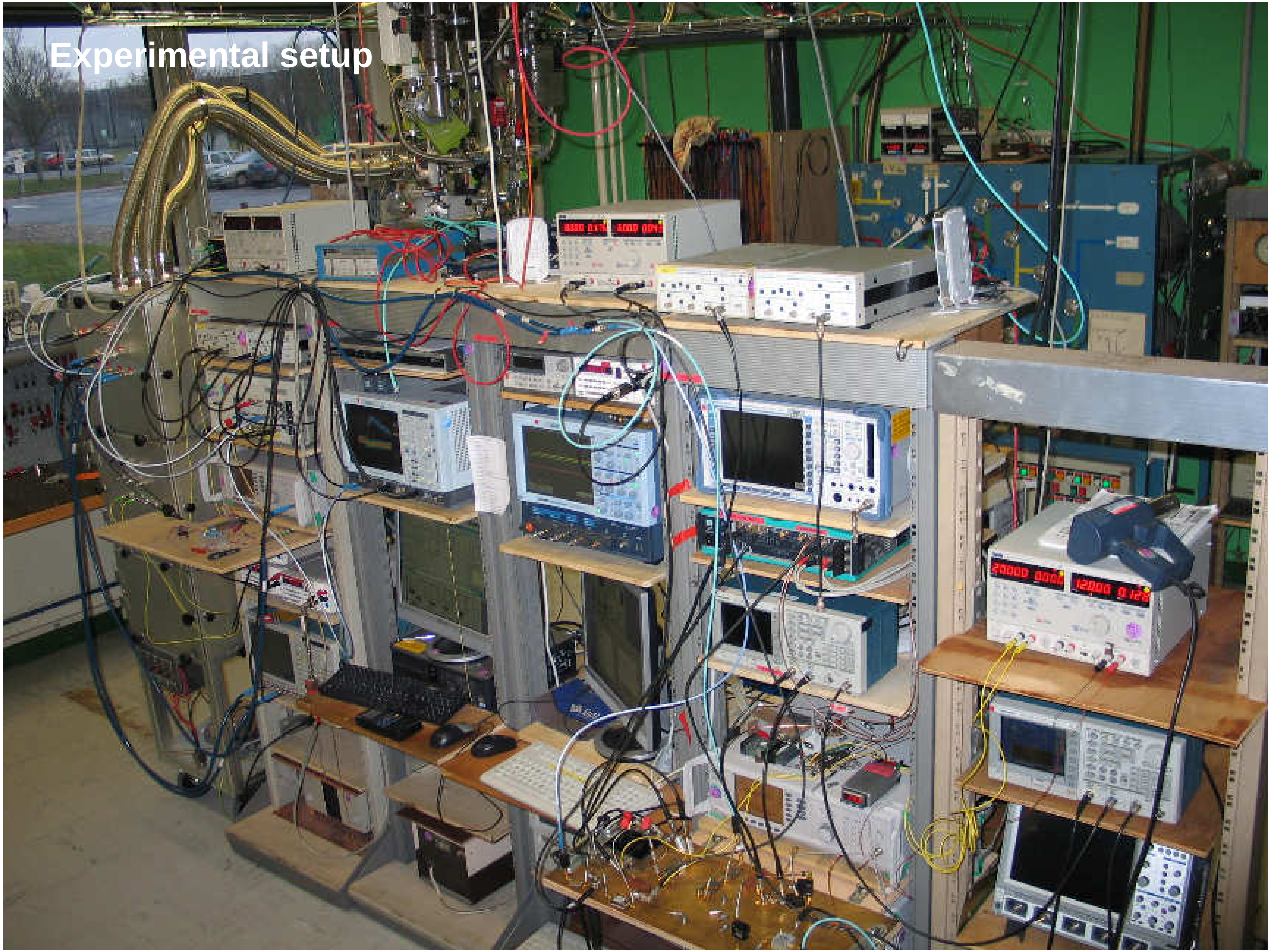
Experimental setup

Dilution fridge
(20 mK)

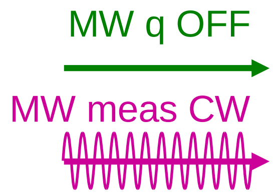


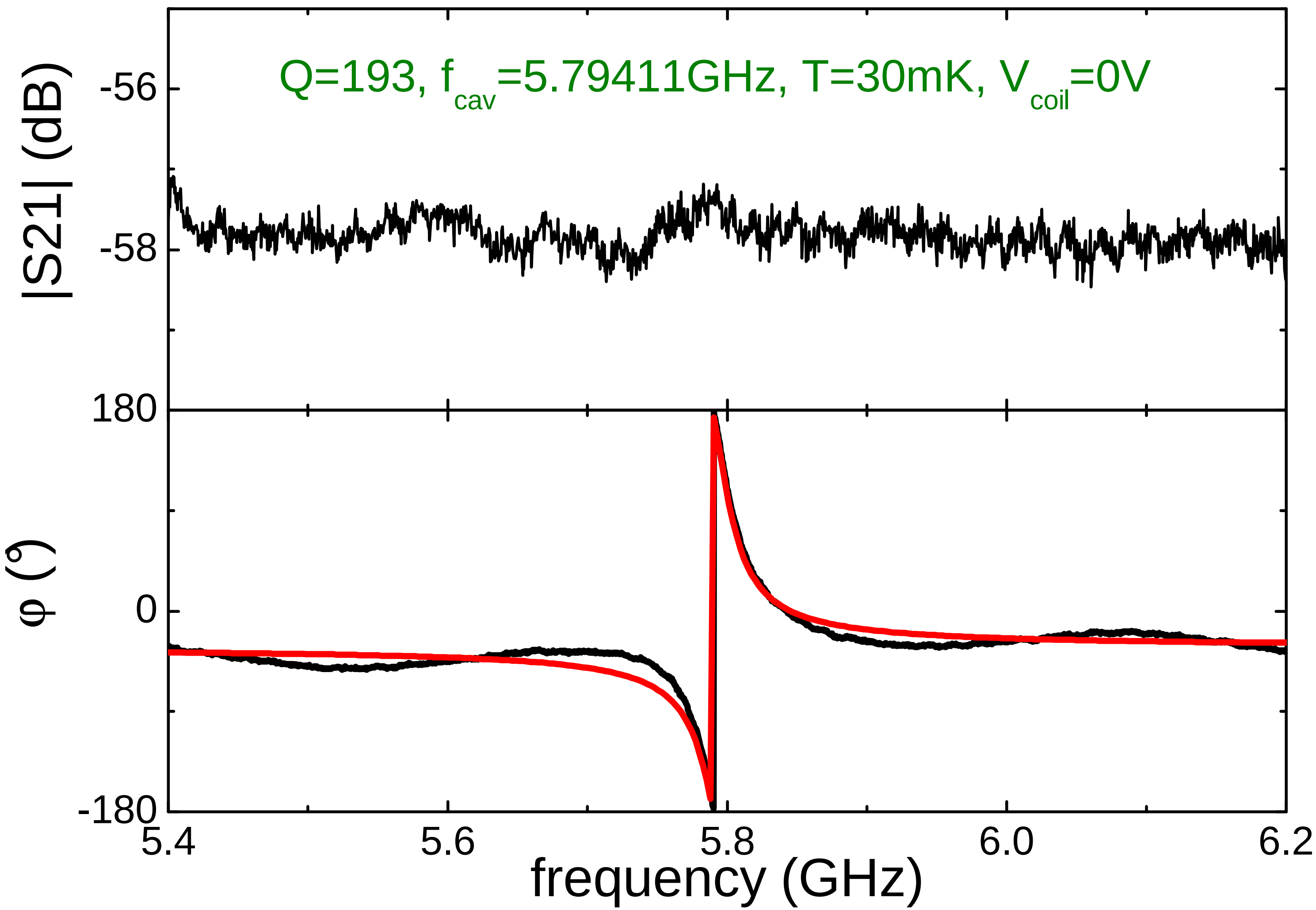
Experimental setup



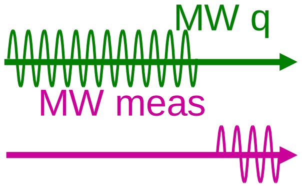

Experimental setup

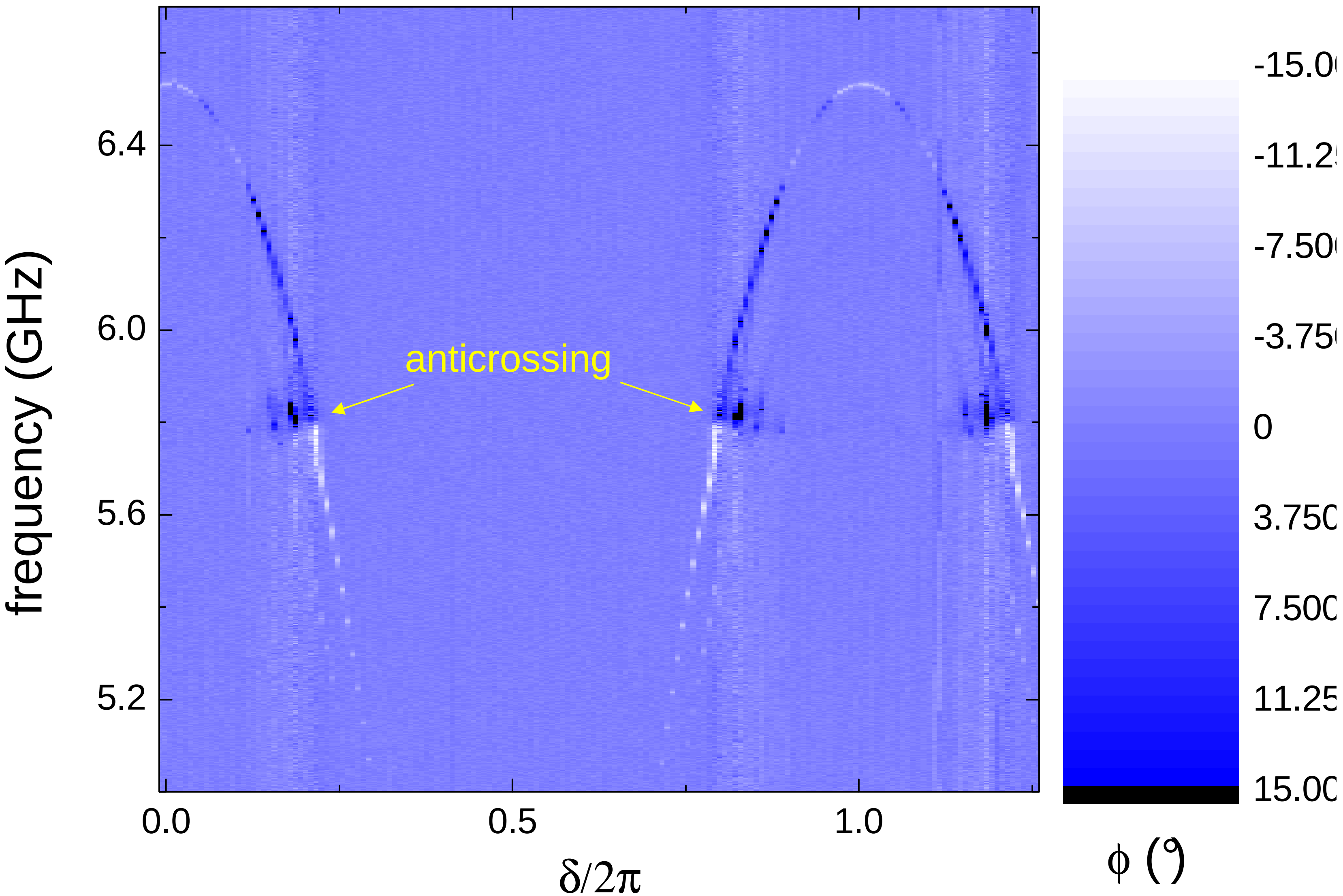
Resonator characterization



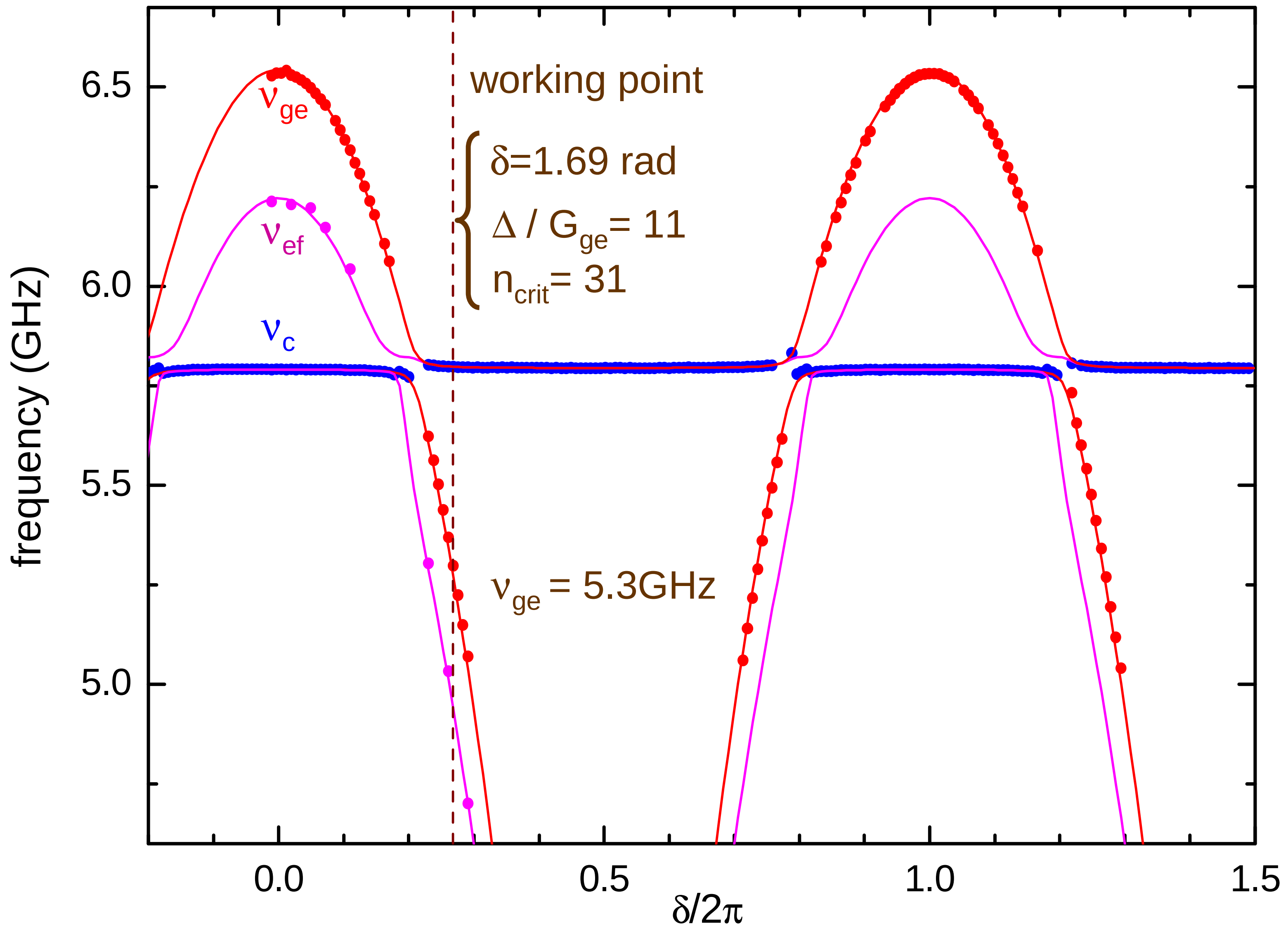


Cooper pair box spectroscopy



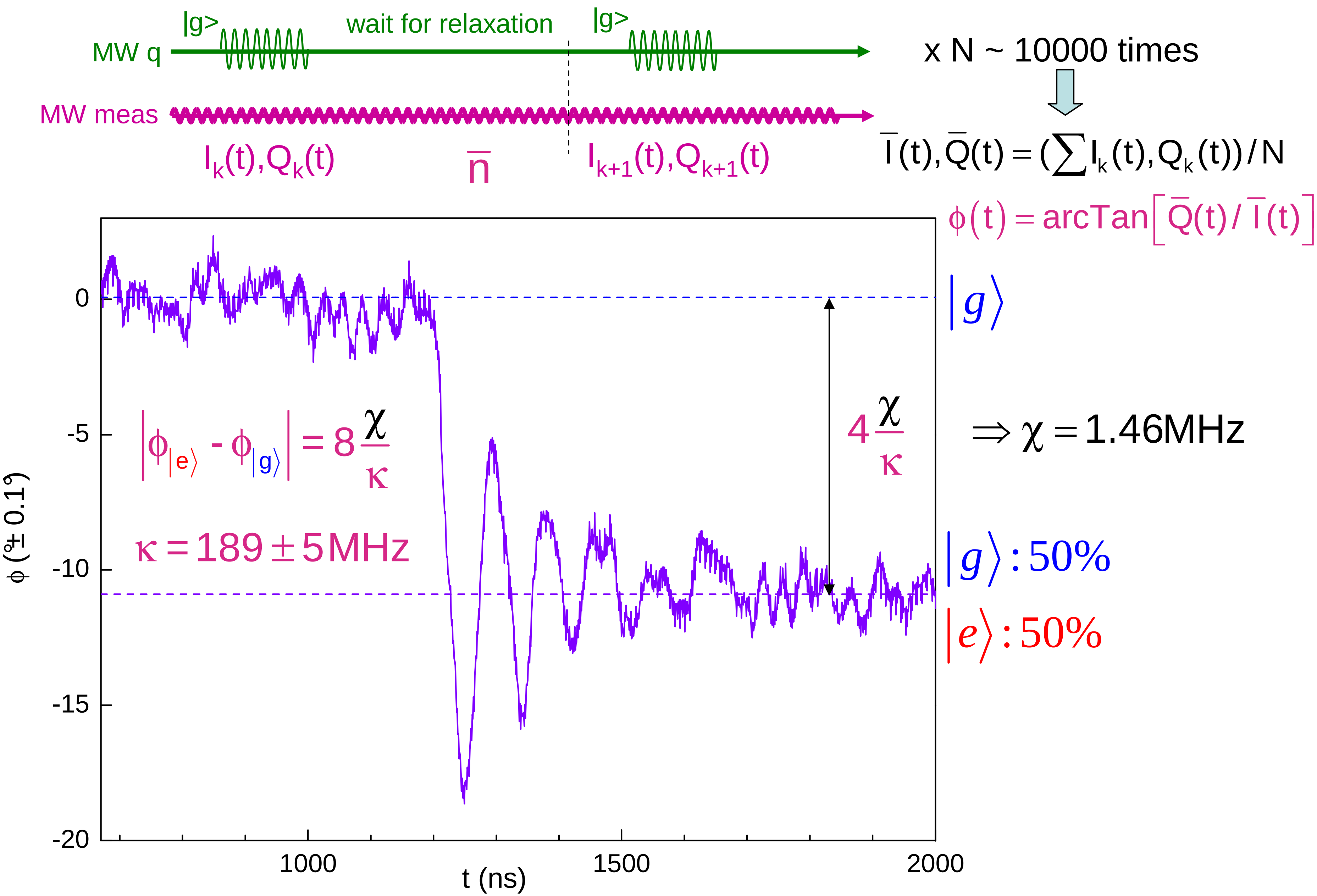


Cooper pair box parameters from spectroscopy fit



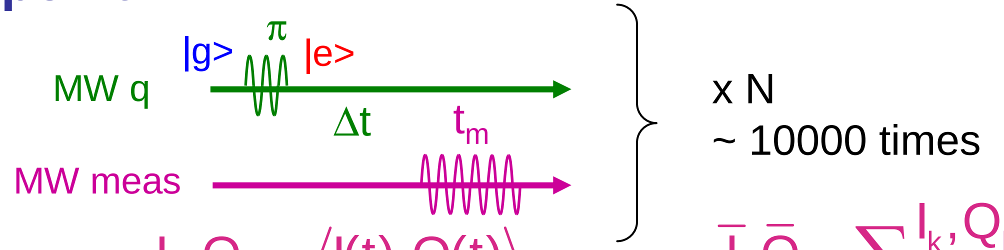
E_c = 1.16 K E_J = 2*10.1 K G_{ge} = 2.50 MHz ν_c = 5.794 GHz

Time domain Rabi oscillations: « real-time » mode and χ determination



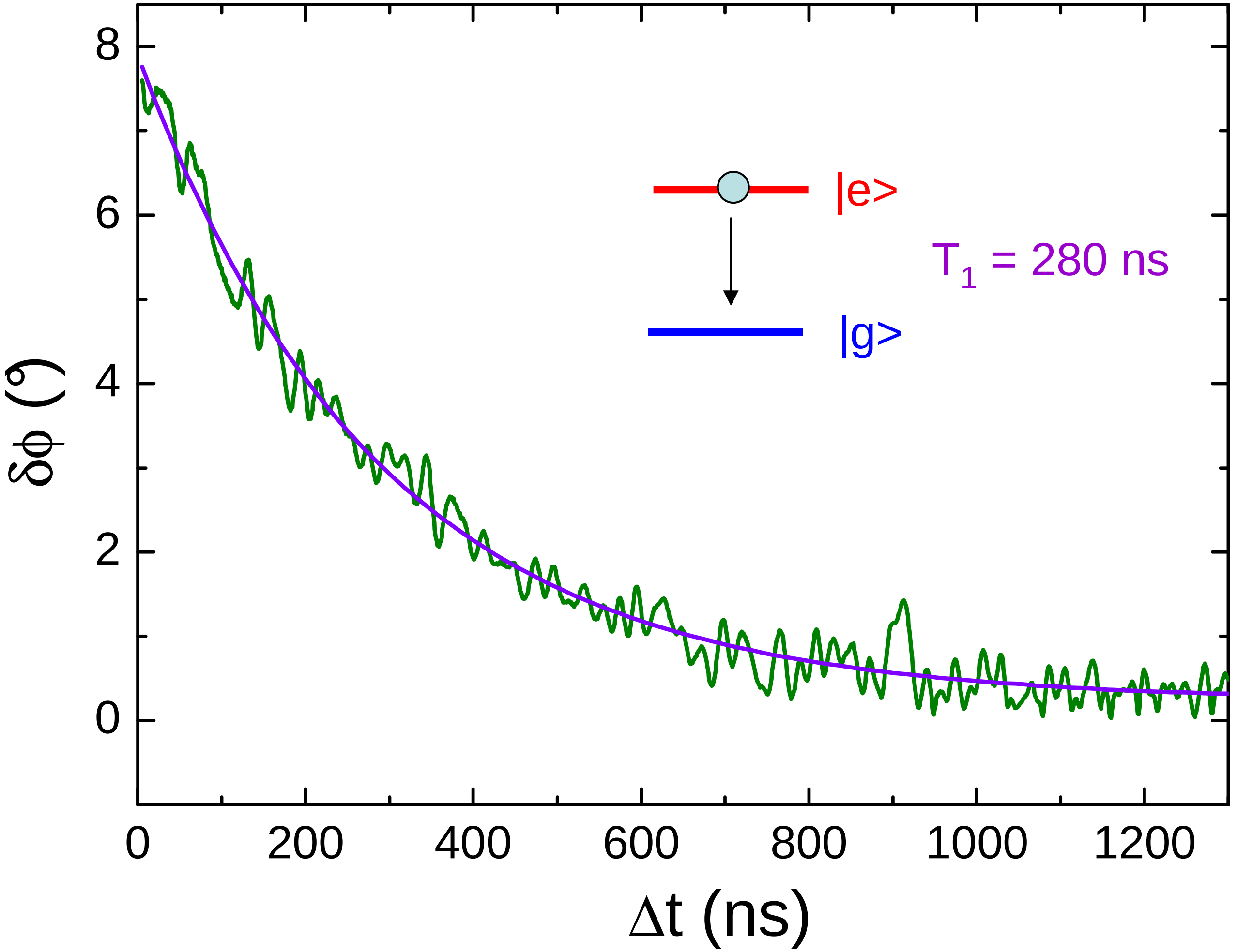
Coherence times at working point

1) Relaxation time T_1



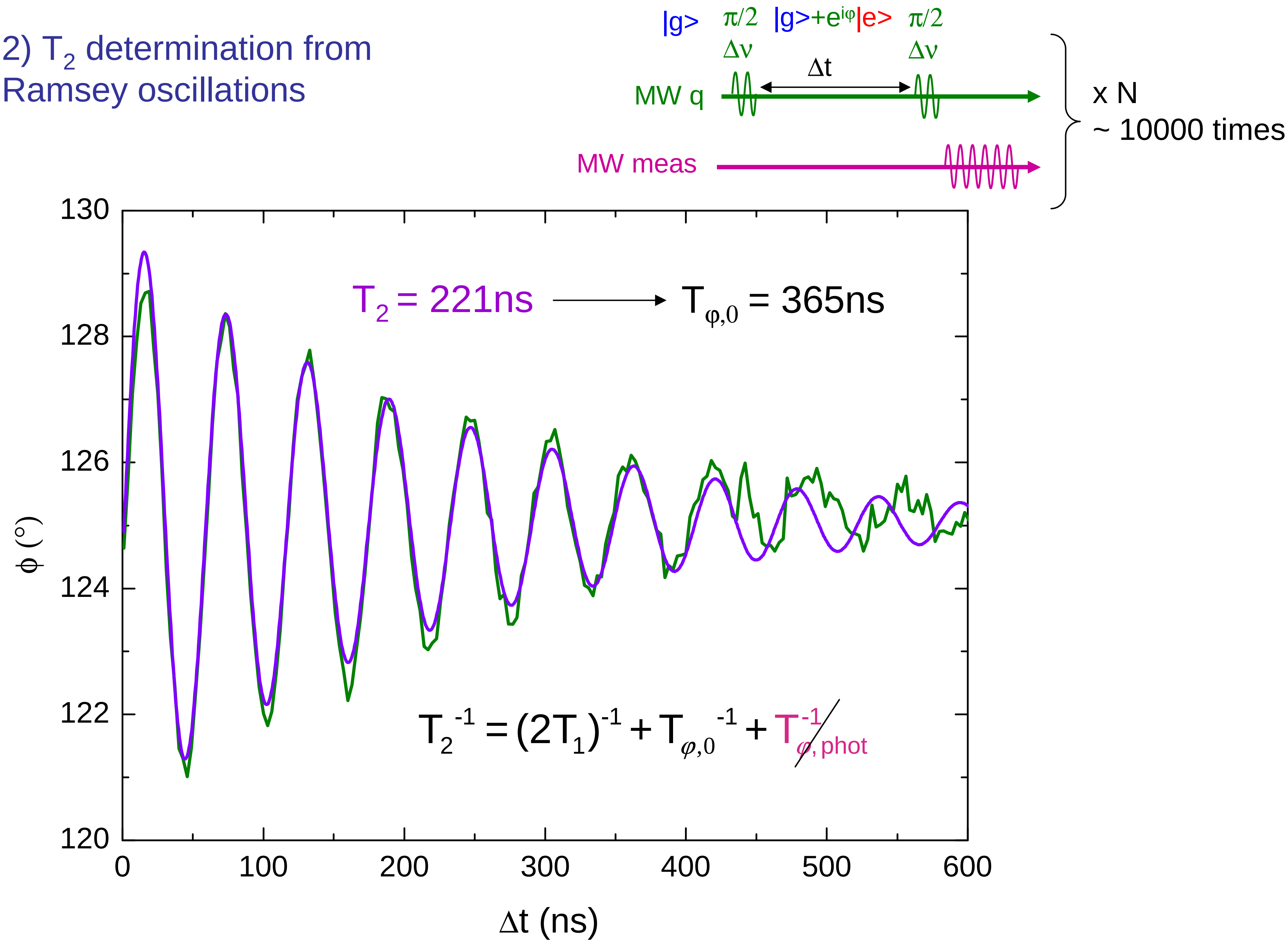
$$I_k, Q_k = \langle I(t), Q(t) \rangle_{t_m} \longrightarrow \bar{I}, \bar{Q} = \sum_k \frac{I_k, Q_k}{N}$$

$$\phi = \mathrm{arcTan}(\bar{Q} / \bar{I})$$

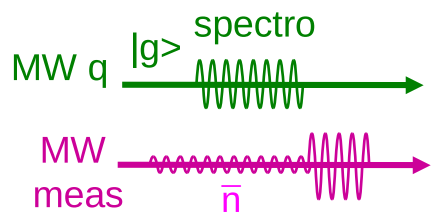


Coherence times at working point

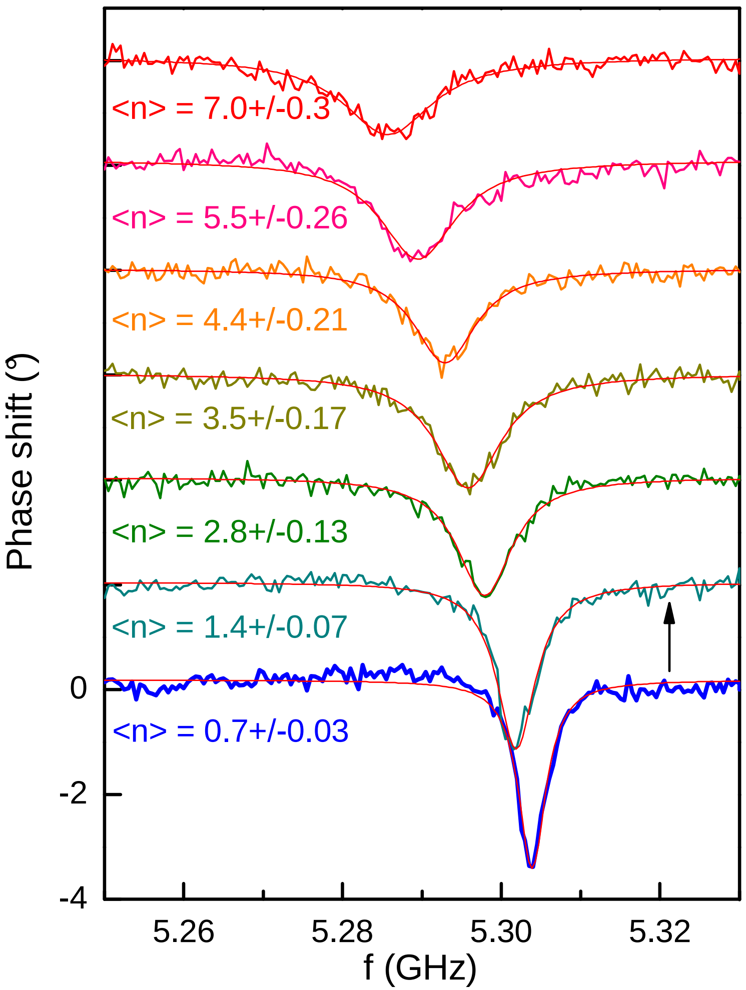
2) T_2 determination from Ramsey oscillations



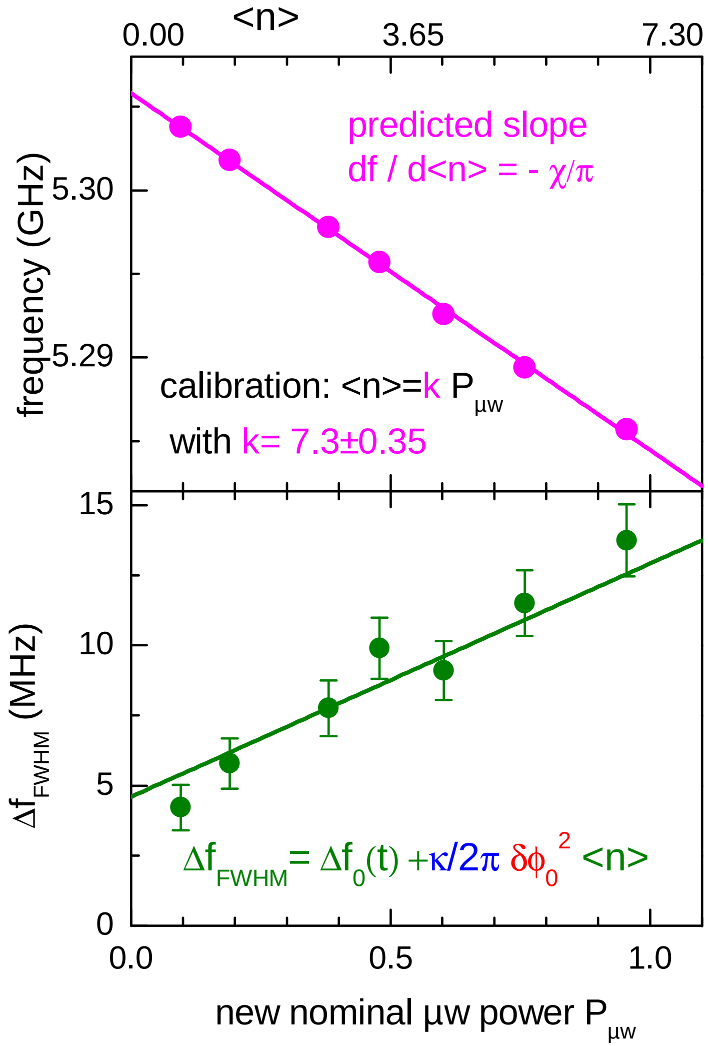
Calibration of photon number from ac-Stark shift



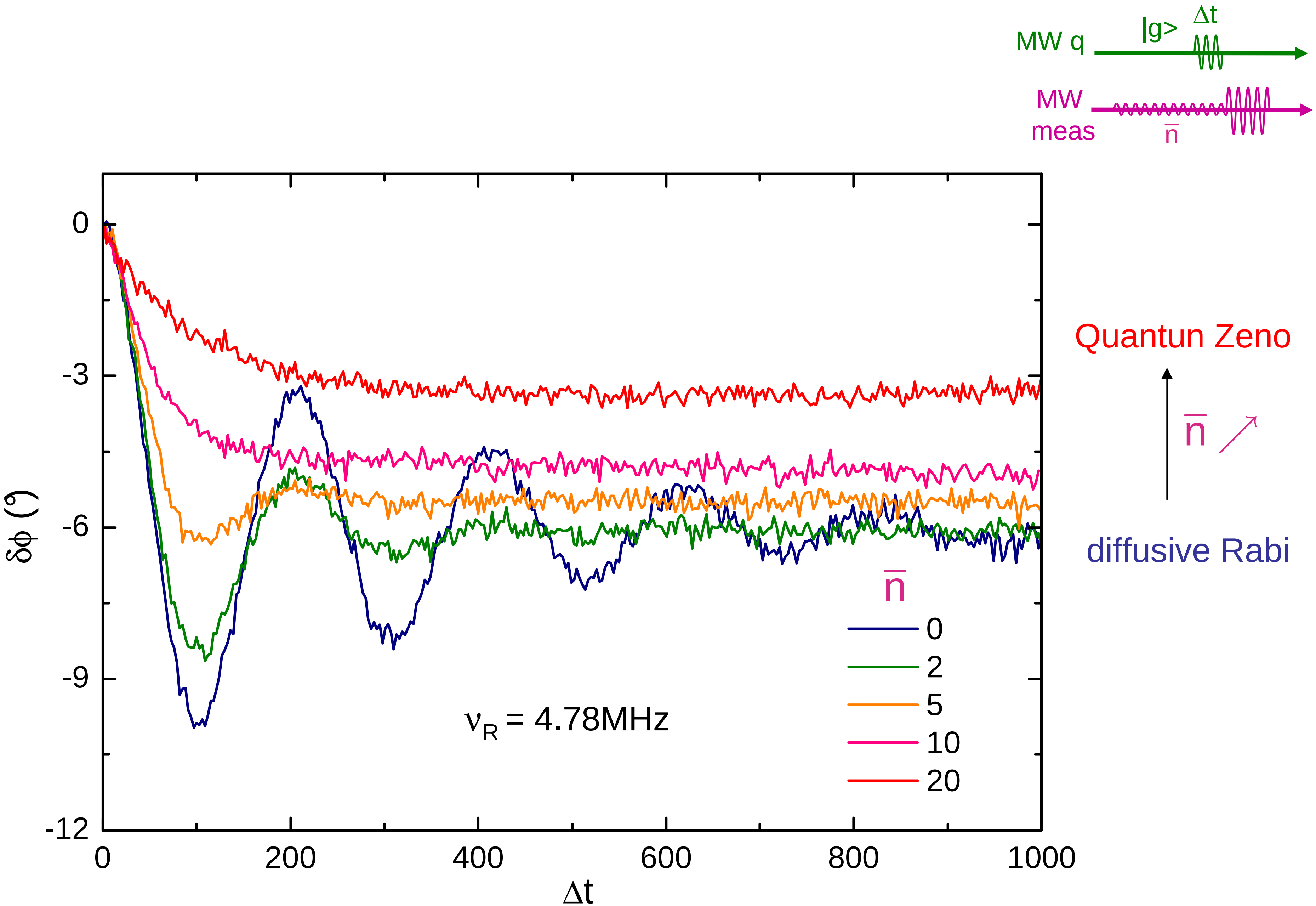
AC stark shift of qubit frequency



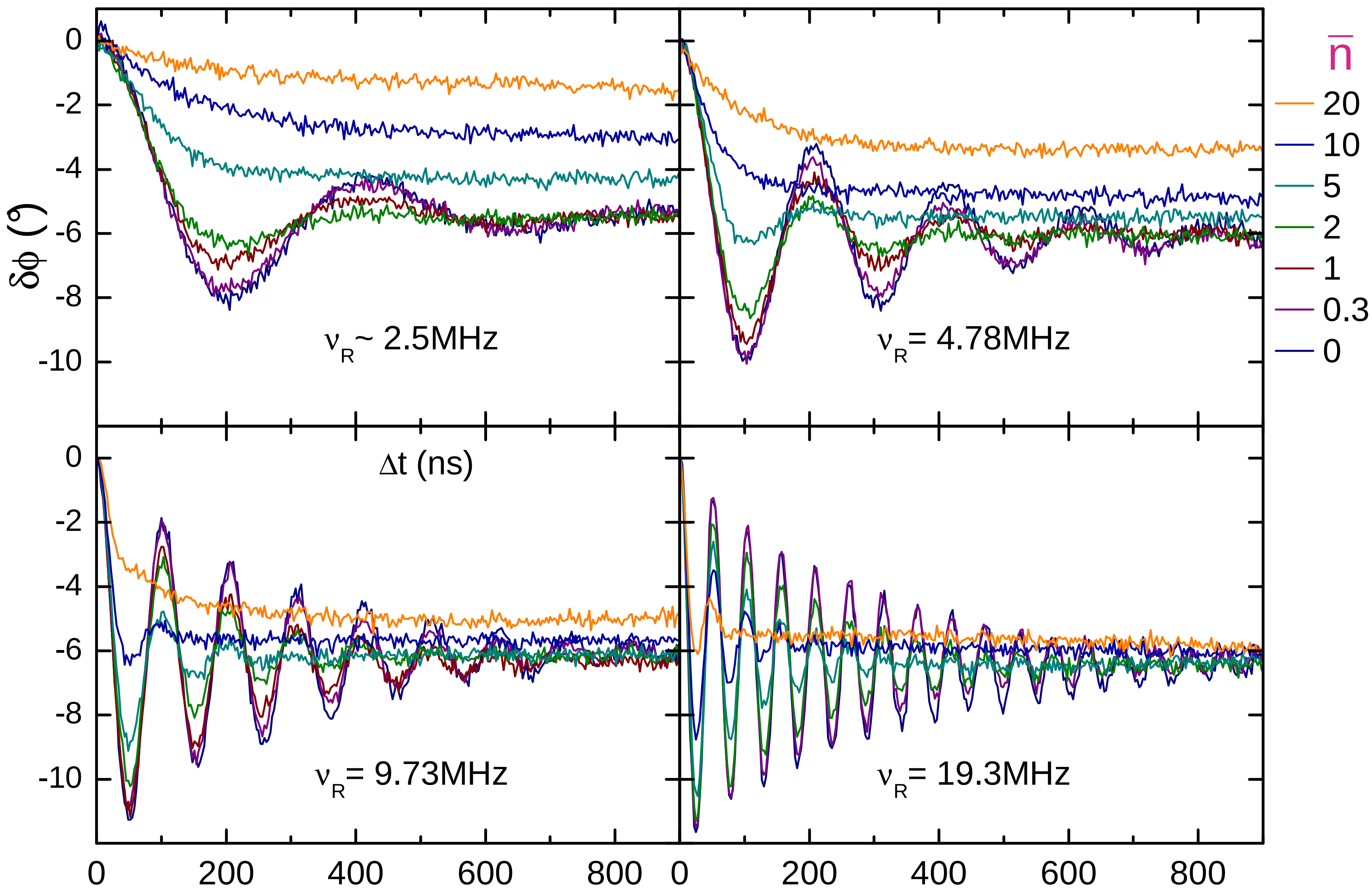
AC stark shift and broadening



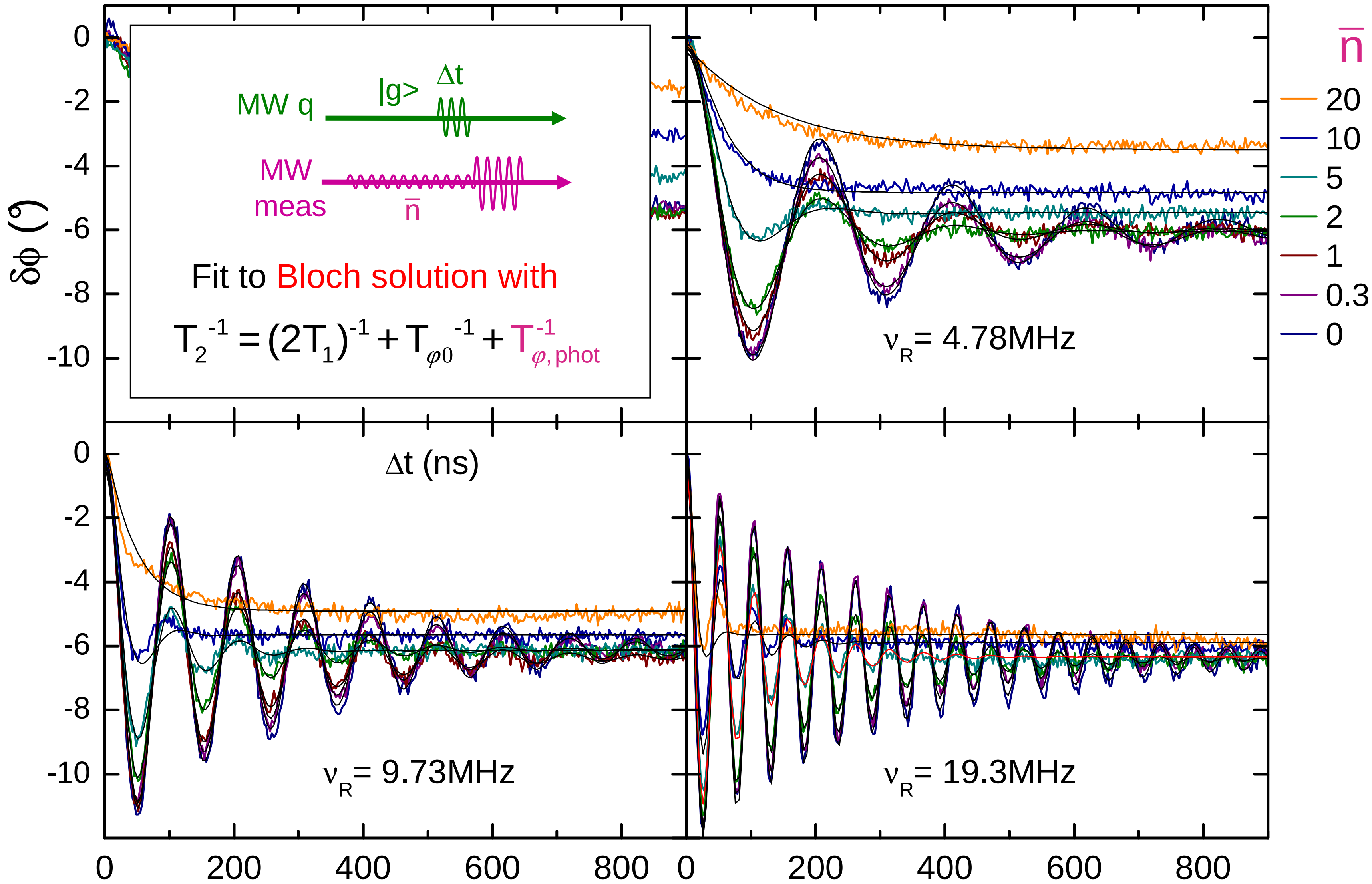
Time domain Rabi oscillations perturbed by resonator field



Time domain Rabi oscillations perturbed by resonator field

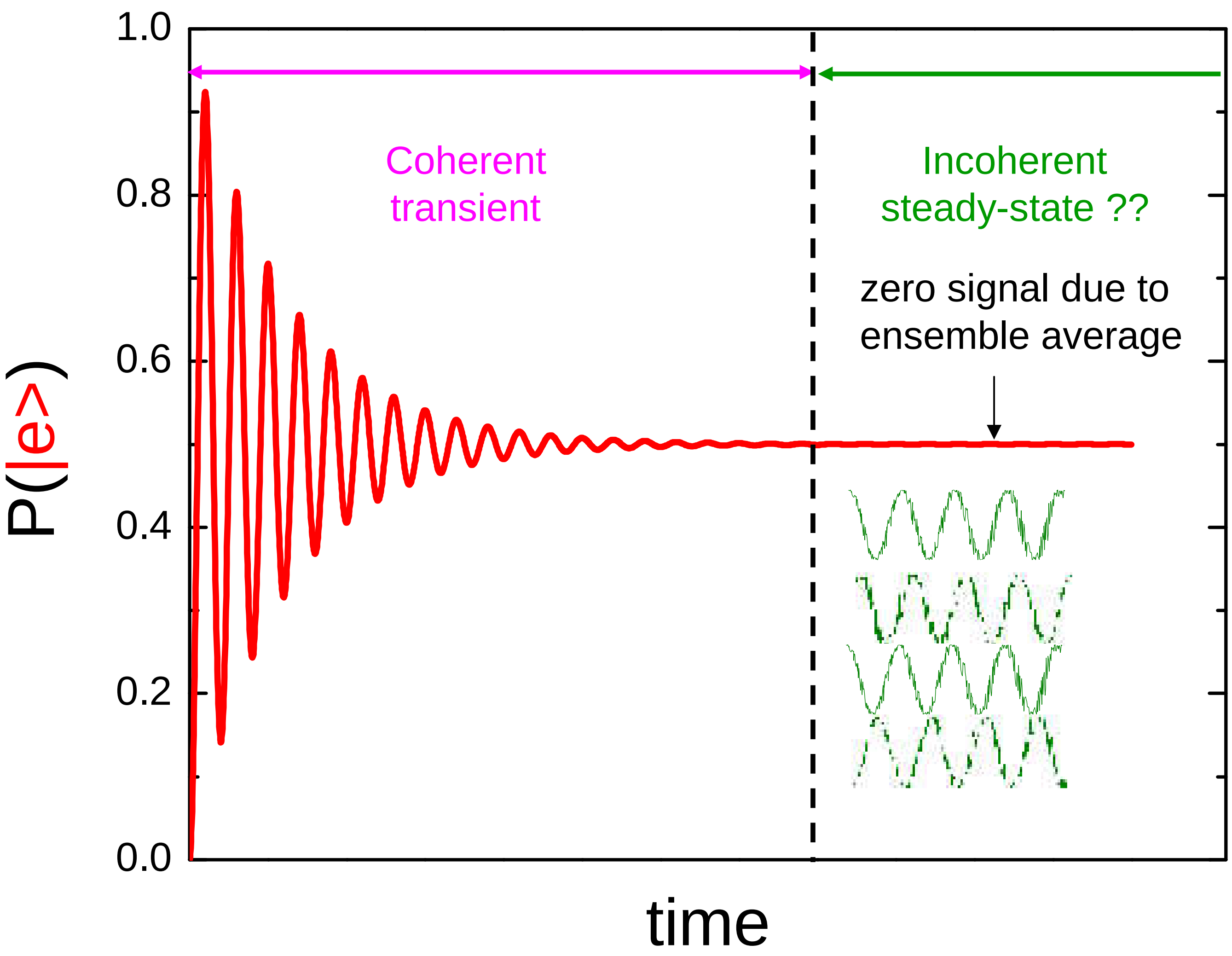


Time domain Rabi oscillations perturbed by resonator field



Ensemble averaging behaviour of Rabi in presence of measuring field well understood

Rabi oscillations in the noise spectrum ?



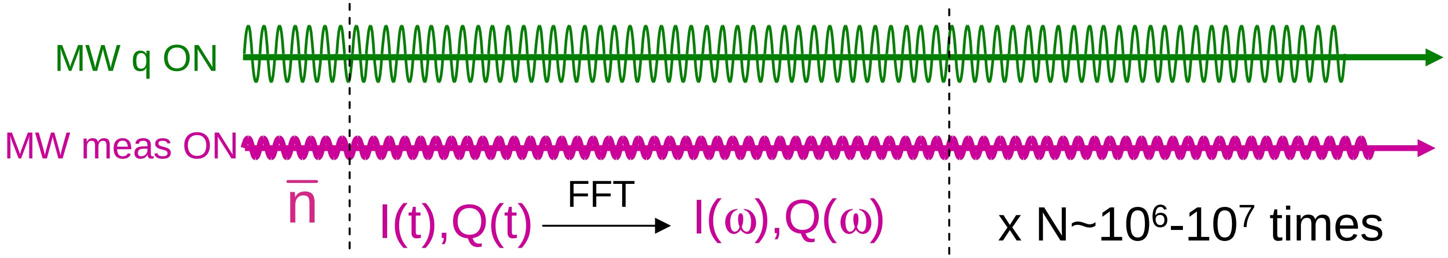
Do not
ensemble average !

Fourier transform first,
and average spectra

Signature of
quantum oscillations
In the noise spectrum

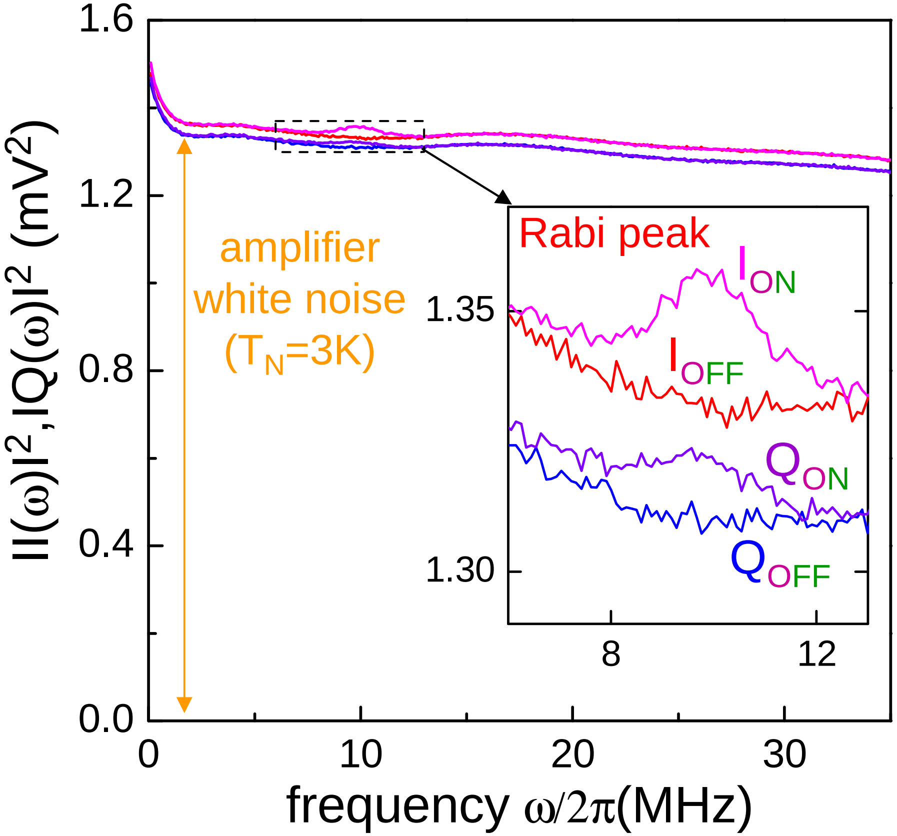
A. Korotkov and D. Averin,
PRB **64** (2001)

Rabi oscillations in the noise spectrum

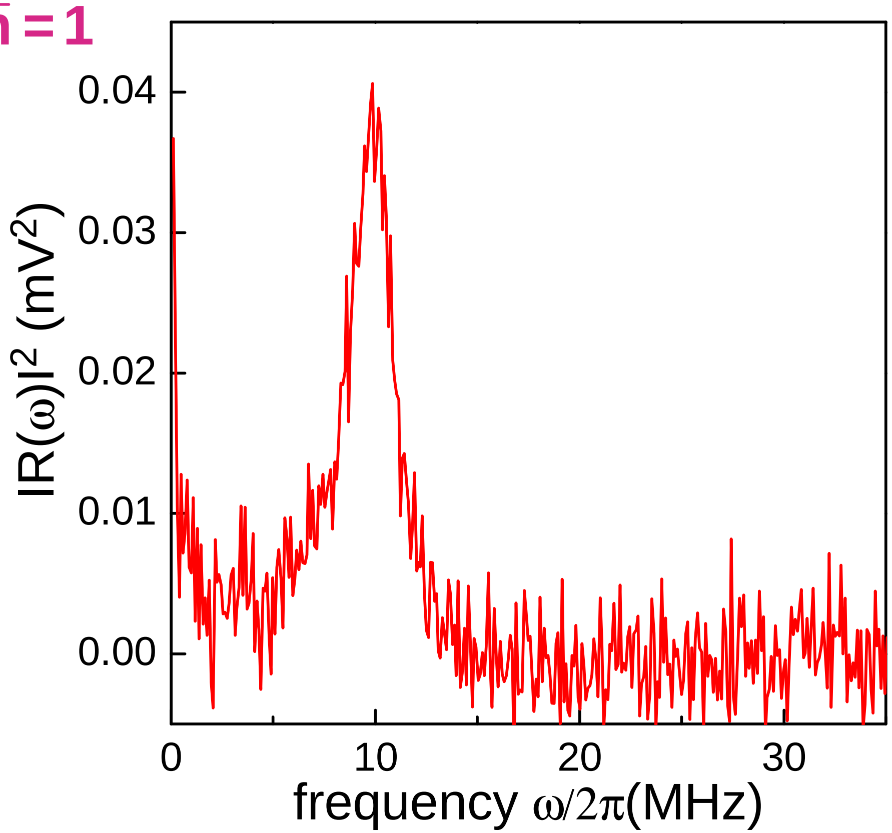


$$\overline{|\mathbf{I}(\omega)|^2}, \overline{|\mathbf{Q}(\omega)|^2}$$

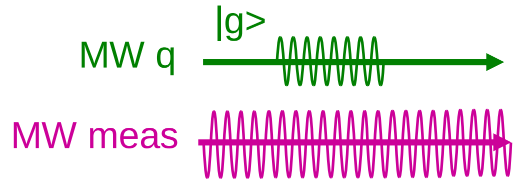
$$|R(\omega)|^2 = |I_{ON}(\omega)|^2 + |Q_{ON}(\omega)|^2 - |I_{OFF}(\omega)|^2 - |Q_{OFF}(\omega)|^2$$

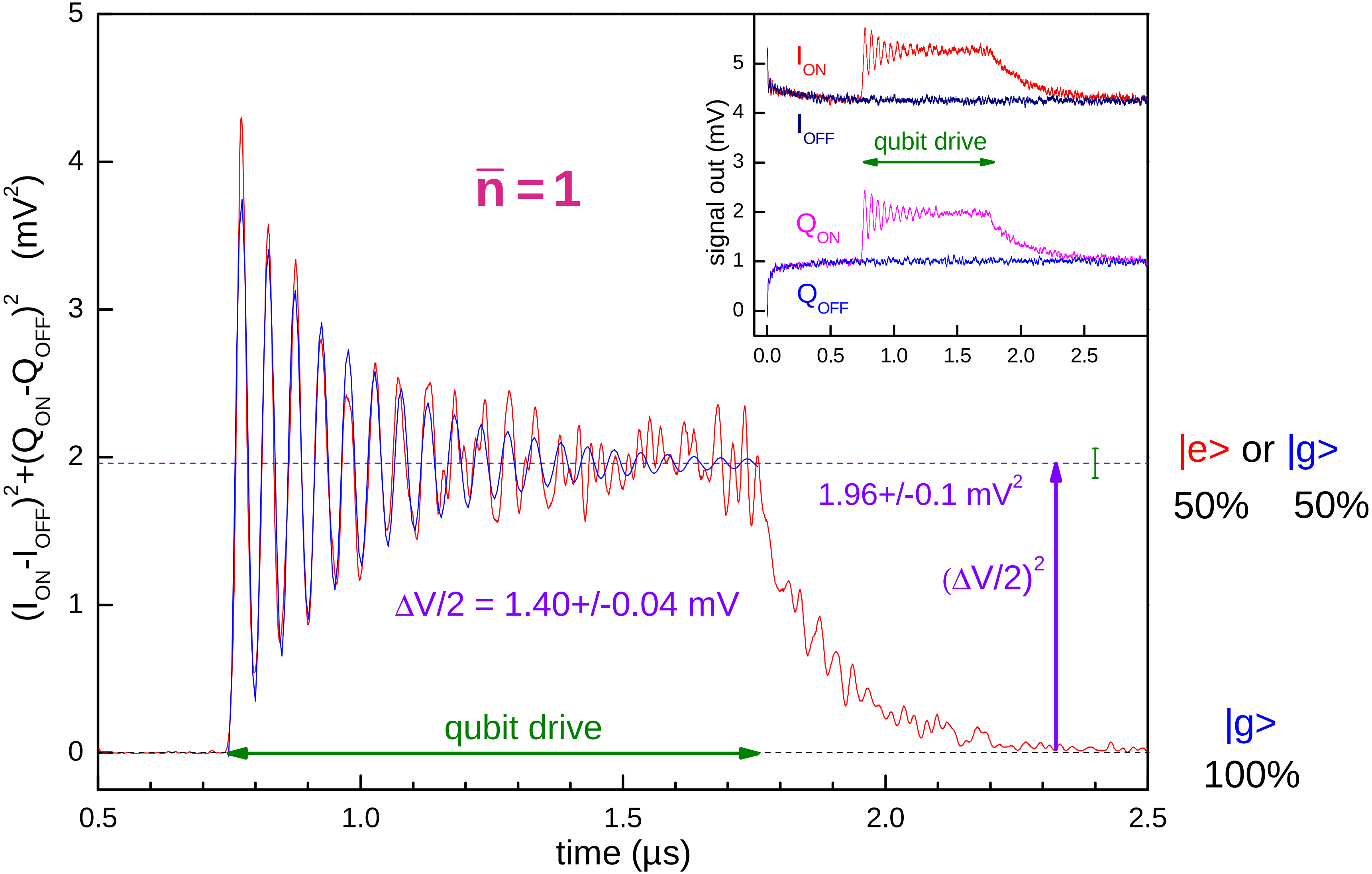


$\bar{n} = 1$

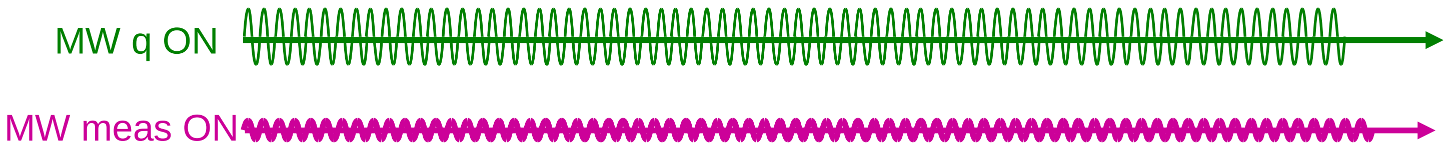


Time domain Rabi oscillations: « real-time » mode and signal calibration

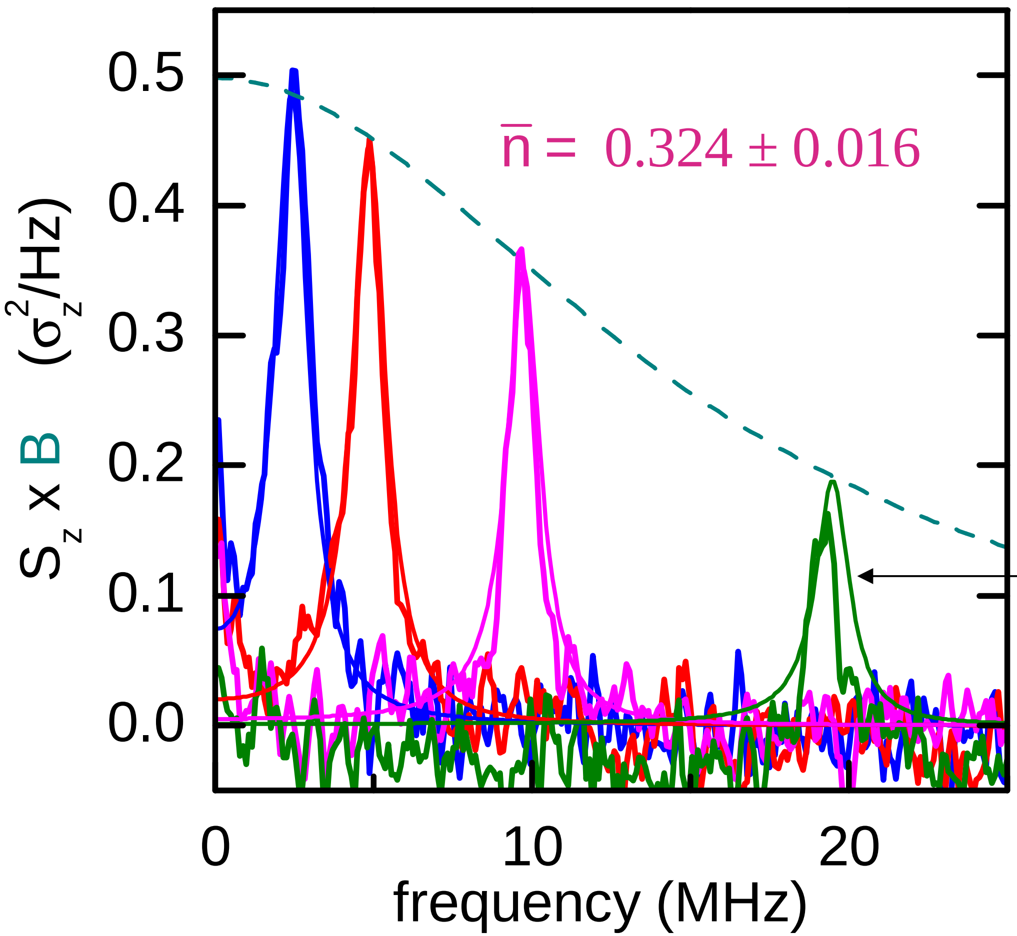




Rabi oscillations in the noise spectrum



Spectral density of noise in units of σ_z



$$S_{\sigma_z}(\omega) = \frac{|R(\omega)|^2}{B(\omega)\bar{n}\left(\frac{\Delta V}{2}\right)^2_{1photon}}$$

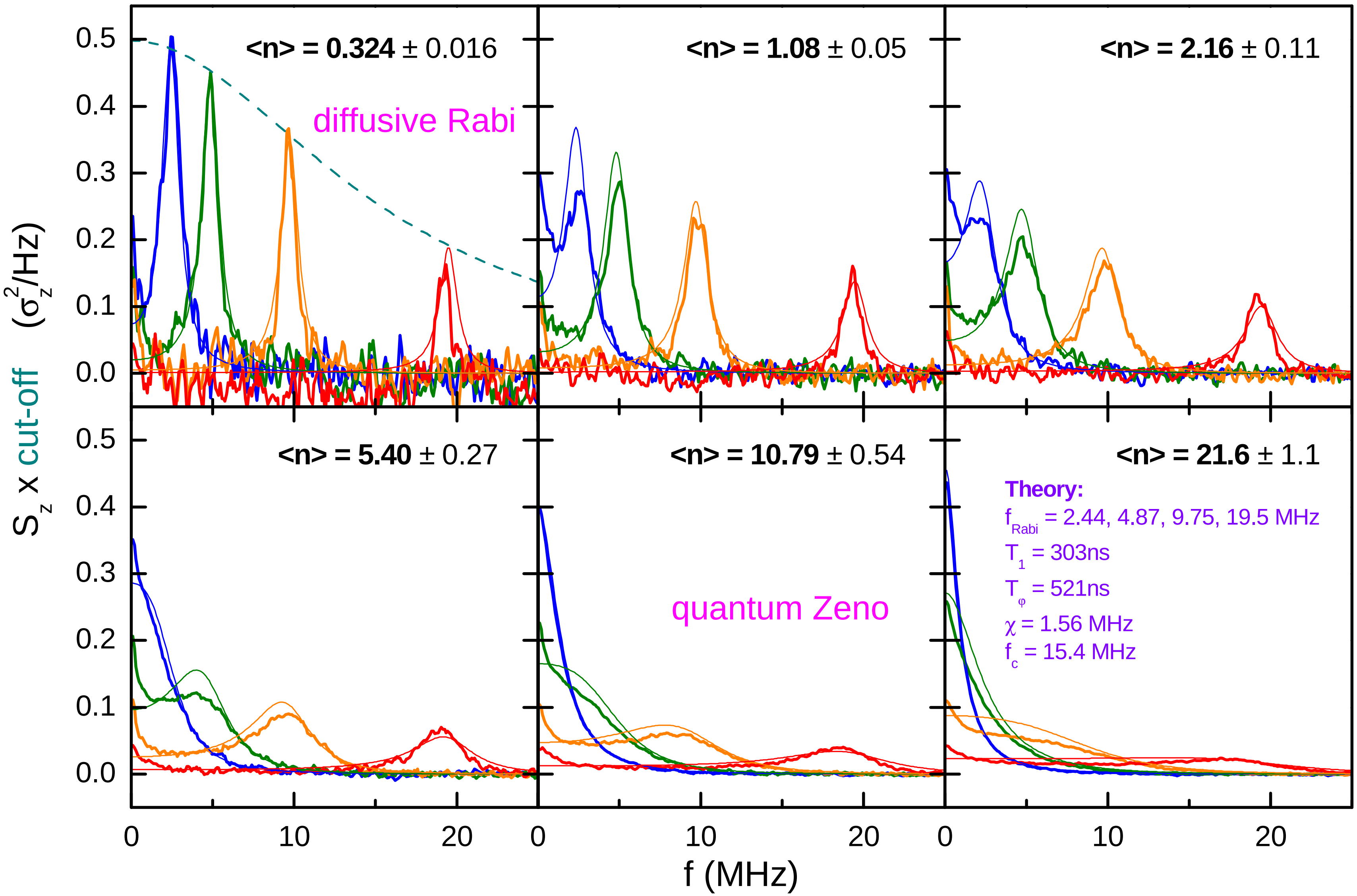
B : cavity cutoff

Theory (A. Korotkov) :

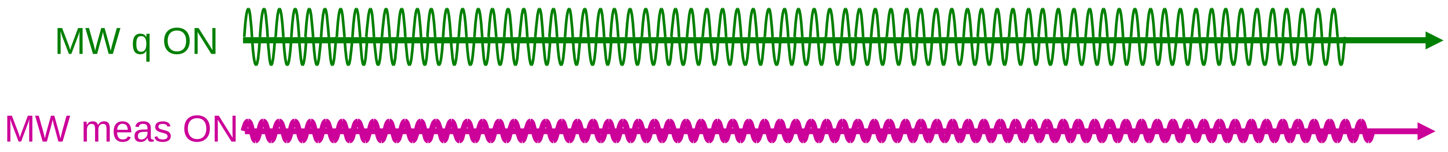
Fourier transform of time-domain Rabi oscillations

Excellent agreement **without adjustable parameter**

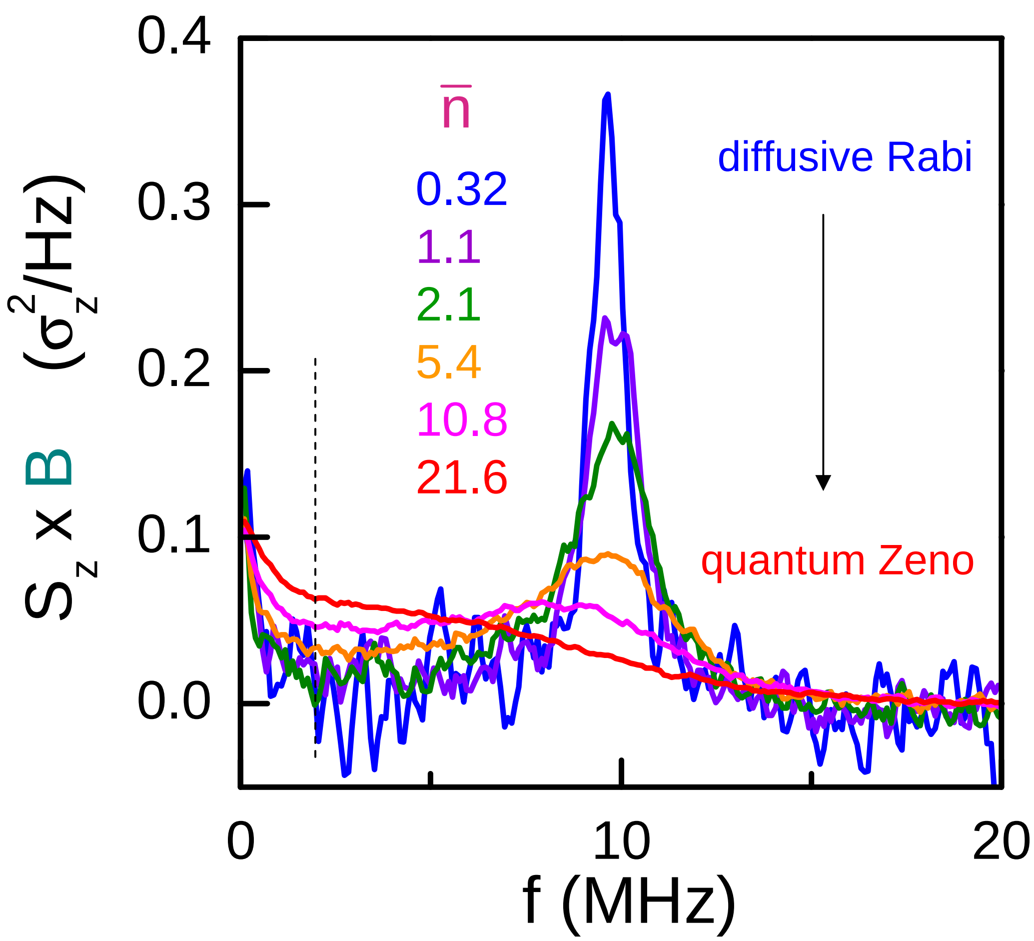
Rabi oscillations in the noise spectrum



Rabi oscillations in the noise spectrum



Spectral density of noise in units of σ_z



$$S_{\sigma_z}(\omega) = \frac{\left|R(\omega)\right|^2}{B(\omega)\bar{n}\left(\frac{\Delta V}{2}\right)^2_{1photon}}$$

B : cavity cutoff

Do we learn something
new compared to time domain?

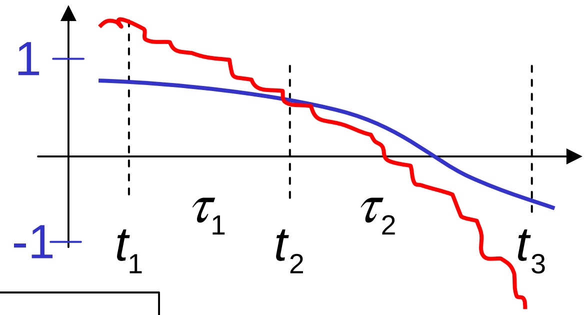
YES! information about quantum
back-action during measurement

Bell inequalities in time of a single degree of freedom

Projective measurement on a statistical ensemble: Garg-Legett (1985)

$$q(t) \overset{\chi}{\Leftrightarrow} V(t) = \frac{\Delta V}{2} q(t) + \eta(t) \qquad S_\eta(\omega) = S_0$$

$\in [-1,1]$



Macrorealism:

1) q(t) is defined at all time $\qquad q(t_1)q(t_2) + q(t_2)q(t_3) - q(t_1)q(t_3) \leq 1$

2) q(t) can be measured in a non-invasive way $\qquad <\eta(t)q(t+\tau)> \xrightarrow{\chi \to 0} 0$

$$K(\tau) = <V(t)V(t+\tau)> = \left(\frac{\Delta V}{2}\right)^2 <q(t)q(t+\tau)> + <\eta(t)q(t+\tau)>$$

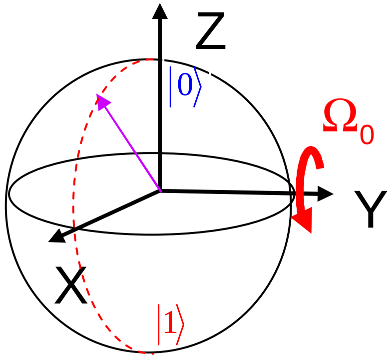
$$\Rightarrow \quad K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \leq \left(\frac{\Delta V}{2}\right)^2$$

Not true for a quantum system: large

Violation of Bell inequalities in time with a single spin

Weak continuous measurement during continuous Rabi oscillation
Korotkov (2000)

$$\sigma_z(t) \overset{\chi}{\Leftrightarrow} V(t) = \frac{\Delta V}{2}\sigma_z(t) + \eta(t) \qquad S_\eta(\omega) = S_0 \qquad \text{Back-action dephasing: } \Gamma = \frac{\Delta V^2}{4S_0}$$

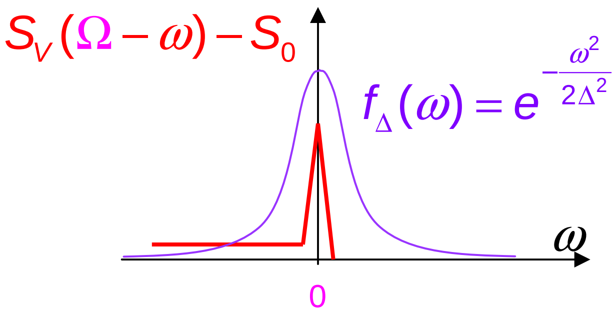


$$K(\tau) = \left(\frac{\Delta V}{2}\right)^2 e^{-\Gamma\tau/2}\left(\cos(\Omega\tau) + \frac{\Gamma}{2\Omega}\sin(\Omega\tau)\right) \qquad \Omega = \sqrt{\Omega_0^2 - \Gamma^2/4}$$

$$K(\tau) + K(\tau) - K(2\tau) = \left(\frac{\Delta V}{2}\right)^2 \left[1 + 2\cos(\Omega\tau)(1 - \cos(\Omega\tau))\right]$$

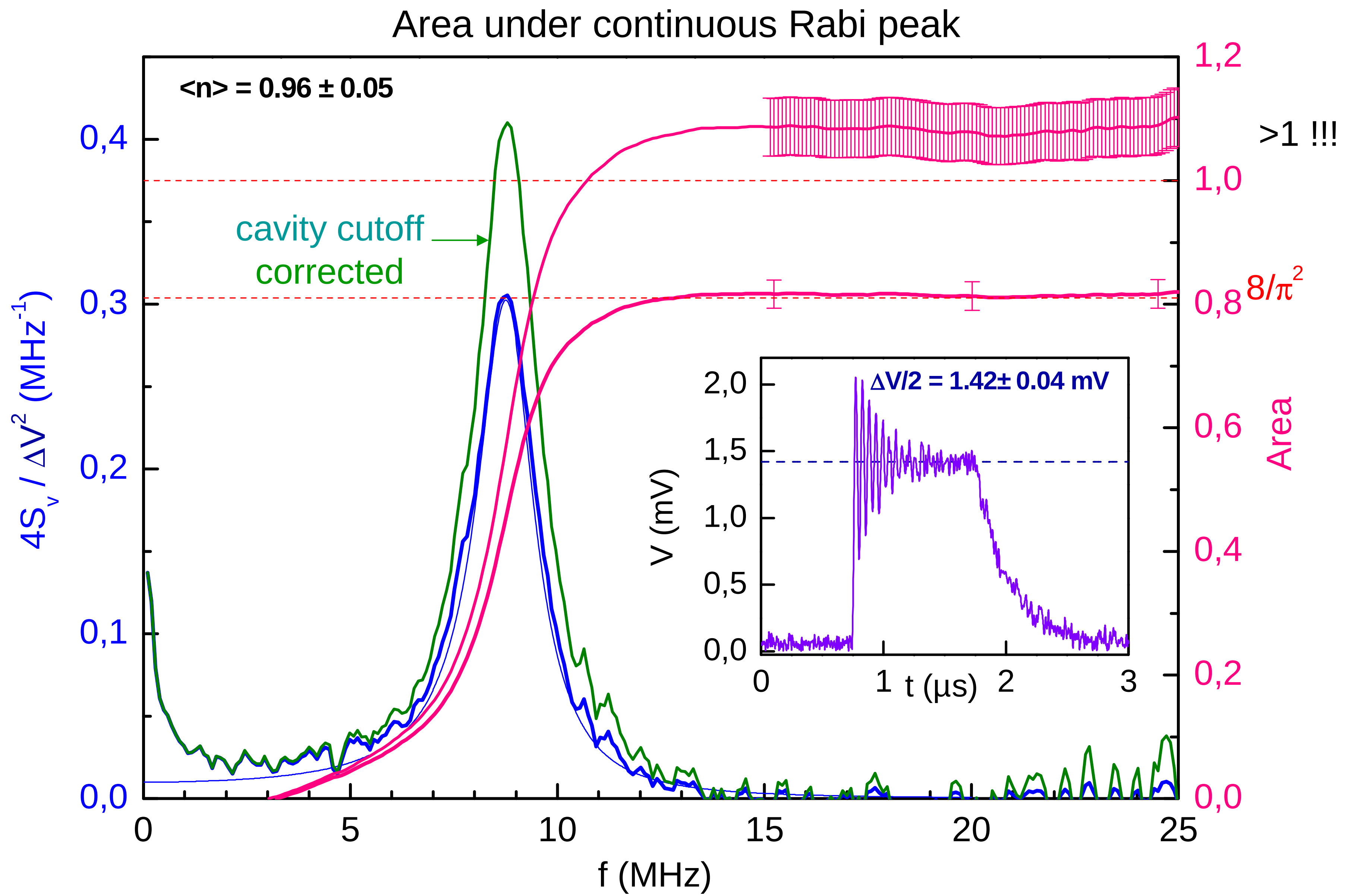
possibly > 1

In the frequency domain: $S_V(\omega) - S_0 = \frac{\Delta V^2}{4} S_z(\omega) = 2\int_{-\infty}^{\infty} K(\tau) e^{i\omega\tau}\, d\tau$



$$\frac{\int_{-\infty}^{\infty} [S_V(\Omega - \omega) - S_0] f_\Delta(\omega) d\tau}{(\Delta V/2)^2} < \frac{8}{\pi^2} + o\left(\frac{\Delta}{\Omega}\right)$$

Experimental violation of Bell inequalities ?

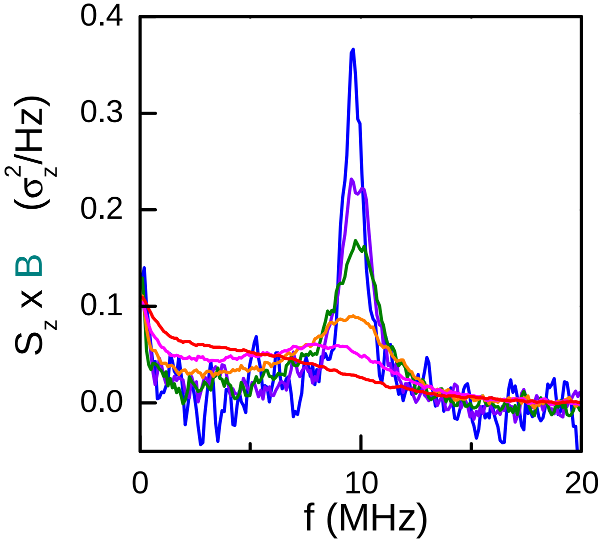


more work needed…

Conclusion and perspectives

Quantum dynamics/measurement in a circuit QED setup

- Quantitative understanding of dephasing of Rabi oscillations during measurement



- Rabi peak in the noise spectrum : coherent dynamics after steady-state reached

- Rabi spectra : from diffusive Rabi oscillations to quantum jumps. Quantitative understanding

-Towards a violation of a weak measurement version of Leggett-Garg inequality?

- Towards quantum feedback experiments with single quantum system ? Need quantum-limited amplifier

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Superconducting circuits based on Josephson junctions can really behave quantum mechnically: a new test bench for Quantum physics?